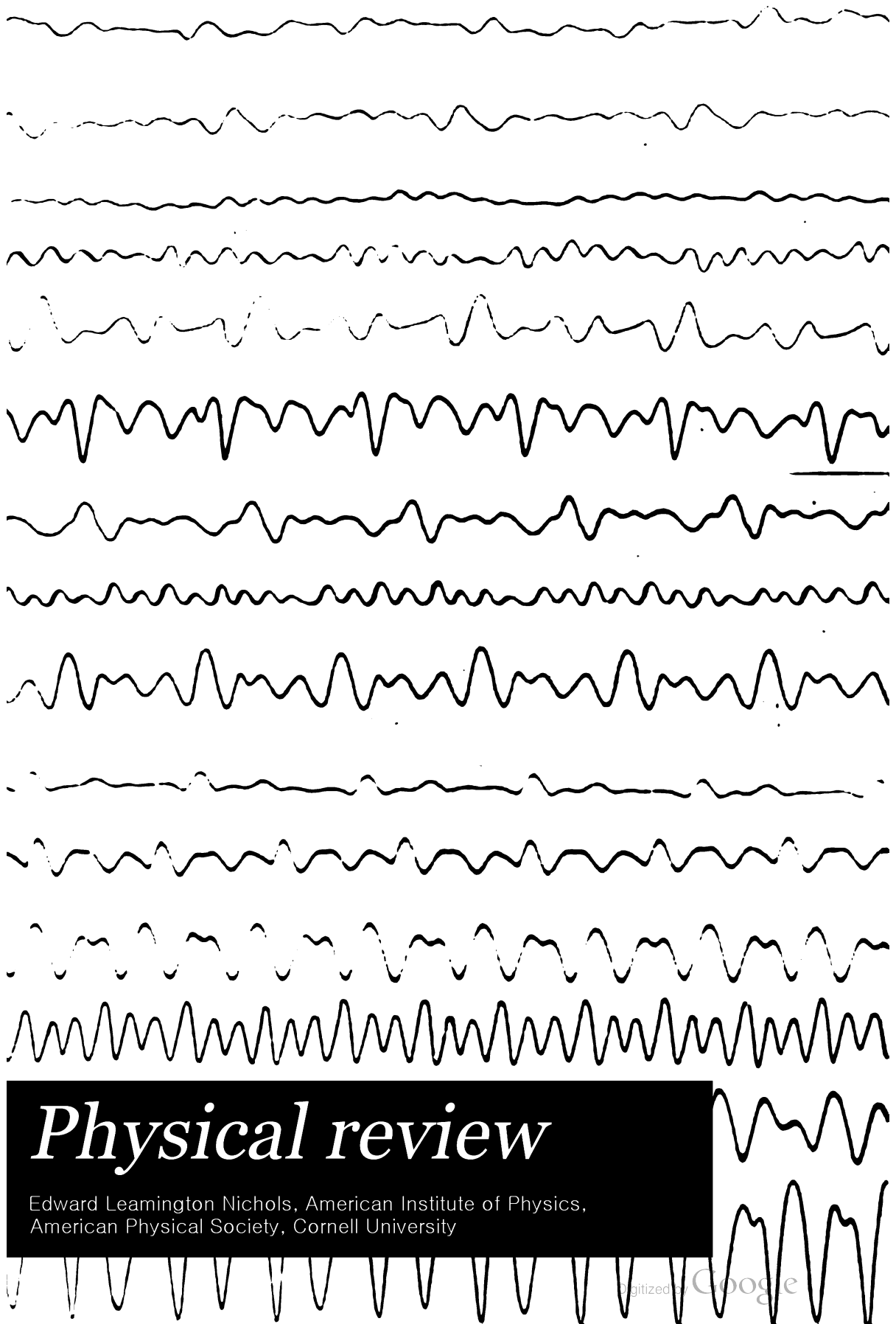

Physical review

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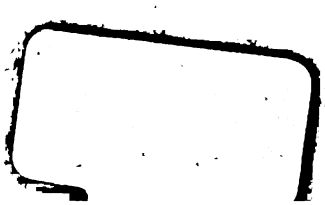

THE PHYSICAL REVIEW

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THE

PHYSICAL REVIEW.

ON THE RELATION BETWEEN DENSITY AND INDEX OF REFRACTION OF AIR.

By Henry G. Gale.

IF a body is compressed or its temperature varied, its density, d, will change and so also will its index of refraction, n. The problem of finding the relation between the quantities d and n is an old one and one attended by some difficulties. In the case of solids and liquids the density can be made to vary only slightly and experiments cannot be conducted over wide limits. In the case of gases the density can be given a wide range, but the index of refraction differs but little from unity, and its variation with changes of density is small.

Gladstone and Dale[1] first proposed the very simple relation,

$$\frac{n-1}{d} = \text{const.} \tag{1}$$

This is the relation which would be expected if transparent substances owe their refractive power to their molecules alone, and if the molecular index of refraction does not change with the density.

Jamin[2] proposed the formula,

$$\frac{n^2-1}{d} = \text{const.} \tag{2}$$

a relation which follows necessarily on the basis of the emission

[1] Gladstone and Dale, Phil. Trans., p. 887, 1858; Phil. Trans., p. 317, 1863.

[2] Jamin, Ann. de Chim. et de Phys., 3e série, T. LII., p. 163, 1858; T. LXI., p. 358, 1861.

theory. For gases this relation is almost the same as that of Gladstone and Dale, since

$$\frac{n^2 - 1}{d} = \frac{(n-1)(n+1)}{d} = \frac{2(n-1)}{d}$$

$n + 1$ being very little different from 2.

Lorentz,[1] on the basis of the electro-magnetic theory of light, has proposed the following formula,

$$\frac{n^2 - 1}{(n^2 + 2)d} = \text{const.} \tag{3}$$

which comes at once on substituting n for D in Clausius' equation $\frac{D+2}{D-1} d = \text{const.}$, where D is the dielectric constant. It is seen that the relation (3) amounts to the same thing as relation (1) in the case of gases since

$$\frac{n^2 - 1}{(n^2 + 2)d} = \frac{(n-1)(n+1)}{(n^2 + 2)d} = \tfrac{2}{3}\frac{(n-1)}{d}$$

because $n + 1$ is very nearly equal to 2, and n^2 differs but very little from unity.

Jamin[2] was among the first to conduct experiments designed to test these relations. He measured the index of refraction of water at different pressures and found $\frac{n^2 - 1}{d}$ very nearly constant, but Mascart[2] showed that Jamin's results satisfied still better the equation of Gladstone and Dale.

Quincke[3] verified the results of Mascart for water and found that for glycerine, Rüböl, carbon disulphide, alcohol and ether, equation (2) always gave too large computed values and equation (3) too small values, while the values computed from equation (1) were nearly correct, being sometines a little too large and sometimes too small.

[1] Lorentz. Wied. Ann., Bd. IX., S. 641.

[2] Mascart, Comptes Rendus, LXXVIII., p. 801; Pogg. Ann., Bd. CLIII., S. 154.

[3] Quincke, Wied. Ann., Bd. XIX., S. 401; Bd. XLIV., S. 774.

Röntgen and Zehnder[1] obtained results in agreement with Quincke's for water, but they found that for the other substances equation (1) gave results consistently a little too large, but nearer than equations (2) and (3).

Quincke attempted to measure the change in the index of refraction of solids with a change of pressure, but the variations he was able to produce were too small to be measured accurately.

Rudberg,[2] Fizeau,[3] Stefan,[4] F. Vogel,[5] Pulfrich,[6] Offret[7] and others have investigated the change in the index of refraction of many crystals and transparent bodies as the temperature was varied over wide limits.

Jamin,[8] Damien,[9] Pulfrich,[10] Rühlmann,[11] Wüllner,[12] B. Walter,[13] Ketteler[14] and others have found that the maximum index of refraction of water occurs at a temperature somewhat below that of maximum density, and have investigated the relation of n and d for water from temperatures below 0° C. to 100° C. They found that the quotient $\frac{n-1}{d}$ diminished slightly as the temperature was increased. The observations of Ketteler and Pulfrich for water give, for sodium light, the following values:

	$\frac{n-1}{d}$	$\frac{n^2-1}{d}$	$\frac{n^2-1}{(n^2+2)}\cdot\frac{1}{d}$
At −10°	.33447	.78061	.20656
At 10°	.33363	.77922	.20618
At 56.8°	.33274	.77464	.20585
At 95.2°	.33189	.76998	.20583

[1] Zehnder, Wied. Ann., Bd. XXXIV., S. 91; Röntgen und Zehnder, Wied. Ann., Bd. XLIV., S. 1 und S. 24.

[2] Rudberg, Pogg. Ann., Bd., XXVI., S. 291.

[3] Fizeau, Ann. de Chim. et de Phys., 3e série, t. LXVI., p. 429; 4e série, t. II., p. 143; Pogg. Ann., Bd. CXIX., S. 87 u. 297.

[4] Stefan, Berichter der Wiener Akademie (2), LXIII., S. 223.

[5] F. Vogel, Wied. Ann., Bd. XXV., S. 87.

[6] Pulfrich, Wied. Ann., Bd. XLV., S. 609.

[7] Offret, Bulletin de la soc. franç. de min., t. XIII., p. 405, 1890.

[8] Jamin, Comptes Rendus, t. XLIII., p. 1191; Pogg. Ann., Bd. C., S. 478, 1857.

[9] Damien, Ann. de l'école norm., sup., 2e série, t. X., p. 257, 1881.

[10] Pulfrich, Wied. Ann., Bd. XXXIV., S. 326, 1888.

[11] Rühlmann, Pogg. Ann., Bd. CXXXII., S. 1 u. 177.

[12] Wüllner, Pogg. Ann., Bd. CXXXIII., S. 1.

[13] B. Walter, Wied. Ann., Bd. XLVI., S. 423.

[14] Ketteler, Wied. Ann., Bd. XXXIII., S. 353 u. 506.

When instead of n the constant A of Cauchy's formula, $n = A + \frac{B}{\lambda^2}$ was used, the following relations were obtained:

	$\frac{A-1}{d}$	$\frac{A^2-1}{d}$	$\frac{A^2-1}{A^2+2}\cdot\frac{1}{d}$
At 10°	.32516	.75867	.20185
At 20°	.32463	.75717	.20159
At 30°	.32445	.75644	.20154

Wüllner found that the best agreement was given by the formula $\frac{A-1}{d} = a + bt$, the constant b being negative for glycerine, alcohol and carbon disulphide, but positive for a saturated solution of zinc chloride in water. Knops,[1] Weegmann[2] and Dufet[3] obtained similar results.

Ketteler[4] proposed the formula $\frac{n^2-1}{n^2+x}\cdot\frac{1}{d} = \text{const.}$, and computed x from the observations of Knops and Weegmann who had extended their investigations to a great number of organic substances. Ketteler found values of x ranging from 2 to 8.4.

On varying the temperature of air, Mascart[5] found that the index of refraction decreased more rapidly than the density with increasing temperature, but von Lang[6] obtained an opposite result by measuring the index of refraction directly from air at a high temperature to air at ordinary temperatures.

Benoit[7] found that for air the diminution of the refraction was exactly proportional to the diminution of density when the temperature was raised, and Chappuis and Rivière[8] obtained the same result for cyanogen.

[1] Knops, Liebigs Ann., CCXLVIII., S. 175.

[2] Weegmann, Zeitschr. für physikal. Chemie, Bd. II., S. 218.

[3] Duet, Journal de Phys., 2e série, t. IV., pp. 389 et 477.

[4] Ketteler, Wied. Ann., Bd. XXX., S. 285; Bd. XXXIII., S. 353 u. 506; Bd. XXXV., S. 662.

[5] Mascart, Ann. Sc. de l'Ecole Norm. sup. (2), t. VI., p. 9, 1877; Comptes Rendus, t. LXXVIII., pp. 617, 679, 801, 1874; t. LXXXVI., pp. 321, 1182, 1878.

[6] v. Lang, Pogg. Ann., Bd. CLIII., S. 448, 1874.

[7] Benoit, Jour. de Phys. (2), t. VIII., p. 451, 1889.

[8] Chappuis et Rivière, Ann. de Chim. et de Phys. (6), t. XIV., p. 5, 1888.

Biot and Arago,[1] working between — 1.5° C. and 25° C., found that the "refractive power" $n^2 - 1$ was proportional to the density when d was computed by the formula, $d = \frac{d_0}{1 + \alpha t}$, α being the temperature coefficient of expansion. Their results, however, satisfy equally well the relation $\frac{n - 1}{d} = \text{const.}$

Lorenz[2] and Prytz[3] found that the relation $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{1}{d}$ held fairly well for many substances though the liquid into the gaseous state —markedly better, in fact, than the relation $\frac{n - 1}{d}$ const. Some of their results are shown in the following tables:

	Liquid.		Gas.
	10°	20°	100°
Ether.	0.30264	0.30287	0.3068
Alcohol.	0.28042	0.28066	0.2825
Water.	0.20615	0.20608	0.2068
Chloroform.	0.17902	0.17909	0.1796

Mascart[4] conducted an extensive research on the effect of pressure on the index of refraction of air, using interference methods. Two tubes were filled with air under pressure, the one always a fixed amount higher than the other, and the relation between density and index of refraction was deduced from the number of fringes which passed as the air in the two tubes was brought to the same pressure. He found that the refraction $n - 1$ increased more rapidly than the pressure; the ratio increased about 1 per cent. when the pressure was increased to about 8 atmospheres. On taking into account variations of density in accordance with Regnault's formula, $d = AH(1 + BH)$, Mascart found that there were still discrepancies.

[1] Biot et Arago, Mem. de la prem. classe de l' Institut, t. VIII., p. 301, 1806; Mem. de l'Acad. des. Sciences, t. VII., p. 301, 1806; Gilberts Ann., Bd. XXV., S. 345, 1807; B. XXVI., S. 79, 1807.

[2] Lorenz, Wied. Ann., Bd. XI., S. 70, 1880.

[3] Prytz, Wied. Ann., Bd. XI., S. 104, 1880.

[4] Mascart, Ann. Sci. de l'Ecole Norm. sup. (2), t. VI., p. 9, 1877; Comptes Rendus, t. LXXVIII., pp. 617, 679, 801, 1874; t. LXXXVI., pp. 321, 1182, 1878.

which, however, he considered were within the limits of observational error.

Chappuis and Rivière[1] kept the pressure in one tube constan and measured the change dn in the index of refraction of the air in the other tube as its pressure was changed by an amount dH, Working with air and carbon dioxide up to twenty atmospheres they found that in every case the refractive power, $n - 1$, was proportional to the density.

Carnazzi,[2] however, working with a prism and a long beam of light, found that the ratio $\frac{n-1}{d}$ showed a decided increase in the case of air and hydrogen as the pressure was increased, but that the ratio decreased with increasing pressures in the case of carbon dioxide.

Since the work which had been done on the relation between density and index of refraction of air with a changing pressure did not appear to be entirely conclusive, an attempt has been made in the work described in this article to get some data on the subject. Interference methods naturally suggested themselves as the best for attacking the problem, but it was found advisable to modify the form of interferometer in common use. The pressure was measured in a simple way by applying the inverse principle of the McLeod gauge. The consideration of the investigation divides itself naturally into four parts: I. The pressure gauge, II. The optical arrangement, III. Observations, IV. Conclusions.

I. THE PRESSURE GAUGE.

The pressure gauge used is outlined in Fig. 1. AB is a steel rod about an inch in diameter, along the axis of which has been drilled a hole, C, about $\frac{1}{32}$ inch in diameter. D and E are steel stopcocks which were carefully ground with fine carborundum to fit as exactly as possible into tapering holes drilled through the rod AB. A piece of heavy copper pressure tubing of fine bore, screwed into one end of the rod and soldered in place, connected the pressure gauge to a large tank which contained air at the pressure to

[1] Chappuis et Rivière, Ann. de Chim. et de Phys. (6), t. XIV., p. 5, 1888.
[2] Carnazzi, Il Nuovo Cimento, 6, p. 385, 1897.

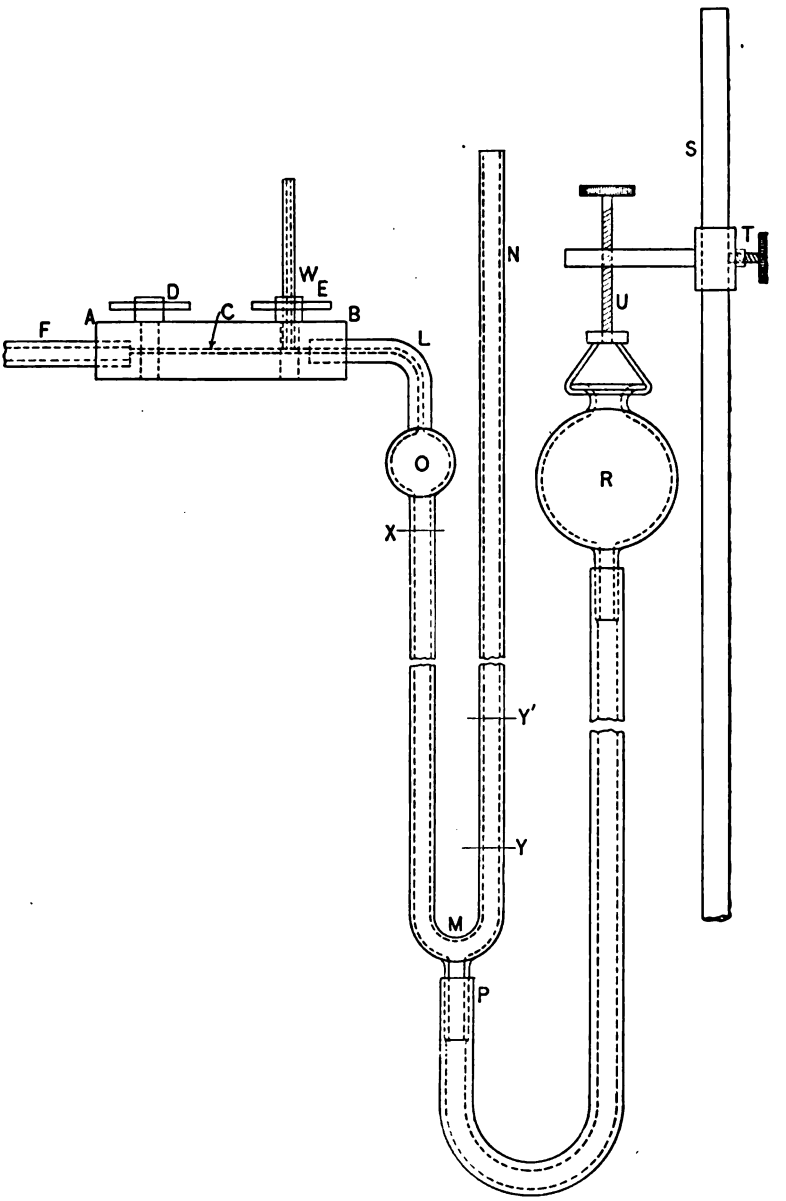


Fig. 1.

be measured. Into the other end of the rod *AB* was fastened one end of a glass tube *LMN* of the form shown in the figure. The bulb *O* had a capacity of about 10 c.c., and the tubes *OM* and *MN* had an internal diameter of about .5 cm. A bulb *R*, of about 500 c.c. capacity, for holding mercury, was connected with *P* by a piece of rubber tubing. The height of the mercury in the tubes *OM* and *MN* was regulated by sliding the clamp *T*, which held the bulb, up and down the rod *S*. A threaded rod *U*, which held the bulb, gave an opportunity for fine adjustment.

The stopcock *D* had a hole through it of small diameter in line with the hole *C*, through *AB*. The stopcock *E* was a three-way cock, having one hole through it in line with *C* and another half way through at right angles to the first. This stopcock was first set across so that the air could pass from *L* to the center of the stopcock and then out through the back of the rod *AB*, through a hole drilled to meet the hole *C*. A glass tube *W* was waxed into this hole to carry off the overflowing mercury.

As the bulb *R* was raised, mercury rose in the tube *NOL*, driving the air before it through the three-way cock *E*, and out into the tube at *W*. When the mercury began to rise in the tube at *W* the stopcock *E* was turned through 45°, thus closing all of its openings, and the bulb *R* was lowered until the mercury stood at some fixed point, as *X*, below the bulb *O*. The mercury then stood in the other arm of the tube at some point *Y*, almost a barometric column below *X*. A very high vacuum could not be expected in *O*, but by measuring with a cathetometer the difference of level between *X* and *Y* and subtracting this difference from the barometric height, the pressure of the gas left in *O* was found. The stopcock *D* was then opened and the small volume *C*, between the stopcocks, was allowed to fill with air at the pressure of the tank. Some little time was allowed for the pressure to adjust itself evenly, the stopcock *D* was then closed, and *E* was turned another 45°. This allowed the small volume of air in *C* at a high pressure to expand into the larger volume *O*, the mercury, of course, being forced down from *X*. The bulb *R* was then raised until the mercury came back exactly to its original position *X*, and a new reading was taken for the difference of level of the mercury in the two arms, the level in the second arm having been raised to some point *Y'*.

From these readings and the barometric height the pressure in the tank may be deduced in the following way: let V be the volume of the bulb and tube down to the point X, and call the pressure of the residual gas P. Let v' represent the volume of the space between the stopcocks, and p' the pressure of the gas in the tank. If the final pressure in the bulb, after the three-way cock has been opened and the mercury brought back to X, is represented by $\bar{P}$, the following equations will hold:

$$VP + v'p' = (V + v')\bar{P},$$

$$p' = \frac{V + v'}{v'}\bar{P} - \frac{V}{v'}P,$$

$$p' = k(\bar{P} - P) + P,$$

when

$$k = \frac{V + v'}{v'};$$

$$\therefore \frac{p'}{k} = \bar{P} - P + \frac{P}{k}.$$

Since $\bar{P} - P$ is equal to $Y' - Y$, this equation shows that the pressure p' in the tank is proportional to the change from Y to Y' if to this change be added the small quantity $\frac{P}{k}$ in which P stands for the residual pressure, which need never be more than a few mm., and k for the ratio of the total volume to the volume before expansion. This ratio need be found only roughly if it is desired to measure simply relative pressures. The value of the ratio in this experiment was about 30.

A further advantage was gained by the use of this form of pressure gauge. Since the density of a gas is not directly proportional to its pressure, if the pressure had been measured directly, the values of the density, computed from them by the simple law of Boyle, would have been too small. But if the gas is allowed to expand first by a definite amount and its pressure is measured at some smaller value, the departures from Boyle's law will be negligible, and the densities at the high pressures will be directly proportional to these lower pressures. This may be shown as follows:

$$M = V_0 D_0 = VD,$$

$$\frac{D}{D_0} = \frac{V_0}{V}.$$

But

$$V_0 P_0 (1 + aP_0 + bP_0^2) = VP(1 + aP + bP^2);$$

$$\therefore \frac{D}{D_0} = \frac{V_0}{V} = \frac{P(1 + aP + bP^2)}{P_0(1 + aP_0 + bP_0^2)};$$

where D_0, P_0, and V_0 represent the density, pressure, and volume under some standard condition, D, P, and V, the same quantities at some high pressure, and $(1 + aP + bP^2)$ is a factor which gives with a high degree of accuracy the departure from Boyle's law. But if the high pressure P is allowed to fall by expansion to some small pressure P', the following relations will hold:

$$VP(1 + aP + bP^2) = V'P'(1 + aP' + bP'^2),$$

and

$$P = \frac{V'P'}{V}\frac{(1 + aP' + bP'^2)}{(1 + aP + bP^2)};$$

$$\therefore D = \frac{D_0 V'P'(1 + aP' + bP'^2)}{VP_0(1 + aP_0 + bP_0^2)}.$$

Since the ratio $\frac{V'}{V}$ is constant, and since P_0, D_0, and the factor $(1 + aP_0 + bP_0^2)$ are constants, the result found above may be written

$$D = CP'(1 + aP' + bP'^2)$$

where C is a constant. If the pressures are less than one atmosphere, the factor $(1 + aP' + bP'^2)$ will not differ sensibly from unity, and the density D at the high pressure will be expressed as proportional to the small pressure P'. The readings on the pressure gauge will, then, be proportional to the densities at the high pressures and not to the high pressures themselves.

II. Optical Arrangement.

The air of which the index of refraction was to be found was confined in one of two similar stout tubes (Fig. 2), the ends of which were closed by pieces of plate glass. Two plates of steel, held to-

gether by bolts running from one end of the tubes to the other, pressed the pieces of plate glass tight against the ends of the tubes. Rubber washers were placed on each side of the glass plates. One of the tubes was connected by copper pressure tubing to the tank which contained the air under investigation, and to the pressure gauge.

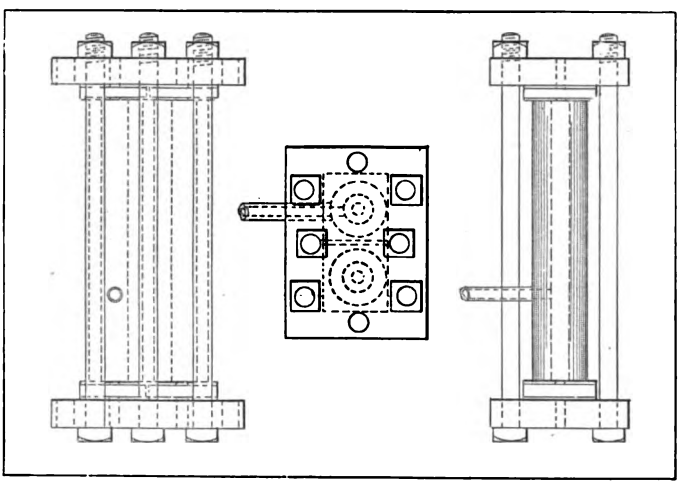


Fig. 2.

It was found convenient to arrange the interferometer mirrors in a way different from that in ordinary use, in a manner which is essentially a modification of Jamin's form. The objection to the latter is that the beams of light cannot be separated to any considerable extent except by the use of very thick plates. On the suggestion of Professor Michelson the mirrors were arranged as indicated in Fig. 3. A and C are "plane parallel" plates, each of which is coated with a thin film of silver, the plate A on the side nearer the source S and the plate C on the side nearer the telescope T. B and D are plane mirrors. Light coming from the source S strikes the thin film on A; half of the light is reflected to B and out through C to T. The rest of the light passes through A to D and is reflected to C. At C it is reflected, in part, by the thin film C to T. When the mirrors are properly adjusted, the conditions are right for interference, since the two parts of the original beam, when they arrive at T, have traveled nearly equal paths.

The tubes described above were placed between these mirrors, so that the light going from A to B passed down one tube, and that going from C to D passed down the other. When the apparatus was in adjustment and the fringes in view in the telescope, if the density of the air in one of the paths was changed, the fringes would shift.

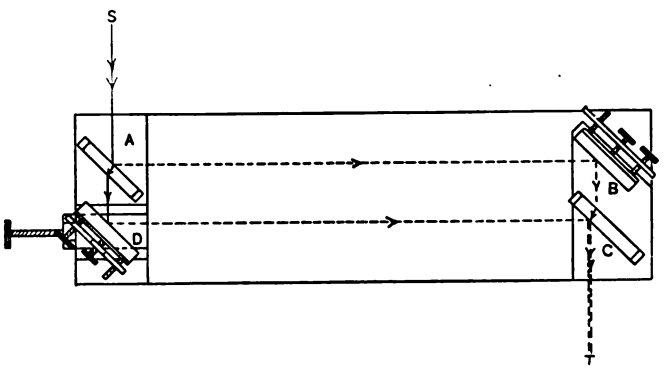


Fig. 3.

Let l equal the length of the tube in the path AB, λ the wave-length in a volume of the green light given out by incandescent mercury vapor in a vacuum tube, let n' and λ' be the index of refraction and wave-length, at a temperature t' and a pressure p'. Let N be the number of wave-lengths in the distance l in a vacuum, and N' the number at a temperature t' and a pressure p'. The following relations hold:

$$N = \frac{l}{\lambda},$$

$$N' = \frac{l}{\lambda'},$$

$$n' = \frac{\lambda}{\lambda'} = \frac{N'}{N},$$

$$n' - 1 = \frac{N'}{N} - 1 = \frac{N' - N}{N}.$$

Since N is constant this shows that the quantity $n' - 1$ under any conditions is proportional to $N' - N$, the number of fringes which

would pass on removing from the tube the air of index n', *i. e.*, the air at a temperature t' and pressure p'.

In practice it was, of course, impossible to remove all the air from the tube, but since the ratio $\frac{n-1}{d}$ was known to be very nearly constant, especially at low pressures, the pressure in the tube was simply allowed to fall to the pressure of the room, and the number of fringes was counted. The small additional number which would have passed if all of the air had been removed was computed in the following way, and added to the observed number. Let n'' and d'' be the index of refraction and density respectively of air at the temperature and pressure of the room, and let N'' be the number of wave-lengths in the tube under the same conditions. Let C represent the value of the constant ratio $\frac{n_0-1}{d_0} = \frac{n''-1}{d''} = C$. Then

$$n'' = Cd'' + 1,$$

$$n'' = \frac{N''}{N},$$

$$n'' - 1 = \frac{N'' - N}{N},$$

$$N = \frac{l}{\lambda_0 n_0};$$

$$\therefore N'' - N = (n'' - 1)N,$$

$$= C\frac{d''l}{\lambda_0 n_0},$$

$$= C'd'',$$

where C' is a constant equal to $\left(\frac{n_0-1}{d_0}\right)\frac{l}{\lambda_0 n_0}$.

The quantity $N'' - N$ is proportional to the density d'' and is equal to the number of fringes which would pass on removing the air at the pressure and temperature of the room, and is therefore the number of fringes to be added to the number counted while the

pressure was falling from p' to p'', in order to get the number which would have passed if the pressure had fallen from p' to 0.

III. OBSERVATIONS.

In the taking of observations the pressure was measured several times, and since only the small volume between the stopcocks was drawn off at each measurement, the pressure in the tank was not sensibly diminished. In the process of counting the number of fringes, the valve at the tank was opened and the tube between the interferometer mirrors and the connecting tubes were allowed to fill. The valve at the tank was then closed, and the union which joined the pressure tubing to the tank was gradually opened and the air in the apparatus allowed to escape. It was not at all difficult to regulate the escape so that the fringes would pass the cross hairs of the observing telescope at any desired rate. On filling the apparatus again and repeating the count, it was found that the number of fringes was slightly less than before, which meant that the pressure in the tank had been slightly reduced by drawing off enough air to fill the tubes. This amount of diminution was constant and it was not difficult to deduce the number of fringes which would have passed at the measured pressure, since this number corresponded to the number of the first count. The succeeding counts simply served as a check on the first one.

After a set of readings at one pressure had been taken, the air was allowed to escape from the tank into the room until the pressure had fallen to some desired point, when another set of readings was taken.

One set of readings, taken at the lowest pressure, is given below.

Temp. 23.8°. Bar. Ht. 74.785.

x	y	y'	$\overline{P}-P$	P	$\overline{P}-P+\frac{P}{K}$
103.5	29.250	40.345	11.095	.535	11.113
"	29.170	40.085	11.085	.455	11.100
"	29.100	40.205	11.105	.385	11.118
"	29.175	40.245	11.070	.460	11.085
"	29.160	40.240	11.080	.445	11.094
"	29.080	40.145	11.065	.365	11.077
"	29.085	40.155	11.070	.370	11.084

Mean—11.096.
Reduced to 0° C.—10.163.

$l = 15.024$ cm. $\lambda_0 = .00005461$. $n_0 = 1.0002928$. $N'' - N = 72.6$.

Fringes 249.0
248.8
248.6
248.4

$N' - N = 321.6$.

$$\frac{N' - N}{\bar{P} - P + \frac{P}{K}} = 31.64.$$

A similar set of readings was taken for each of the other pressures, and the results are collected in the table below. In the first column is the approximate pressure in atmospheres; in the second column the pressure as read on the gauge, reduced to 0° C. The third gives the number of fringes, $N' - N$, and the fourth column gives the ratio

$$\frac{N' - N}{\bar{P} - P + \frac{P}{K}}$$

which is proportional to $\frac{n - 1}{d}$.

Atmos.	Press.	$N' - N$	$\frac{N - N'}{\bar{P} - P + \frac{P}{K}}$	
4.	9.989	316.7	31.70	31.67
	10.146	321.2	31.66	
	10.163	321.6	31.64	
7.2	18.281	579.2	31.68	31.70
	18.365	582.7	31.72	
10.6	26.932	852.6	31.66	
14.2	35.990	1142.1	31.69	
19.2	48.780	1545.1	31.68	

IV. Conclusions.

There is probably not an error of more than .1 in any of the numbers of fringes. The errors in the pressures however may run as high as one or two tenths of 1 per cent. Since the variations in the values of the ratio at different pressures do not amount to more than .2 per cent., any apparent variations in the value of the ratio may be due to errors of observation. It seems, therefore, that if there is any departure from the law of Gladstone and Dale up

to twenty atmospheres in the case of air, this departure does not amount to more than about .1 per cent. The ratio between Lorentz's equation and that of Gladstone and Dale is so nearly equal to a constant, $\frac{2}{3}$ that an attempt to compare the two would be useless.

It is desired to express thanks to Professor Michelson, both for suggesting the experiment and for valuable advice and encouragement throughout the work. Thanks are due to Mr. F. B. Jewett for the drawings which accompany this article, and to Dr. Mann and Dr. Millikan for revision of the manuscript and checking of results.

MEASUREMENT OF THE INTERNAL RESISTANCE OF GALVANIC CELLS.

By C. H. Ayres, Jr.

IN the following paper it is proposed to set forth a new method of measuring electrolytic resistance, and to show that, in the case of certain galvanic cells, the *true* internal resistance is not a function of the current passing through the cell.

It is often questioned whether or not a galvanic cell has a fixed resistance, even at constant temperature; and many measurements have been made[1] which have been interpreted to show that the internal resistance decreases as the current passing through the cell increases. Where these measurements were made by steady-current methods, it has been impossibe to separate changes in electromotive force from changes in resistance, and in these cases the experimental results may be explained by assuming changes in electromotive force fully as well as by assuming changes in resistance. Where alternating currents have been used either the reristance has remained constant, or the variations have been much less pronounced than those shown by steady currents. Using an alternating current, F. Kohlrausch[2] found a value for the internal resistance which remained constant while the resistance in the bridge and the intensity of the current sent through the primary of the induction coil were varied between wide limits. Even with alternating currents Uppenborn[3] and Greef[4] found the resistance to depend on the current. Later, however, Haagn,[5] by a modification of Kohlrausch's method, found the internal resistance of galvanic cells to be independent of the current. The method here em-

[1] Streintz, Wied. Ann., 49, p. 571, 1893. Carhart, Physical Review, II., p. 392, 1895. Richarz, Wied. Ann., 47, p. 567, 1892.

[2] Pogg. Ann., Jubeland, p. 220, 1874. Pogg. Ann., 154, p. 1, 1875. Wied. Ann., 6, p. 1, 1878. Wied. Ann., 11, p. 653, 1880.

[3] Electrotech. Ztschr., 1891, p. 157.

[4] Greef Dissert. Marburg, 1895.

[5] Zeitschr. für Phys. Chem., 23, p. 97, 1897.

ployed, which is a bridge method, differs from the preceding in that the resistance and the capacity of the cell are separately, but in the final adjustment, simultaneously balanced, the self-induction being reduced to a negligible quantity. While attention is here directed to measurements of resistance, it may be noted that the method gives also the capacity of the cell.

THE METHOD.

The method here used is a modification of Kohlrausch's method, and is shown schematically in Fig. 1. In the arms AC and BC of a wheatstone bridge, the resistance r_1 and r_2 are in series with the condensers c_1 and c_2, respectively. AB is the bridge wire; I, an induction coil; and T the telephone used in obtaining a balance. For the sake of convenience, this arrangement shall be referred to as the "capacity bridge." a and b are the segments into which the slider, N, divides the bridge wire AB. When the induction coil is running and the system so adjusted that there is silence in the telephone, the double relation,

$$\frac{r_1}{r_2} = \frac{a}{b} = \frac{c_2}{c_1} \qquad (A)$$

is satisfied. If r_1 is the resistance to be measured, the capacities c_1 and c_2 may be fixed, and r_2, a, and b varied to satisfy the above relation. r_1 is then given by the equation

$$r_1 = \frac{a}{b} \cdot r_2. \qquad (B)$$

This method was chosen because by it the internal resistance of a galvanic cell may be measured when no steady current is flowing through the cell as well as when the cell is sending a current through any desired resistance.[1] Thus if a battery is inserted between A and c_1, no steady current can flow because of the condenser c_1, and hence, by means of an alternating current and telephone, the resistance may be obtained when the battery is delivering no current. The battery may then be shunted by a resistance S, the combined

[1] The method employed by Haagn, though different from the one here used, also afforded this advantage.

resistance of the battery and shunt measured, and, by the law of shunts, the internal resistance of the battery calculated. A second reason for the choice of this method will be shown farther on in this paper.

The induction coil I was a small one, such as in ordinarily used with a Kohlrausch bridge. The resistance of the primary coil was 2 ohms and that of the secondary about 30 ohms. The primary was interrupted by the device known as Neeff's hammer. The length of the spring carrying the hammer could be varied, its greatest length being twice its shortest. In order that the noise from the interrupter might not disturb the observer when using the telephone, the induction coil was placed at some distance from the remainder of the apparatus, in a room by itself.

The bridge wire AB (Fig. 1) was a meter long, and, on different occasions, had a resistance of from 3 to 5 ohms.

In the first series of experiments, the condensers c_1 and c_2 were made of paper and their capacities were varied from 10 to 40 microfarads. In a later series of experiments, mica condensers of 30 microfarads each were used.

Several telephones were always at hand, and that one was chosen which seemed best suited to the conditions of the experiment under consideration. The one most used was a Swiss telephone of about 80 ohms resistance. Sometimes, however, an American telephone, having a resistance of about 1 ohm, was used.

The method of constructing the variable resistance, r_2, may be readily explained by reference to Fig. 1. Between C and E is a gap, g, into which a resistance coil of any desired size may be inserted. $EGHF$ is a manganine wire, about two meters in length and of about 1.25 ohms resistance, fastened at E and F, and stretched over the two posts G and H. MM' is a trough carrying a drop of mercury connecting the two parts, EG and HF, of the wire. The resistance in g, together with the resistance of EM' and MF, makes

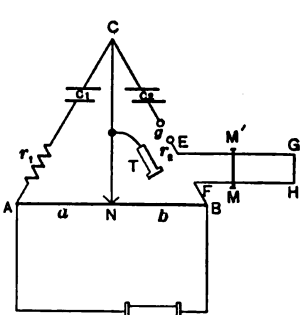


Fig. 1.

the resistance r_2. r_2 could be varied, therefore, by changing the resistance in g and by sliding MM' along the wire. By amalgamating the wire before each experiment, an excellent contact between the mercury and the wire was secured.

The resistance to be measured was inserted in the arm AC between A and c_1, and its value calculated equation from (B). In order to determine the resistance r_2, it was thrown from the arm of the capacity bridge into an arm of a wheatstone bridge, where it was compared with a standard resistance by means of a steady current and galvanometer. Fig. 2 shows the connections of the two bridges. G represents the galvanometer and E the cell used with the second bridge. The transfer of r_2, from one bridge to the other was effected by means of a two-way

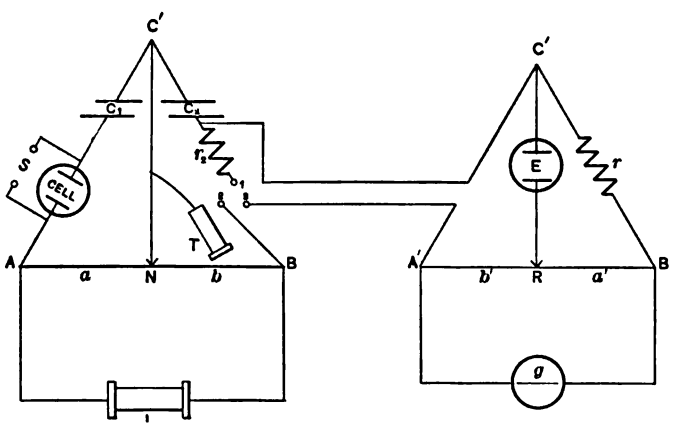


Fig. 2.

switch, which consists of three mercury wells and a heavy copper strap. When wells 1 and 2 were connected by the strap, r_2 was in the capacity bridge; and when the strap connected wells 1 and 3, r_2 was in the second or steady-current bridge. This second bridge was made entirely of copper, in order to reduce the thermo-electric effects to a minimum; but as r_2 was manganine, not copper, and thermo-electric currents might therefore be expected, the galvanometer was connected to the two ends of the bridge wire, its circuit being always closed, while the battery was placed across the bridge, its circuit being open, except for short intervals of time.

In this way, all trouble from thermoelectric effects was avoided.

It is not a difficult operation to satisfy the double relation

$$\frac{r_1}{r_2} = \frac{a}{b} = \frac{c_2}{c_1}. \qquad (A)$$

One has merely to move the slider, N, along the bridge wire until a minimum is found, then vary r_2 so as to improve the minimum; then again move the slider, and so on until silence is obtained. Silence having been obtained, r_2 is put into the copper bridge and compared with a standard resistance. If r is the resistance against which r_2 is balanced in the copper bridge, a' and b' the segments of the bridge wire, then

$$r_2 = \frac{b'}{a'} \cdot r$$

and, by equation (B),

$$r_1 = \frac{b'}{a'} \cdot r \cdot \frac{a}{b} \qquad (C)$$

a and b being the segments of the wire on the capacity bridge.

When metallic resistances, free from self-induction, are used in the bridge, perfect silence is obtainable in the telephone. This is not always the case when a liquid resistance is inserted in one arm of the bridge. When polarization takes place in the liquid cell, a disturbing effect is produced. This effect we shall refer to as the capacity effect of the cell, for it has been shown [1] that a polarized cell may be regarded as a resistance in series with a capacity. When such an effect exists, in the arrangement of the bridge here used, it is combined with the capacity of the condenser with which it is in series, and causes no error in the determination of resistance. If a current of a single frequency were used in working the bridge, it would always be possible to obtain silence in the telephone, but when the current is made up of vibrations of different frequencies, only a minimum sound is obtainable, because the capacity effect of a cell is different for different frequencies and it is impossible to balance for all frequencies at once.[2] Suppose the relations (A) are satisfied for metallic resistance r_1 and r_2. If any other metallic re-

[1] Varley, Phil. Mag., 4, 41, p. 310, 1871. Kohlrausch and Holborn, "Leitvermögen der Elektrolyte," p. 67.

[2] M. Wien, Wied. Ann., 42, 593, 1891; 47, 626, 1892; 58, 37, 1896; 59, 267, 1896.

ristance r_1' is substituted for r_1, silence may again be obtained by varying the resistance r_2, leaving the slider, N, unmoved, for c_1 and c_2 remain unchanged and

$$\frac{a}{b} = \frac{c_2}{c_1}.$$

Now suppose the resistance r_1' to be a liquid resistance in which polarization takes place. The capacity of the arm AC will no longer be c_1, but c_1', a combination of c_1 and the capacity effect of the cell. It will not now be possible to obtain a minimum by varying r_2 only, but the slider N must also be moved, for the segments a and b of the bridge wire must satisfy the relation

$$\frac{a}{b} = \frac{c_2}{c_1'}.$$

Even in case the capacity effect of the cell causes a considerable displacement of the position of the slider corresponding to a minimum, the resistance of the cell may be accurately measured, for a minimum is determined by the double relation

$$\frac{a}{b} = \frac{c_2}{c_1'} = \frac{r_1'}{r_2},$$

and r'_1 is given by the equation

$$r_1' = \frac{a}{b} \cdot r_2.$$

In this respect, the form of the bridge under consideration affords an advantage over the Kohlrausch bridge in which a displacement of the minimum causes an error in the determination of resistance. This advantage formed the second reason for the development of the method.

RESULTS AND DISCUSSIONS.

Experiments with Paper Condensers.—The cell to be measured was always inserted in the arm AC so that its resistance corresponded to that which has been called r_1. Care was taken to make the resistance of the connecting wires in the arms AC and BC as small as possible. In order to determine whether or not these wires caused serious error in the measurement of resistance, standard coils were measured. The results showed that resistance varying from

one to three ohms could be measured with an accuracy of a few tenths of one per cent. The results given here are all above one ohm, and are probably accurate to within .5 per cent. Allowance was made for the resistances of the end connections of the bridge wires. In the copper bridge, in which the wire was of very low resistance, the end connection at A' (Fig. 2) was equivalent to the resistance of .7 cm. of the bridge wire, while that at B' was equivalent to .8 cm. On the capacity bridge, the connection resistance at A was equivalent to .19 cm. of the bridge wire, and that at B to .33 cm. In the experiments with mica condensers, the connection resistances were so reduced as to be negligible.

The first cell measured was a modified Daniel cell, composed of copper in sulphate of copper and zinc in sulphate of zinc. The copper electrode and the sulphate of copper were placed in a large glass jar, and into the solution of copper sulphate was sunk a porous cup containing the zinc and the zinc sulphate. The cell is shown in the adjoining figure, and will be referred to by its commercial name, "Excello Cell." In the tables of observations, given below, T represents the temperature of the cell, as given by a thermometer, the bulb of which was placed in the porous cup; N and R denote the positions of the sliders on the capacity and copper bridges, respectively; S denotes the resistance by which the cell was shunted, and b is the value found for the resistance of the cell. When no shunt is used, the symbol ∞ is placed in column S.

Fig. 3.

It is noticeable that the setting of the slider was practically the same for all these measurements. This shows that if the capacities c_1 and c_2 of the paper condensers changed at all during the experiment, the changes were such that the ratio c_2/c_1 remained constant.

Next the cell was shunted by resistances varying from 100 to 3 ohms, and its resistance measured.

In these experiments, the alternating current in the wires leading from the induction coil was less than .005 ampère, of which only a small part went through the cell. Thus the current used in

$C_1 = C_2 = 20$ *microfarads. August* 2, 1900.

T	N	R	S	b
25.6	51.20	38.40	100	1.700
25.6	51.20	37.98	∞	1.704
25.6	51.20	38.27	100	1.708
25.6	51.20	38.10	∞	1.693
25.6	51.20	38.47	100	1.698
25.6	51.20	38.03	∞	1.698
25.6	51.20	38.40	100	1.700
25.6	51.20	38.00	∞	1.700
25.5	51.20	38.41	100	1.700
25.5	51.20	38.02	∞	1.698
25.5	51.20	38.41	100	1.700
25.5	51.20	37.94	∞	1.704
25.5	51.20	38.40	100	1.700
25.4	51.20	37.98	∞	1.704
25.4	51.20	38.44	100	1.698
25.4	51.20	37.97	∞	1.704
25.4	51.20	38.44	100	1.698
25.4	51.21	37.88	∞	1.709
25.4	51.21	38.42	100	1.700
25.4	51.22	38.00	∞	1.700

Mean of open circuit resistance = 1.701 ohms. Mean of closed circuit resistance = 1.700 ohms.

$C_1 = C_2 = 20$ *microfarads. August* 3, 1900.

T	N	R	S	b
23.7	51.20	39.57	100	1.613
23.7	51.20	39.16	∞	1.611
23.7	51.20	39.85	50	1.616
23.7	51.20	39.15	∞	1.611
23.7	51.20	40.10	40	1.616
23.7	51.20	39.20	∞	1.608
23.7	51.20	40.52	30	1.604
23.7	51.20	39.18	∞	1.610
23.7	51.20	41.16	20	1.603
23.7	51.20	39.15	∞	1.612
23.7	51.20	42.81	10	1.611
23.7	51.20	39.21	∞	1.607
23.7	51.10	49.52	3	1.604
23.7	51.20	39.25	∞	1.605

measuring the resistance was not merely alternating, but also extremely small in comparison with the steady current.

The following table contains the results obtained on LeClanche, Daniell, and Excello cells:

	$c_1 - c_2 = 20$ m.f.			$c_1' - c_2 = 40$ m.f.		
	Excello Cell No. 1.	Excello Cell No. 2.	Excello Cell No. 3.	LeClanche Cell.	Daniell Cell No. 1.	Daniell Cell No. 2.
∞	1.603			1.584	1.613	1.184
100	1.602	1.325	1.092	1.540		
50	1.603					
40	1.603		1.072	1.575		
30	1.603		1.095	1.561	1.611	
20	1.604		1.095	1.551	1.620	
10	1.600	1.321	1.096			1.181
3	1.603		1.110			

Figures for Excello and LeClanche cells represent single observations, while those for the Daniell cells are means of a number of observations taken in succession. It was impossible to obtain a reliable minimum when low shunts were used on the LeClanche cells. In all these experiments, the minimum was sharp and its position was readily found. In the case of the Excello cells, it was found that if the zinc electrode was kept clean and well amalgamated the minimum was not only sharp, but symmetrical, as is the case when metallic resistances are used. Moreover, when the zinc electrode of these cells was clean, the position of the minimum was practically the same as for metallic resistances. This shows that there was no disturbing effect due to polarization. In the case of the LeClanche and Daniell cells, the minimum was displaced from 1 to 2 mm. from that given by metallic resistances. Under these circumstances, of course, the effect being different for different frequencies, the minimum was not symmetrical.

Experiments with Mica Condensers.

The mica condensers used in this series of experiments had a capacity of 30 microfarads each. The resistance of the wires connecting the various sections was very small and as nearly as possible the same in both condensers. Before beginning these experiments, the two bridges were thoroughly overhauled and the resistances of the end connections of the bridge wires were made

negligible. Measurements on known metallic resistance, made to test the accuracy of the bridge, gave results correct to one-twenty-fifth per cent. In the experiments with this apparatus, the cells were measured first with no shunt and then with a shunt of 1 ohm. In order to keep the condition of the cells as steady as possible, they were shunted by a resistance of 10 ohms during the time between the different observations.

The table below contains the results obtained from five different cells. The zinc electrode of Daniell cell No. 1 consisted of a rod about three inches long and three-sixteenths of an inch in diameter. The zinc electrode of Daniell cell No. 2 was in the shape of a rectangle about 10 centimeters long and 2 centimeters wide. The electrode in the Excello cells, as well as those of the Daniell cells given in the first table, were of the size ordinarily used in these cells.

S.	Excello Cell No. 1.	Excello Cell No. 2.	Daniell Cell No. 1.	Daniell Cell No. 2.	Columbia Dry Cell.
∞	.6133	.5838	2.650	2.044	.1151
1	.6123	.5845	2.651	2.027	.1151
∞	.6138	.5843	2.656	2.036	.1157
1	.6117	.5855	2.652	2.052	.1151
∞	.6112	.5863	2.659	2.036	.1147
1	.6117	.5863	2.651	2.027	.1158

The results given above show that the resistance was the same whether the cell was in open or closed circuit, within high limits of accuracy. However, it is to be noted that the condition of a cell, of course, depends upon its previous history, among other things upon any prolonged current that may have been flowing, insofar as this actually alters the cell. What is here shown is that the resistance of a cell is not a function of the current strength at the time, a result contrary to the conclusions of most other observers.

ON THE THEORY OF THE BRIDGE.

Thus far we have considered only the apparatus used and the results obtained in actual experiment. It will now be of interest to consider the relations between the current in the telephone and the magnitude of the various quantities used in the bridge, for from these relations we may see what conditions are necessary for accurate

work. Our arrangement of the bridge is represented in Fig. 1. In the arms AC and BC the resistance r_1 and r_2 are in series with the capacities c_1 and c_2, respectively. T denotes the telephone, and I the induction coil used with the bridge. Let r_3 and r_4 be the resistances of the segments AN and NB of the bridge wire AB. For the sake of clearness, the various quantities will be considered under separate heads.

Capacity.—In order to form an idea of how much capacity must be used to render accurate measurements of resistance possible, it is necessary to write down the expression for the current in the telephone. For the sake of simplicity, assume the resistance and self-induction of the main branch, AIB, to be so large that changes in the capacities c_1 and c_2 do not materially alter the current supplied to the bridge. If the current in the main branch is assumed to be

$$\cos(nt) = \tfrac{1}{2}(e^{int} + e^{-int}),$$

the current in the telephone at any time may be found by taking one-half the sum of the currents in the telephone when currents e^{int} and e^{-int} are separately taken as the currents in the main branch.

If there is a current e^{int} in the main branch, the current in the telephone is given by

$$Z' = e^{int}\frac{a_2a_3 - a_1a_4}{(a_2 + a_4)(a_1 + a_3) + a(a_1 + a_2 + a_3 + a_4)} \qquad (1)$$

where

$$a_1 = r_1 + \frac{1}{inc_1},$$

$$a_2 = r_2 + \frac{1}{inc_2},$$

$$a_3 = r_3,$$

$$a_4 = r_4,$$

and

$$a = r + inL,$$

r being the resistance, and L the inductance of the telephone. Putting these values for the a's in equation (1), we obtain,

$$Z' = \frac{e^{int}(A + iB)}{1 - C + iD} \qquad (2)$$

where

$$A = n^2 c_1 c_2 (r_1 r_4 - r_2 r_3),$$

$$B = n(r_3 c_1 - r_4 c_2),$$

$$C = n^2 k_1 k_2 c_1 c_2 + n^2 r c_1 c_2 (k_1 + k_2) + n^2 L(c_1 + c_2),$$

$$D = n[K_2 c_2 + k_1 c_1 + r(c_1 + c_2) - n^2 L(k_1 + k_2) c_1 c_2],$$

$$k_1 = r_1 + r_3$$

and

$$k_2 = r_2 + r_4.$$

Multiplying the numerator and denominator of (2) by $(1 - C) - iD$, we obtain,

$$Z = e^{int} \frac{(A + Bi)[(1 - C) - iD]}{(1 - C)^2 + D^2}$$

or

$$Z' = e^{int}(A'' + iB'') \tag{3}$$

where

$$A'' = \frac{A(1 - C) + BD}{(1 - C)^2 + D^2} \tag{4}$$

and

$$B'' = \frac{B(1 - C) - AD}{(1 - C)^2 + D^2} \tag{5}$$

If e^{-int} is taken as the current in the main branch, the current in the telephone is given by

$$Z'' = e^{-int}(A'' - iB''),$$

and the current in the telephone corresponding to a current cos (nt), in the main branch, is

$$Z = \tfrac{1}{2}(Z' + Z'')$$

$$= A'' \frac{e^{int} + e^{-int}}{2} + B'' \frac{e^{int} - e^{-int}}{2} i$$

$$= A'' \cos(nt) - B'' \sin(nt)$$

whence

$$Z = \sqrt{A''^2 + B''^2} \cos(nt - \varphi) \tag{6}$$

where

$$\tan \varphi = -\frac{B''}{A''}.$$

When $r_1/r_2 = r_3/r_4$, we have $A = 0$, and when $c_1/c_2 = r_4/r_3$, $B = 0$. When both A and B are zero, A'' and B'' are zero; and, therefore, by (6), $Z = 0$. Now if it is desired to measure resistances there must be a disturbance in the telephone whenever $r_1/r_2 \neq r_3/r_4$, even if $c_1/c_2 = r_4/r_3$; that is, even if $B = 0$. It is, therefore, desirable to study the relations that must exist between the resistances, capacity, and inductance in order that the current in the telephone may be detected as long as the resistances do not satisfy the condition $r_1/r_2 = r_3/r_4$. The work is much simplified, while its usefulness is not at all impaired, by the assumption $B = 0$. If $B = 0$,

$$A'' = \frac{A(1 - C)}{(1 - C)^2 + D^2},$$

and

$$B'' = \frac{-AD}{(1 - C)^2 + D^2},$$

hence, if the maximum value of Z is called $\bar{Z}$, we may write

$$\bar{Z} = \sqrt{\left[\frac{A(1 - C)}{(1 - C)^2 + D^2}\right]^2 + \left[\frac{-AD}{(1 - C)^2 + D^2}\right]^2},$$

or

$$\bar{Z} = A[(1 - C)^2 + D^2]^{-\frac{1}{2}}. \qquad (7)$$

Now

$$A = n^2 c_1 c_2 (r_1 r_4 - r_2 r_3);$$

therefore, A increases as the product $c_1 c_2$, and, if C and D are small in comparison with unity as is the case when $c_1 c_2$ are small, $\bar{Z}$ will increase as the product $c_1 c_2$. In this case evidently $\bar{Z}$ may be increased by increasing r_3 and r_4, *i. e.*, by increasing the resistance of the bridge wire. Let us now assume that $c_1 = c_2$, as was the case in all the experiments made with this arrangement of the bridge, and write

$$P = \frac{1 - C}{c_1^2},$$

and

$$Q = \frac{D}{c_1^2},$$

or

$$P = \frac{1}{c_1^2} - \left[n_2 k_1 k_2 + n^2 r(k_1 + k_2) + \frac{2n^2 L}{c_1}\right] \qquad (8)$$

and

$$Q = n\left[\frac{k_1 + k_2 + 2r}{c_1} - n^2L(k_1 + k_2)\right]. \tag{9}$$

Then equation (7) becomes

$$Z = \frac{n^2c_1^2}{c_1^2}(r_1r_4 - r_2r_3)\left[P^2 + Q^2\right]^{-\frac{1}{2}},$$

or

$$Z = n^2(r_1r_4 - r_2r_3)\left[P^2 + Q^2\right]^{-\frac{1}{2}}. \tag{10}$$

Consider c_1 the only variable in the right hand side of equation (10). Z is largest for that value of c_1 which makes $[P^2 + Q^2]^{\frac{1}{2}}$ smallest, or for that value which makes

$$D_{c_1}P \cdot P + D_{c_1}Q \cdot Q = 0 \tag{11}$$

Putting in (11) the values of P and Q given by (8) and (9), we obtain,

$$ac_1^3 - a_1c_1^2 + a_2c_1 - a_3 = 0 \tag{12}$$

where

$$a = n^4L(k_1^2 + k_2^2)$$
$$a_1 = n^2[k_1^2 + k_2^2 + 2r(k_1 + k_2) + 4r^2] + 4n^4L_2$$
$$a_2 = 6n^2L$$

and

$$a_3 = 2.$$

It is evident that equation (12) always has at least one real, positive root. Hence for any given resistances and inductance there is always at least one value for c_1 that makes $\overline{Z}$ a maximum.

Let us now study the variations of $\overline{Z}$ with c_1 for the special case in which $c_1 = c_2$, $r_3 = r_4 = 1$, $r_1 = 1 + x$, $r_2 = 1$, $r = 1$, $L = \frac{1}{2}\,10^{-4}$, and $n = 10^3$, these values being chosen because they are of the same order of magnitude as those that were used occasionally in actual experiment. x is here the amount by which r_1 deviates from satisfying the relation $r_1/r_2 = r_3/r$. For this case,

$$k_1 = 2 + x;$$
$$k_2 = 2;$$
$$A = 10^6c_1^2x;$$
$$C = 10^6(8 + 3x)c_1^2 + c_1 10^2$$

and

$$D = 10^3\left[6c_1 + xc_1 - 50(4 + x)c_1^2\right].$$

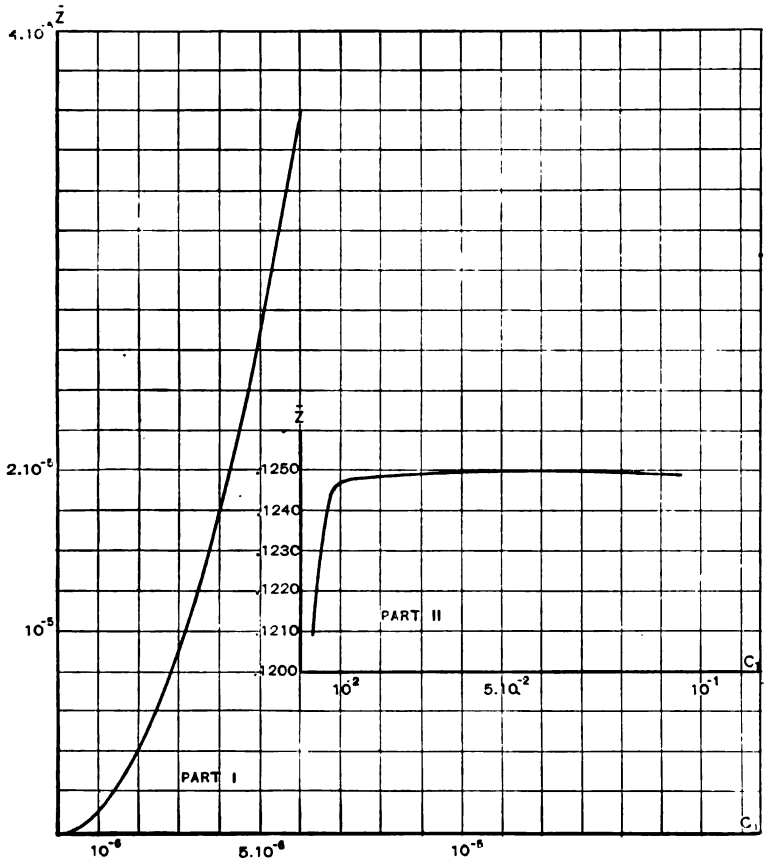


Fig. 4.

If x is small in comparison with unity, we may write,

$$C = 8 \cdot 10^6 \cdot c_1^2 + c_1 \cdot 10^2,$$

and

$$D = 10^3 \left[6c_1 - 2 \cdot 10^2 \cdot c_1^2\right].$$

This makes, by (8) and (9),

$$P = \frac{1}{c_1^2} - 10^2 \left[10^4 \cdot 8 + \frac{1}{c_1}\right]$$

and

$$Q = 10^8 \left[\frac{6}{c_1} - 2 \cdot 10^2\right].$$

By equation (10),

$$\bar{Z} = 10^6 x[P^2 + Q^2]^{-\frac{1}{2}}. \qquad (13)$$

To find the value of c_1 for which $\bar{Z}$ is a maximum, we substitute for P and Q in the equation,

$$D_{c_1}P \cdot P + D_{c_1}Q \; Q = 0.$$

This gives,

$$4 \cdot 10^8 c_1 - 2001 \cdot 10^4 c_1^2 + 3 \cdot 10^2 c_1 - 2 = 0. \qquad (14)$$

By Sturm's theorem, equation (14) is seen to have but one real root; this root lies between $c_1 = 10^{-1}$ and $c_1 = 10^{-2}$. Hence there is but one value of c_1 which renders $\bar{Z}$ a maximum. The relation between c_1 and $\bar{Z}$ is shown by Fig. 4. This curve is given in two parts, each part having a scale of its own. Part I. shows the rapid increase of $\bar{Z}$ with c_1, when c_1 is small. Part II. shows the maximum value of $\bar{Z}$, which corresponds very nearly to $5 \cdot 10^{-2}$ farads. Between Parts I. and II. a portion of the curve, having no peculiar points, is omitted. The table below gives a few values of c_1, together with the corresponding values of $\bar{Z}$.

c_1	$\bar{Z}$	c_1	$\bar{Z}$
0	0	10^{-2}	$.1248x$
10^{-5}	$10^{-6}x$	2.10^{-2}	$.1249x$
2.10^{-5}	$4.10^{-6}x$	5.10^{-2}	$.1250x$
10^{-3}	$10^{-4}x$	10^{-1}	$.1249x$
10^{-4}	$9.2x10^{-3}$	1	$.1249x$
10^{-3}	$.109x$	∞	$.1249x$

The table shows that $\bar{Z}$ at the maximum is only slightly greater than the limit approached as c_1 increases without limit; that is, $\bar{Z}$ is never much larger than it would be if resistances only were used in the bridge. The value of c_1 corresponding to a maximum of $\bar{Z}$ is, in the case under consideration, exceedingly large. It is, therefore, fortunate that accurate work may be done long before the best condition is attained. With a bridge wire of two or three ohms resistance, and with capacities of from twenty to forty microfarads, a resistance as low as half an ohm may be measured with an accuracy of .5 per cent.

If in place of the telephone, we have a non-inductive conductor of resistance, r, across the bridge,

$$P = \frac{1}{c_1^2} - n^2 k_1 k_2 - n^2 r(k_1 + k_2),$$

and

$$Q = n[k_1 + k_2 + 2r]c_1^{-1}.$$

In this case,

$$Dc_1 P \cdot P + Dc_1 Q \cdot Q = 0,$$

becomes

$$c_1^2[4r^2n^2 + 2rn^2(k_1 + k_2) + n^2(k_1^2 + k_2^2)] + 2 = 0.$$

Since the roots of this equation are always imaginary, there is no value of c_1 for which $\bar{Z}$ is a maximum. In fact,

$$\bar{Z} = n^2(r_1 r_4 - r_3 r_2)\left[\frac{1}{c_1^4} + \frac{G}{c_1^2} + H\right]^{-\frac{1}{2}}$$

where

$$G = n^2(k_1^2 + k_2^2) + 4n^2r^2$$

and

$$H = n^4 k_1 k_2 + n^4 r^2(k_1 + k_2)^2 + 2n^4 r k_1 k_2(k_1 + k_2).$$

And it is evident that $\bar{Z}$ increases when c increases, as would naturally be expected.

Inductance.—That there is a value of the inductance, L, of the telephone which renders $\bar{Z}$ a maximum when the resistances and capacity in the bridge are fixed, may be seen by placing the derivative, with respect to L, of the right-hand side of the equation (10) equal to zero. This gives the equation,

$$D_L P \cdot P + D_L Q \cdot Q = 0,$$

an equation which is linear as regards L. Putting in the values of P and Q, we obtain,

$$L = \frac{c_1(k_1^2 + k_2^2) + \frac{2}{n^2 c_1^2}}{4 + n^2(k_1 + k_2)^2 c_1^2}. \qquad (14)$$

From this equation, it is evident that the smaller c_1, the larger the inductance required to make $\bar{Z}$ a maximum. That the resistance of the telephone does not enter into this expression for the inductance is not surprising, for we have assumed that we could vary the inductance of the telephone without varying its resistance.

$$k_1 = k_2 = 2,$$

$$c_1 = 4 \cdot 10^{-5},$$

and

$$n = 10^3,$$

we find

$$L = .0124 \text{ henry.}$$

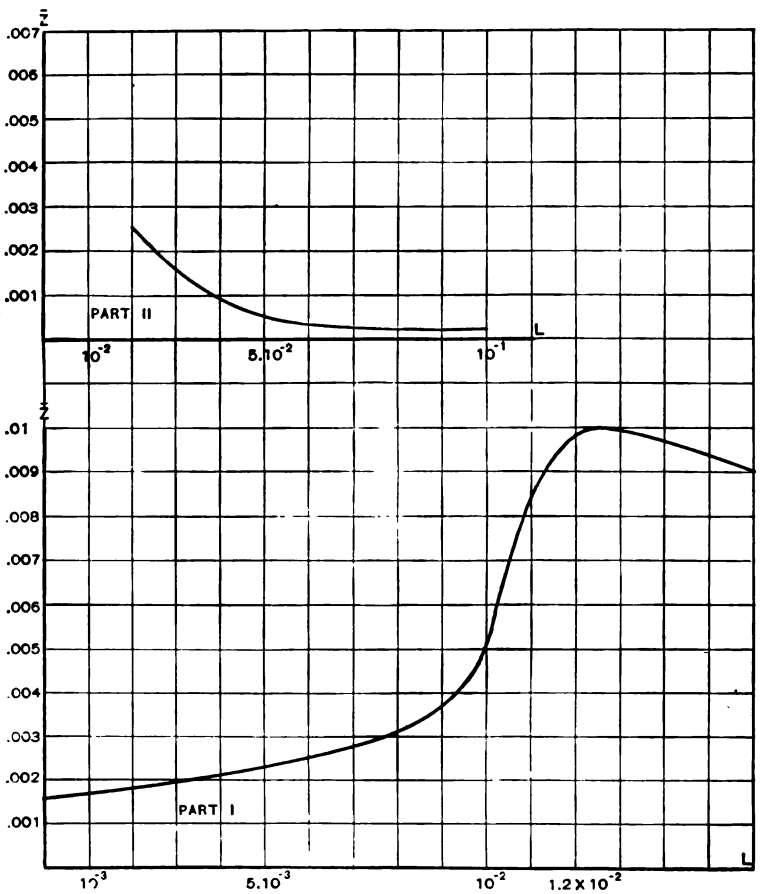


Fig. 5.

Fig. 5 shows the relation between L and Z as calculated from equation (13) on the assumption that

$$r_2 = r_3 = r_4 = r = 1 \text{ ohm.}$$

$$r_1 = 1 + x,$$

$$n = 10^3,$$

and

$$c_1 = 40 \text{ microfarads.}$$

This curve is given in two parts, having different scales. Part I. contains the maximum and is of chief interest. There is a small and unimportant part of the curve omitted between Parts I. and II. The following are a few values of L, with the corresponding values of $\bar{Z}$.

L	$\bar{Z}$	L	$\bar{Z}$
0	$.00157x$	$1.24.10^{-2}$	$.0100x$
10^{-5}	$.00158x$	2.10^{-2}	$.00257x$
10^{-4}	$.00158x$	5.10^{-2}	$.000506x$
10^{-3}	$.00170x$	10^{-1}	$.000230x$
10^{-2}	$.00525x$	∞	$.0000$

Frequency.—In order to see if, by varying n, keeping the resistances and inductance constant, a value of n can be found for which $\bar{Z}$ is a maximum, it will be well to write equation (7) in the form,

$$\bar{Z} = c_1^2(r_1r_4 - r_2r_3)[P_1^2 + Q_1^2]^{-\frac{1}{2}} \tag{15}$$

where

$$P_1 = \frac{1}{n^2} - [k_1k_2c_1 + rc_1^2(k_1 + k_2) + 2Lc_1],$$

and

$$Q = \frac{k_1c_1 + k_2c_1 + 2rc_1}{n} - nL(k_1 + k_2)c_1^2,$$

c_1 being taken equal to c_2, as before.

To make $\bar{Z}$ a maximum, we make

$$D_nP_1 \cdot P_1 + D_nQ_1 \cdot Q_1 = 0. \tag{16}$$

Substituting the values of P_1 and Q_1 in this equation, we obtain, after reduction,

$$n^6L^2(k_1 + k_2)^2c_1^4 + n^2[4Lc_1 - 4r^2c_1^2 - 2rc_1^2(k_1+k_2) - k_1^2c_1^2 - k_2^2c_1^2] - 2 = 0.$$

Or, if we let $n_1 = n^2$

$$n_1^3L^2(k_2 + k_1)^2c_1^4 + n[4Lc_1 - 4r^2c_1^2 - 2rc_1^2(k_1 + k_2) - k_1^2c_1^2 - k_2^2c_1^2] - 2 = 0. \tag{17}$$

If, for example, we take

$$c_1 = 3 \cdot 10^{-5},$$

$$L = \tfrac{3}{2}\ 10^{-4},$$

and

$$k_1 = k_2 = 2,$$

equation (17) reduces to

$$n_1^3 = \frac{10^{28}}{145\ 8},$$

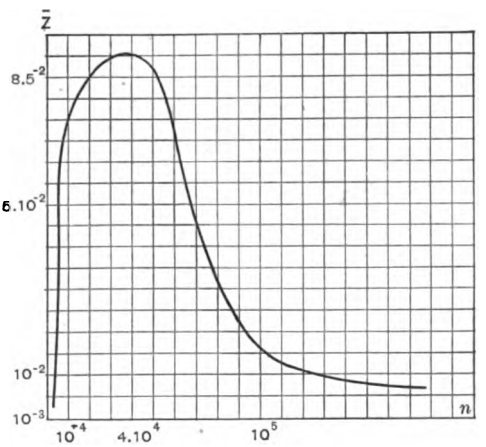


Fig. 6.

or

$$n_1^3 = \frac{10^{27}}{146};$$

hence

$$n_1 = \frac{10^9}{(146)^{\frac{1}{3}}}$$

and

$$n = \frac{10^3}{(146)^{\frac{1}{3}}},$$

$$n = 4 \cdot 10^4.$$

And the number of complete vibrations per second is given by $\frac{n}{2\pi}$, or $\frac{2.10^4}{\pi} = 6366.$

Fig. 6 shows the relation between Z and n as calculated from equation (13); it being assumed that $k_1 = k_2 = 2$,

$$c_1 = 3 \cdot 10^{-5},$$

and

$$L = \tfrac{8}{2}\, 10^{-4}.$$

The following table shows a few of the values of n, together with the corresponding values of $\bar{Z}$.

n	$\bar{Z}$	n	$\bar{Z}$
0	0	10^5	$1.6.10^{-2}x$
10^2	$9.10^{-6}x$	10^6	$1.7.10^{-3}x$
10^3	$9.10^{-4}x$	10^7	$1.7.10^{-4}x$
10^4	$7.10^{-2}x$	10^8	$1.7.10^{-5}x$
4.10^4	$8.6.10^{-2}x$	∞	0

It is to be noticed that $\bar{Z}$ increases rapidly at first as n increases, reaching a maximum when n is $4 \cdot 10^4$, and then decreases as n increases, approaching the value of zero as n increases without limit.

The above treatment shows how the accuracy of measurement depends upon the relations between inductance, capacity, resistance, and frequency of current used in the bridge.

ON THE DIELECTRIC CONSTANT OF DILUTE ELECTROLYTIC SOLUTIONS.

BY A. DE FOREST PALMER, JR.

IN 1892 E. Cohn[1] published the results of a series of determinations of the refractive index of water and dilute solutions of sodium chloride for electromagnetic waves of frequency about 10^8 per second. With an exciter of the well-known Hertz type he set up a system of stationary waves along two parallel wires suspended partly in air and partly in a long tank containing water or a salt solution. A comparison of the wave-lengths in the two sections of the wires, determined by a modification of Ruben's bolometer method, gave the results tabulated below where λ is the conductivity of the water or solution, l_a the half wave-length in air, l_w the half wave-length in water or the solution, and n the index of refraction. The temperature was always between 16.8° and 17.2°.

Substance.	$\lambda \times 10^6$	l_a	l_w	n	n^2
Water.	7.9	292.4	34.1	8.57	73.5
NaCl solution.	140.0	292.4	33.8	8.65	74.8
"	484.0	292.4	33.0	8.86	78.5

When the conductivity was greater than 532×10^6 he was unable to determine the position of the nodes in the solution owing to the increased damping of the oscillations.

A much more extensive and thorough investigation of the same subject, by a similar method, was published by P. Drude[2] in 1896. He used an oscillator of the Blondlot type and determined the position of the nodes along two parallel wires by the aid of Zehnder's vacuum tube. Great care was taken to develop a method of observation as free as possible from consistent errors and to apply suit-

[1] E. Cohn, Wied. Ann., 45, p. 370, 1892.
[2] P. Drude, Wied. Ann., 59, p. 17, 1896.

able corrections for such errors as could not be eliminated. Containing vessels, for the liquids experimented upon, of various dimensions and materials were tried and their effect on the results determined. After a comprehensive series of observations he was led to the conclusion that, up to a conductivity $\lambda = 5.3 \times 10^{-3}$, the square of the index of refraction of a solution varies less than one per cent. from the value for pure water. Solutions of copper sulphate and sodium chloride were used and the frequency of the electromagnetic waves was about 4×10^8 per second. For higher conductivities he found that n^2 decreased when the concentration was increased and reached a value about 10 per cent. less than that for pure water when λ equalled 40×10^{-3}.

In 1899 W. D. Coolidge,[1] using Drude's method, found the wave-length in water and in a copper sulphate solution of 6.5 times as great conductivity to be the same within 0.2 mm. at a temperature 19.2°. His mean result for the half wave-length in water was 66.184 cm., and in the $CuSO_4$ solution 66.202 cm.

Ernest Lecher,[2] in a footnote to an article on the "Measurement of Dielectric Constants by Means of Hertz Oscillations," states that the capacity of a condenser, measured by his method with Ruhmkorff oscillations, was just the same whether a vessel between its plates contained distilled water or a ten per cent. solution of sulphuric acid. The condenser plates were 28 cm. apart and the vessel containing the liquid was 27 cm. long.

E. B. Rosa[3] in 1891 measured the force between a fixed and a movable electrode by means of a torsion balance and found that it decreased when hydrant water or minute quantities of acid or copper sulphate were added to the distilled water surrounding the electrodes. The difference of potential required for these measurements was furnished by a gravity battery and was mechanically reversed about fifty times per second. He thinks that the observed decrease in force was due to polarization of the electrodes.

The investigations of Drude and Coolidge indicate that the index of refraction of a dilute electrolytic solution, for electromagnetic

[1] W. D. Coolidge, Wied. Ann., 69, p. 134, 1899.
[2] Ernest Lecher, Wied. Ann., 42, p. 152, 1891; Phil. Mag. (5), 31, p. 181, 1891.
[3] E. B. Rosa, Phil. Mag. (5), 31, p. 188, 1891.

waves, is independent of the concentration and the results of Cohn do not appear of sufficient weight to controvert this conclusion. Hence, assuming Maxwell's relation $k = n^2$, we might expect the dielectric constant of such a solution to be the same as that of pure water. But since this relation has been shown to fail in many instances, this deduction cannot be rigorously maintained without corroborative evidence from direct determinations of the dielectric constants of solutions. Published results of such determinations are meagre and inadequate, but they indicate that the change in dielectric constant with concentration is very small in comparison with the corresponding change in conductivity.

After a preliminary experimental and theoretical study of several of the best known direct methods, it was decided that small variations in dielectric constant could be most accurately determined by measuring the force between two electrodes immersed in the solution and maintained in the same relative position. The principal sources of error in such a method may be expected to arise from the polarization of the electrodes, from changes in their relative position, due to convection currents in the liquid or to inaccuracies in adjustment, and from errors in determining their difference in potential. The extent to which these difficulties have been overcome will appear in the following discussion. The first apparatus that gave intelligible results consisted of two electrometers, constructed on the attracted disk type, connected in parallel between the earth and one terminal of the spark-gap of a Hertzian plate-oscillator, having a frequency of about 4×10^7 vibrations per second.

The movable electrode or needle of one of the electrometers was shaped like the longitudinal section of a dumb-bell and suspended at its center of gravity by a fine platinum rhodium wire. The aluminum disks, of which it was made, were 3.8 cm. in diameter and 0.27 mm. thick, and their centers were 5 cm. apart in a horizontal line. The plane of the needle was vertical, and, in the zero position, parallel to and midway between the planes of the two fixed electrodes. These electrodes were slightly greater in area than the disks, and were placed adjacent to opposite ends of the needle, so that when charged they tended to rotate it in the same direction. An inverted glass bell-jar was clamped against the lower surface of the bed-plate

of the instrument and filled nearly to the level of the electrodes with strong sulphuric acid to absorb the moisture from the inclosed air. A small glass rod was suspended from the center of the needle by a platinum wire, and served to damp its vibrations. The other electrometer was constructed in essentially the same way, except that the electrodes were copper cylinders, about 2.9 mm. in diameter, with their axes lying in a horizontal plane and the acid and damper were dispensed with. A funnel passing through a hole in the bed-plate permitted the introduction of the liquid experimented upon, and a stop-cock in the bottom of the bell-jar facilitated its removal.

Both suspensions were provided with concave mirrors so adjusted that, when the needles were in the zero position, the two images of an illuminated wire, placed above the center of a scale some distance in front of the electrometers, were superposed on the middle division of the scale. When the oscillator was put in action both images were deflected from the center in the same direction and after their position became sufficiently steady they were caused to return to the center by suitably turning the torsion heads at the top of the suspensions. The action of the oscillator was then interrupted and the angles of torsion calculated from the resulting scale deflection of the images. When a sufficient number of similar independent observations had been made the solution in the second electrometer was changed and the operation repeated.

Owing to variations in the intensity of the oscillating current and to irregularities in the intervals between the wave trains sent through the electrometers it was at first found quite impossible to bring the two images together on the central division of the scale. The former difficulty was due to the varying condition of the surfaces of the spark balls of the oscillator and was to a great extent eliminated by rotating these balls at a high speed in opposite directions in a plane parallel to the spark gap. This operation apparently produced an effect similar to that of a strong air blast though the gap or of a strong magnetic field perpendicular to it and in the present instance was much more convenient than either of these devices. The latter difficulty was due to irregularities in the action of the interrupter to the induction coil that energized the oscillator. After extensive experiments with various well-known

forms of break, the following construction was found to give by far the most satisfactory results. A german silver wire, 0.8 mm. in diameter and 88 cm. long, was stretched between the poles of a powerful horseshoe magnet, and a platinum mercury contact was attached to its center and very near the magnet. The primary current of the induction coil was passed through this wire in such a direction that it was raised by the magnetic field and the circuit was broken. The tension of the wire could be varied at will through a long range by a guitar string winch, and the relative time of closed and open circuit could be varied by raising or lowering the mercury cup by a screw adjustment. The mercury was covered with alcohol, and a Willyoung adjustable condenser was connected in parallel with the contact. When the instrument was in good condition and properly adjusted no spark was produced at the break, and the interruption could be maintained regular for an indefinite time. This form of break has been used by several investigators, but I am uncertain who first introduced it.

With these appliances the two images could be adjusted to the center of scale within an error of one mm., but they could be held there only a short time. Calling A the observed angle of torsion for the electrometer containing the solution and B that for the other eletrometer the ratio A/B is proportional to the ratio of the forces acting on the needles when in the zero position and any change in this ratio when the solution is changed indicates a corresponding difference in the dielectric constants of the solutions. Distilled water of conductivity about 4×10^{-6} and ten copper sulphate solutions varying in concentration from 5×10^{-5} to 4×10^{-4} gram equivalents per liter were tested in this way. The values for A/B thus obtained were very irregular not only for the different solutions but to a slightly less extent for the same solution. The average values for the very dilute solutions showed a small irregular increase with the conductivity but when the concentration was greater than about 2×10^{-4} they were irregularly distributed about the mean value for distilled water. The temperature was so nearly constant during the series of observations that the irregularities could not be attributed to its variations and it seemed possible that they might be due to changes in the constant of the electrometer, caused by

completely emptying and refilling it between each set of observations, or to corrosion of the copper electrodes. These electrodes were consequently replaced by platinum ones of a similar construction and the solutions were made in the electrometer itself, by adding small quantities of a solution of copper sulphate to the water it contained, thus leaving the suspended system entirely undisturbed. After each addition of the salt sufficient time was allowed to elapse for the solution to become uniform before observations were undertaken. The results were similar to those before obtained and only slightly less irregular. They are graphically represented in Fig. 1

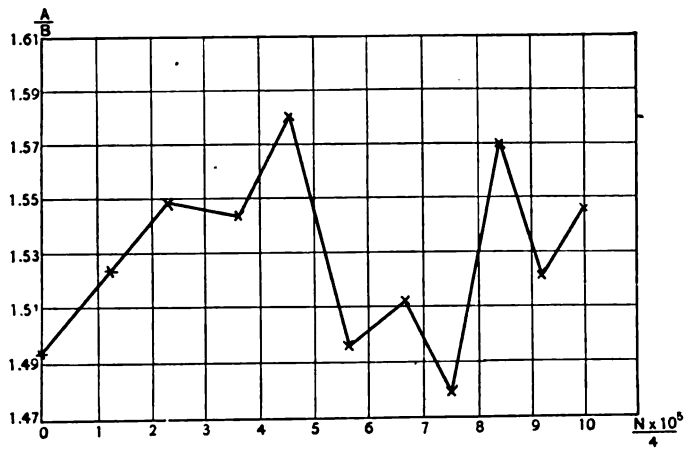


Fig. 1.

where the abscissas are proportional to the concentration N in gram equivalents per liter and the ordinates are proportional to the ratio A/B.

Although these results indicate a slight increase of dielectric constant with the concentration the method appears incapable of giving a trustworthy determination of the relation. This failure is believed to be due to several causes. In the first place the oscillator became less regular in its action as the conductivity of the solutions increased and frequently ceased to act altogether so that some of the observations may have been made with Ruhmkorff in place of Hertzian currents. In this case, on account of the small superficial area of the electrodes, an appreciable polarization may have been

produced. After some of the settings a slow drift of the image, from the electrometer containing the solution, was noticed that may have been due to the subsidence of such a polarization. At all events the direction of the drift was harmonious with such an explanation. Small convection currents in the air electrometer or slight brush discharges from the comparatively sharp edges of the needle may have introduced irregular errors. Furthermore the fixed and movable electrodes of this instrument were placed very near together in order to make its capacity nearly equal to that of the other electrometer and it is obvious that small errors in the position of the needle would produce relatively large errors in the observed force. Since the difficulty experienced in setting the needle in the zero position increased with the conductivity of the solution this source of error probably accounts for a large part of the observed irregularity. It is apparent that errors of setting would have had less effect on the result if electrometers of the quadrant type had been used in place of those of the attracted disk type. The latter type was adopted because I had been unable to devise a high frequency oscillator that would produce wave trains of sufficient energy to cause readable deflections with the quadrant form and with low frequencies polarization completely masked the results. A slight change in one of the oscillators already tried, without success, obviated this difficulty and after some further preliminary work the disposition of apparatus about to be described was adopted and found to give satisfactory results.

Fig. 2 is a diagram of the oscillating system and electrometer

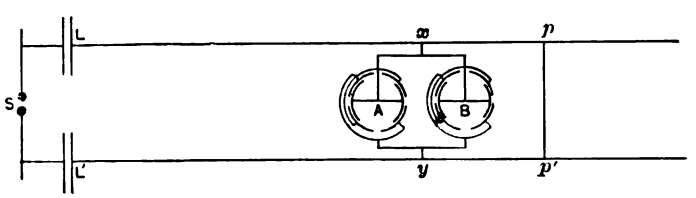


Fig. 2.

connections. Two large Leyden jars L, L' were placed on glass insulating stands and their outside coatings connected to an adjustable spark gap S. The parallel parts of these connections were brass rods, about 12 mm. in diameter, 20 cm. long and 10 cm.

apart, fitted with right angle clamps at their free ends that supported shorter rods 6 mm. in diameter and terminating in brass balls 2.3 cm. in diameter. The inside coatings of the jars were connected with two parallel copper wires, about eight meters long and 20 cm. apart, that extended in an inclined plane from the oscillator to the ceiling of the room. One pair of quadrants, the needle and the brass case of each of the electrometers A and B were connected together and to a point x on one of the parallel wires about 2.5 meters from L. The remaining quadrants were insulated from the case and connected to the point y on the other wire equidistant from L'. A bridge pp' was arranged to slide freely on the wires and its motion was controlled by a flexible cord passing over pulleys in the ceiling of the room and extending to the left hand end of the observing scale. When it was in contact with xy the observed electrometer deflections were very small, but as it was drawn away they gradually increased and finally exceeded the length of the scale. The frequency of electrical oscillation of the system was about 10^6 vibrations per second and it was not appreciably affected by the changes in position of the bridge necessary to produce any desired deflection within the limits of the scale. When first set up the inside coatings of the jars were connected to the spark gap and the outside coatings to the parallel wires, but with this arrangement the energy of the oscillations was so small or the damping so great that no deflection could be obtained when the conductivity of the liquid in either electrometer was greater than that of pure water. With the reverse connection appreciable deflections were produced in liquids of conductivity as high as that of a decinormal solution of copper sulphate. The cause of this great difference in action is not apparent, but it may be due to differences in effective insulation.

The two electrometers A and B were made as near alike as possible and each was filled with water, or a solution, so that their electrostatic capacities were nearly equal. The connecting wires for corresponding parts were of equal length and diameter and were so arranged that the two branches of the circuit had nearly the same self-induction. Each electrometer was supported by a brass bed plate, 15 cm. in diameter and 5 mm. thick, rigidly attached, by means of three leveling screws, to a substantial wooden frame that

was held in position on a brick pier by a heavy weight. A brass tube 2 cm. in diameter and 30 cm. long was supported over a hole in the center of the plate by a flange and shoulder, and carried at its upper end a torsion head from which the needle was suspended by a platinum rhodium wire about 20 cm. long and 0.05 mm. in diameter. A small rectangular window near the base of this tube was closed by plate glass and served to admit light to a small concave mirror on the suspended system. The fixed electrodes were in the form of shortened quadrants of a cylindrical surface 9 cm. in diameter and 3 cm. high and were supported by brass shanks 15. mm. in diameter that passed up through hard rubber plugs screwed into holes in the bed plate. The insulating spaces between their adjacent edges were about 4 mm. wide. The needle was bent to coincide with two opposite quadrants of a concentric cylindrical surface 7 cm. in diameter and 2 cm. high and the shank by which it was supported carried a small binding screw at its upper extremity by which it was clamped to the lower end of a short brass wire soldered to the suspending wire. Both cylindrical surfaces were normal to the bed plate and their axes coincided with the suspension. The needle and electrodes were cut from sheet German silver 0.2 mm. thick and were thoroughly plated with platinum and platinized before being bent into shape. They were so constructed that all soldered connections came above the surface of the liquid in the electrometer. A glass bell-jar, about 11 cm. in diameter and 700 cc. capacity, with a stopcock in its closed end was clamped in an inverted position against the under surface of the bed plate and surrounded by a cylindrical copper vessel that served to shield the electrodes from external electrostatic influences and to convey a current of water from the city mains for the purpose of regulating the temperature when necessary. Generally, however, the temperature of the room was more constant than that of the water and the latter device was seldom used. The temperature was indicated by a mercury thermometer supported by the bed plate with its bulb very near the fixed electrodes and as far as possible from the needle.

The suspensions of the two electrometers were 30.5 cm. apart and the line joining them was parallel to and 177.8 cm. in front of an observing scale, 150 cm. long, supported by a wooden stand on

a brick pier and so placed laterally that a normal erected to its central division would pass midway between the electrometers. Directly above this normal a fine steel wire was stretched vertically across a small window in the supporting stand and illuminated by an incandescent electric lamp and a condensing lens. The concave mirrors on the suspensions were so adjusted that the two images of this wire were superposed on the central scale division when the electrometer needles were symmetrically situated with respect to the fixed electrodes. The images were very sharp and slightly less in width than the division marks of the scale, so that readings could be easily made to fractions of 1 mm. when desirable. The motion of each torsion head was controlled by an endless belt of fine steel wire that engaged a wooden lever arm attached to the head and, after passing over a system of fixed pulleys, circulated several times about a small wooden reel attached to the scale support. The belts were kept taut by brass spiral springs and worked so smoothly when the reels were turned that no tremor of the images were observed during the motion. The stretched wire interrupter described above, the condenser, and the switch for the primary circuit of the induction coil were placed within easy reach of the observer at the scale. The electrometers and scale were enclosed in a tent of dark cloth supported by a wooden frame attached to the floor of the room and mechanically insulated from the brick piers.

The relative conductivity of the solutions used was determined by Kohlrausch's telephone method with the aid of two pairs of small platinum test electrodes similar to the "Tauchelektroden" for poorly conducting liquids, described by Kohlrausch and Holborn.[1] These electrodes were supported by the bed plates of the electrometers, in a position diametrically opposite to the thermometers, and were insulated from them by glass tubes covered with paraffin. They were kept in a fixed position throughout the investigation and could be insulated or connected to either arm of the resistance bridge by suitably placing copper connectors in a mercury commutator constructed from a large block of paraffin.

After carefully cleaning the electrodes and bell-jars the electrometers were assembled and filled with distilled water until its sur-

[1] Kohlrausch and Holborn, "Leitvermögen der Elektrolyte," edition of 1898, p. 18.

face was about one cm. above the top edges of the electrodes and 3 cm. below the bed plate. Determinations of the conductivity of the water showed that it increased about 30 per cent. per day and this was judged to be due to imperfect removal of the salts of the plating bath from the surface of the platinized electrodes by the extended washing in hot and cold water to which they had been subjected. Consequently the electrometers were allowed to remain undisturbed about one week and were then refilled with fresh distilled water. This operation was repeated at nearly equal intervals for three months and measurements then show that the rate of increase in conductivity had been greatly reduced. Nearly equal volumes of all the solutions investigated were used so that the free surface always occupied the same position and produced the same effect on the measurements.

The method of operation in taking the observations was as follows: The image of the illuminated wire due to one of the electrometers (say B) was set at a convenient position on the scale. The interrupter was started and when its pitch became regular the bridge pp' was moved along the parallel wires till the image B came to the central scale division (75th in this instance), keeping B stationary by slight back and forward motions of the bridge, corresponding to the irregularities in the intensity of the oscillating current, the image from electrometer A was brought into coincidence with it by turning the proper torsion head. This adjustment could be easily made to a small fraction of a millimeter and maintained constant for an indefinite time by slight motions of the bridge. The primary circuit of the induction coil was then broken and after the images had come to rest their position on the scale was recorded. Observations of the temperature and conductivity of the solutions in the electrometers were made and recorded. From six to ten sets of observations were taken in this manner, starting each time with the image from B in a different position, in order to obviate prejudice in making the settings. About 2 cc. of a centinormal solution of copper sulphate were then added to the water in electrometer A and the apparatus allowed to remain undisturbed for about twenty-four hours in order that the salt might become uniformly distributed. On succeeding days similar series of observations were made and the concentration

of the solution was increased by successive additions of from 2 cc. to 4 cc. of centinormal or decinormal solutions of copper sulphate until six different solutions had been tested. The solution in A was was then kept constant and eight different copper sulphate solutions were formed in B and tested in a similar manner. After this series of observations B was emptied, thoroughly rinsed out and refilled with fresh distilled water. Similar sets of observations were made on this water and on five different potassium chloride solutions formed as described above. It will be noticed that in the first series of observations the solutions in A were compared with water in B, while in the last two series the comparison was between various solutions in B and a constant solution in A. This procedure was adopted since it was found that the conductivity of the dilute solution in A was much more constant than that of the water.

When the two images were superposed on the 75th scale division the needles of the electrometers were symmetrically situated with respect to the fixed electrodes and the electrostatic forces acting on them were balanced by the torsion of the suspending wires. Since the displacements were always small the torsional couples produced by the wires were proportional to the angles through which they were twisted and when the current was turned off the images were deflected along the scale a distance depending on the magnitude of the torsion. The formulæ ordinarily used in reducing scale readings to angular measure are obviously inapplicable when the apparatus is arranged as above described since the normals to the minors are not perpendicular to the scale when the images are superposed on the 75th division. It is, however, easy to show that the angle of torsion δ_a corresponding to a deflection of the image from electrometer A amounting to x_a scale divisions is given by the expression

$$\delta_a = \tfrac{1}{2} \tan^{-1} \frac{S}{1 + T\frac{x_a}{A^2}} \cdot \frac{x_a}{A^2}$$

where S is the normal distance between the scale and the line joining the electrometer suspensions, A the distance from the minor to the 75th scale division, T one half the distance between the suspen-

sions, and x_a is measured on the opposite side of the normal to the 75th division to that on which the electrometer is situated. Similarly for a deflection x_b on the same side of this normal as electrometer B.

$$\delta_\beta = \tfrac{1}{2} \tan^{-1} \frac{S}{1 - T\frac{x_b}{A^2}} \cdot \frac{x_b}{A^2}.$$

Expanding these expressions in ascending powers of x/A^2

$$\delta_a = \frac{S}{2A^2} x_a - \frac{ST}{2A^4} x_a^2 - \frac{S(S^2 - 3T^2)}{6A^6} x_a^3 + \cdots$$

$$\delta_\beta = \frac{S}{2A^2} x_b + \frac{ST}{2A^4} x_b^2 - \frac{S(S^2 - 3T^2)}{6A^6} x_b^3 + \cdots$$

From the dimensions of the apparatus it follows that $S = 177.8$ cm., $T = 15.25$ cm. and $A = 178.5$ cm. Substituting these values

$$\delta_a \times 10^4 = 27.9\, x_a - .0134\, x_a^2 - .000283\, x_a^3$$

$$\delta_\beta \times 10^4 = 27.9\, x_b + .0134\, x_b^2 - .000283\, x_b^3.$$

A table of the values of $\frac{1}{2}\delta_a \times 10^4$ and $\frac{1}{2}\delta_\beta \times 10^4$ was calculated from these formulæ for values of x_a and x_b varying by one or two centimeters from zero to 60 cm. and the results were plotted to form curves with ordinates proportional to the angles and abscissas proportional to the deflections. The reduction of the observations was made with the aid of these curves and a series of calculations for points well distributed along the curves showed that the results obtained by this method never differed by so much as one tenth of one per cent. from the results calculated directly from the formulæ.

Since the two electrometers were symmetrically connected to the same points of the parallel wires they were acted upon by equal differences of potential when the oscillator was in action. Hence, if δ_a and δ_β represent the observed angles of torsion when the dielectric constants of the contained liquids are κ_a and κ_β respectively it is obvious that

$$\frac{\delta_a}{\delta_\beta} = \frac{C_a \kappa_a}{C_\beta \kappa_\beta} \qquad (1)$$

where C_α and C_β are constants depending on the dimensions of the electrometers and the elastic properties of the suspending wires. Similarly for angles δ_α' and δ_β' corresponding to liquids of dielectric constant κ_α' and κ_β',

$$\frac{\delta_\alpha'}{\delta_\beta'} = \frac{C_\alpha \kappa_\alpha'}{C_\beta \kappa_\beta'}. \tag{2}$$

Hence dividing (2) by (1)

$$\frac{\delta_\alpha'}{\delta_\beta'} \cdot \frac{\delta_\beta}{\delta_\alpha} = \frac{\kappa_\alpha'}{\kappa_\alpha} \cdot \frac{\kappa_\beta}{\kappa_\beta'} \tag{3}$$

from which either ratio $\kappa_\alpha'/\kappa_\alpha$ or $\kappa_\beta/\kappa_\beta'$ can be determined when the other is known.

Owing to slight differences in the temperature of the liquids experimented upon it become necessary to reduce the observations to some standard temperature and the necessary corrections were calculated in the following manner. Write

$$\kappa_\alpha = \kappa(1 + \beta_\alpha t_\alpha)$$

where κ_α is the value of κ when the liquid is at temperature t_α and

$$\beta_\alpha = \frac{1}{\kappa} \frac{\partial \kappa}{\partial t}.$$

Similar expressions for the other κ's may be written

$$\begin{aligned} \kappa_\alpha' &= \kappa'(1 + \beta_\alpha' t_\alpha') \\ \kappa_\beta &= \kappa_0(1 + \beta_\beta t_\beta) \\ \kappa_\beta' &= \kappa_0'(1 + \beta_\beta' t_\beta'). \end{aligned}$$

Since the temperature differences were small and the liquids used were all water or moderately dilute solutions, the temperature coefficients β_α, β_α', etc., were assumed to be the same and equal to that for pure water at the mean temperature of the observations. Granting this assumption

$$\frac{\kappa_\alpha}{\kappa_\beta} = \frac{\kappa}{\kappa_0}\left\{1 + \beta(t_\alpha - t_\beta)\right\}$$

and

$$\frac{\kappa_\alpha'}{\kappa_\beta'} = \frac{\kappa'}{\kappa_0'}\{1 + \beta(t_\alpha' - t_\beta')\}.$$

Hence (3) becomes

$$\frac{\delta_\alpha'}{\delta_\beta'} \cdot \frac{\delta_\beta}{\delta_\alpha} = \frac{\kappa'}{\kappa} \cdot \frac{\kappa_0}{\kappa_0'} \cdot \frac{1 + \beta(t_\alpha' - t_\beta')}{1 + \beta(t_\alpha - t_\beta)}. \tag{4}$$

When B was used as the comparison electrometer the liquid in it was left undisturbed and if we neglect the possibility of small changes in its dielectric constant due to a slight increase in conductivity we have as a first approximation $\kappa_0 = \kappa_0'$ and

$$\frac{\kappa'}{\kappa} = \frac{\delta_\alpha'}{\delta_\beta'} \cdot \frac{\delta_\beta}{\delta_\alpha} \cdot \frac{1 + \beta(t_\alpha - t_\beta)}{1 + \beta(t_\alpha' - t_\beta')} \tag{5}$$

where κ' is the dielectric constant of the solution experimented upon and κ that of the water from which it was made. β was taken equal to $-.004$ which is very nearly the mean of the best published determinations of this constant for temperature changes similar to those observed. When the functions of the two electrometers are reversed and A is used as comparison electrometer the reduction formula is the same as (5) with the α's and β's interchanged.

After the first series of observations had been made and reduced it was found that the value of the ratio $\delta_\alpha/\delta_\beta$ depended on the manner in which the test electrodes were connected to the resistance bridge. This effect was undoubtedly due to currents induced in the connecting wires by the oscillating current in the electrometer circuit and the following observations show that it was constant so long as the connections were unchanged. Three sets of connections were used, P_1, test electrodes in A and B connected to adjacent bridge arms; P_2, electrodes in A and a rheostat similarly connected, and P_3, electrodes in B and a rheostat similarly connected. When both electrometers were filled with water at temperature 25.°7 the observations lead to the values of $\delta_\alpha/\delta_\beta$ given in Table I.

TABLE I.

Connection.	P_1	P_2
$\frac{\delta_\alpha}{\delta_\beta}$	1.045	1.031
	1.044	1.033
	1.047	1.037
	1.048	1.035
	1.045	1.033
Mean	1.046	1.034

The scale deflections from which these results were calculated range from $x_a = 29.1$ cm. to $x_a = 56.9$ cm. and from $x_b = 27.4$ cm.

to $x_b = 51.5$ cm. Unfortunately this set of observations did not involve the connection P_2 and the corresponding value of $\delta_\alpha/\delta_\beta$ could only be obtained by calculation from the mean value of $\delta_\alpha'/\delta_\beta'$ corresponding to a solution for which the ratio κ'/κ had been determined from the observations with connections P_1 and P_3. The solution for which the errors of observation were smallest was adopted for this purpose and the result thus obtained was $\delta_\alpha/\delta_\beta = 1.05$ for connection P_2. Table II. exhibits the observations and calculated results for a solution of copper sulphate of conductivity 3.86 times that of the water from which it was made. The letters in the column headed C indicate the connection of the test electrodes that existed when the corresponding scale deflections x_a and x_b were observed and the given values of κ'/κ were calculated from $\delta_\alpha'/\delta_\beta'$ with the acid of the value of $\delta_\alpha/\delta_\beta$ for corresponding

TABLE II.

x_a	x_b	$\delta_\alpha'/\delta_\beta'$	C	κ'/κ
56.3	51.2	1.037	P_2	.987
53.7	49.6	1.024	P_3	.990
51.6	46.6	1.050	P_2	1.000
47.2	43.6	1 033	P_3	.999
44.8	40.7	1.054	P_2	1.002
40.7	37.8	1.034	P_3	1.000
39.3	35.9	1.050	P_2	1.000
35.8	33.3	1.036	P_3	1.003
34.1	31.3	1.049	P_2	.999
39.5	36.8	1.033	P_3	.999

connections. The mean error of the individual values of κ'/κ, calculated from the average value in the usual manner, is .0052 and since this is somewhat greater than the average of the mean errors for the other sets of observations, the results given in the table convey a fair idea of the general accuracy of the observations and reductions.

During the last two series of observations, A was used as comparison electrometer, and contained a dilute solution of copper sulphate. The test electrodes were always insulated while the electrometer settings were made, and hence but one value of $\delta_\alpha/\delta_\beta$ was obtained when B was filled with distilled water. More or less ex-

tensive sets of observations on various solutions of copper sulphate and potassium chloride lead to the results given in Table III. The

TABLE III.

D	S	t_1	t_2	λ_1	λ_2	κ'/κ	E	N
2	$CuSO_4$	26.0	26.0	1.11	1.69	.998	.0034	8
3	"	26.3	26.3	1.40	2.92	1.010	.0037	8
4	"	25.9	26.0	1.61	3.86	.998	.0052	10
5	"	25.2	25.3	1.74	4.68	1.003	.0028	10
6	"	24.3	24.3	1.73	4.64	1.002	.0053	6
8	"	23.9	23.9	2.00	5.63	1.009	.0021	9
9	$CuSO_4$	23.8	23.8	1.04	2.95	1.000	.0024	10
10	"	22.9	22.9	1.04	3.62	1.004	.0023	10
11	"	23.1	23.1	1.12	4.29	1.006	.0028	8
12	"	23.0	23.1	1.16	7.07	1.003	.0052	8
13	"	22.1	22.5	1.15	10.62	1.002	.0046	7
15	"	23.6	24.0	1.27	18.45	.986	.0177	4
16	"	24.9	25.5	1.28	19.74	.994	.0059	7
17	"	25.2	25.9	1.30	32.45	1.007	.0180	6
19	KCl	25.3	25.3	1.10	3.87	.997	.0047	8
20	"	24.8	25.0	1.15	7.65	.986	.0023	6
22	"	25.1	25.7	1.25	11.68	.990	.0056	6
23	"	25.3	25.8	1.27	14.70	.989	.0046	6
24	"	25.5	25.9	1.31	20.79	.989	.0141	6

independent series of observations are separated by horizontal spaces, and the days of July, 1901, on which the observations were made are indicated in column D. The salts in the solutions examined appear under S. t_1 is the temperature of the solution in the comparison electrometer, and t_2 that of the solution in the test electrometer. λ_1 is the conductivity of the former solution relatively to its value when the ratio $\delta_\alpha/\delta_\beta$ was determined, and λ_2 is the conductivity of the latter solution relatively to that of the water from which it was made. κ'/κ is the mean of N independent determinations of the ratio between the dielectric constants of the latter solution and distilled water, and E is the mean error of the individual values of κ'/κ These results were derived from the observations by equation (5), on the assumption that the dielectric constant of the liquid in the comparison electrometer remained constant, except for temperature changes, during the several series of observations. Hence, if κ'/κ

had been found to vary with λ_2 corrections would have been necssary on account of the observed variationin λ_1. But, since λ_2 varied much more rapidly than λ_1 and the variations of κ'/κ from unity were of the same order of magnitude as the errors of observation, this correction becomes unnecessary. The uniformly low values obtained in the third series are probably due to an error of about one per cent. in the determination of the ratio $\delta_\alpha/\delta_\beta$ when the test electrometer contained distilled water.

The conductivity of the distilled water from which the solutions were made, expressed as the reciprocal of the resistance in ohms of a column one centimeter long and one square centimeter in section, was about 7×10^{-6}. Consequently the maximum conductivity of the solutions tested was not over 2.5×10^{-4} which, for the copper sulphate solutions, corresponds to a concentration of about 0.003 gram equivalents per liter.

Considering the low conductivity of the solutions, the large area of the electrodes, and the high frequency of the oscillating current, the polarization might be expected to be vanishingly small and in fact the observations gave no evidence of such a disturbance. The electrometer needles were nearly dead beat and the images from the mirrors on the suspensions always came to rest in fifteen or twenty seconds without the least sign of drift due to subsiding polarization. During the adjustment of the images to the central scale division no gradual variation of the forces acting on the needles such as would be produced by accumulating polarization was observed. On account of irregularities in the action of the oscillator some of the settings occupied much more time than others and hence offered an opportunity for polarization to produce an increased effect, but the results calculated from these observations never differed from those occupying less time by more than the ordinary errors of observation.

When the images had come to rest after the current was turned off they remained perfectly steady so long as the apparatus was left undisturbed. Consequently the convection currents in the liquids must have been inappreciable.

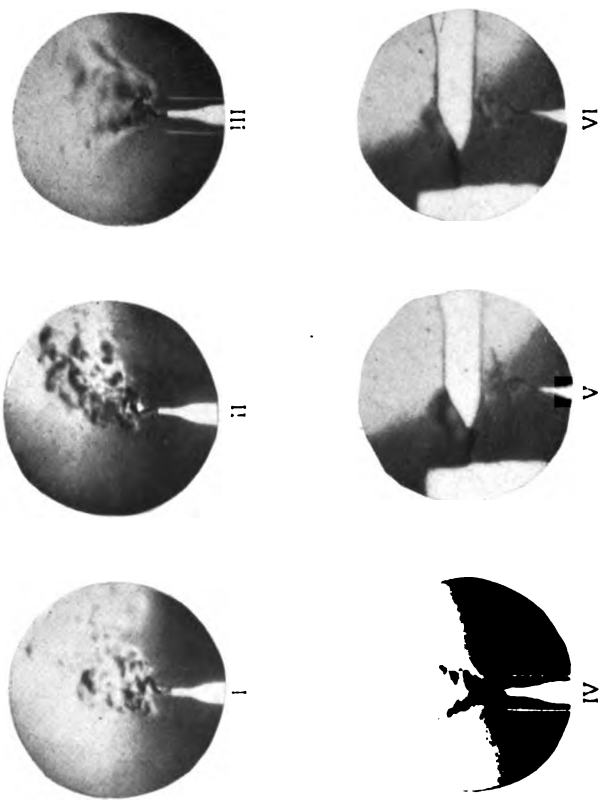
In order to determine whether the deflections were influenced by changes in the torsional rigidity of the suspending wires due to the

heating effect of the currents they transmitted the period of torsional vibration of the needle of electrometer A was determined by comparison with the pendulum of the laboratory clock with the aid of a double chronographic record. The mean of a large series of observations gave 5,965 seconds when no current was passing through the suspension and 5,967 seconds when it was conducting a current larger than that used during any of the electrometer settings. Since the difference between these values lies within the probable errors of observation it is evident that the heating effect was negligible.

An inspection of the values of κ'/κ given in Table III., is sufficient to show that they do not differ from unity by greater amounts than might have been expected if distilled water had been used in the electrometers throughout all the observations in place of the solutions actually used. Hence it is highly probable that the dielectric constant of dilute electrolytic solutions, of conductivity less than 2.5×10^{-4}, is the same as that of pure water in electric fields varying at the rate of 10^6 oscillations per second.

It is obvious that the degree of electrolytic dissociation of such solutions is very large and the observed lack of change in dielectric constant may be due to this fact but the point cannot be definitely decided until determinations have been made with much more concentrated solutions. Some such observations have been made but they are not sufficiently accurate to lead to definite results and I shall therefore defer the discussion of them until I have had time to carry out a more thorough investigation of the subject with an apparatus modified to render it more adaptable to measurements with highly conducting liquids.

PHYSICAL LABORATORY, BROWN UNIVERSITY,
PROVIDENCE, R. I., Nov. 30, 1901.



TUFTS: ACTION OF SOUND WAVES ON UNIGNITED GAS JETS.

A STUDY OF THE ACTION OF SOUND WAVES ON UNIGNITED JETS OF GAS.

By F. L. Tufts.

IN this work the Schlerin method of investigation, used and described by Toepler, C. V. Boys, R. W. Wood and others, was applied to the study of the behavior of unignited jets of gas when subjected to the action of sound waves. Photographs were taken, giving a number of instantaneous positions of a jet of unignited illuminating gas when acted on by the sound waves from an ordinary organ pipe. The orifice from which the gas issued was rectangular in shape and was 4.0 mm. by 0.5 mm. in cross section. It was placed about 6 cm. from the open end of an organ pipe which spoke a fundamental note of 100 vibrations per second. A thin rubber diaphragm was placed across the inside of the pipe at the node of the fundamental note, and thus protected the jet from the action of direct currents of air while it did not interfere with the speaking of the pipe. The jet tube was placed so as to expose the broad side of the gas stream to the to-and-fro motion of the air particles in the sound wave.

The photographs taken showed the sinuous shape of the jet near the orifice, the fan-shaped spreading of the gas particles as they recede from the orifice and, for low velocities of efflux, the division of the jet into two.

Plates I. to IV. are reproductions of some of the photographs taken. Plates I. and II. show two successive positions of the gas stream. In Plates III. to IV. the velocity of efflux was less and the tendency of the stream to divide into two is evident. This is particularly prominent in Plate IV. The sinuous structure of the stream near the orifice is noticeable in all of the photographs, and in most of them traces of it can be found even in the diffused upper portions. The disturbed condition shown in the upper portions of

Plates I. and II. is probably due to air currents in the room. The two pins, shown at the sides of the jet tube in Plates III. and IV., are just one centimeter apart and serve to give an idea of the actual magnitude of the phenomena.

A simple geometrical construction, shown in Fig. 1, gives positions for the gas stream which agree well with the actual positions shown in the photographs. The assumptions upon which the construction is based are the following:

1. Each particle of gas as it issues from the tube moves forward in a straight line, and with a velocity which is the resultant of the combination of the velocity of efflux of the particle and the instantaneous value of the harmonically varying velocity of the air particles about the jet at the instant the gas particle issues from the orifice.

2. This resultant velocity decreases in value as the gas particle recedes from the orifice.

In Fig. 1 the ordinate, *jd*, represents the velocity of eflux of the gas particles. The abscissæ, *dg*, *df*, *de*, *dc*, etc., represent suc-

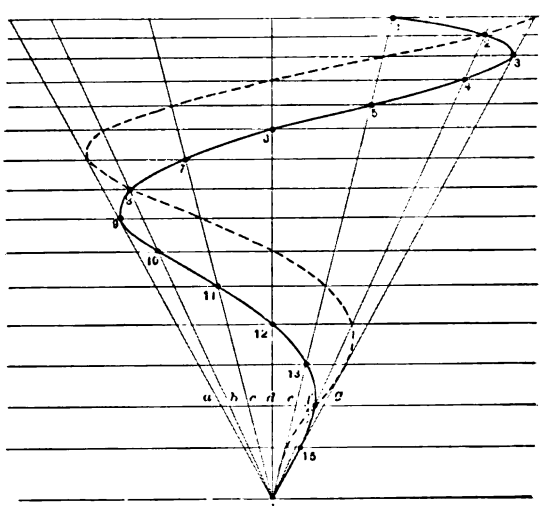


Fig. 1.

cessive values of the harmonically varying velocity of the air particles, due to the sound wave. The lines *jg*, *jf*, *je*, etc., represent the resultant velocities of successive gas particles as they issue from

the orifice. The decrease in the velocity of the gas particles on receding from the tube is taken account of in the spacing of the horizontal lines. The lengths of a radial line intercepted between successive horizontal lines are assumed to be proportional to the velocities with which the gas particle moves as it recedes from the tube.

Suppose the organ pipe used gives a fundamental note of 100 complete vibrations per second, and it is desired to find the positions of successive gas particles following one another from the orifice at intervals of 1/1200 of a second. An inspection of the figure will show that if the distance traveled by a given particle during 1/1200 of a second after leaving the orifice is represented by the line, *j* 15, then the particles which preceded this one at intervals of 1/1200 of a second will occupy the positions marked 14, 13, 12, 11, etc. A curved line connecting these points represents the position of the gas stream at a given instant. Two twelve-hundredths of a second later the position of the gas stream will be that of the dotted line in the figure. The curves thus obtained are very similar to those actually observed in the photographs.

This construction also explains the apparent division of the gas stream into two when the velocity of efflux is small. A low velocity of efflux would be represented by a closer spacing of the horizontal lines in the figure. Under such conditions it is evident that the gas particles would be crowded closer together along the radial lines *jf*, and *jb*, than along any other radial lines and would thus give the gas stream the appearance of dividing into two. The photographs which show a division of the gas stream, show traces in each jet of a structure like that at 2, 3, 4, in the figure.

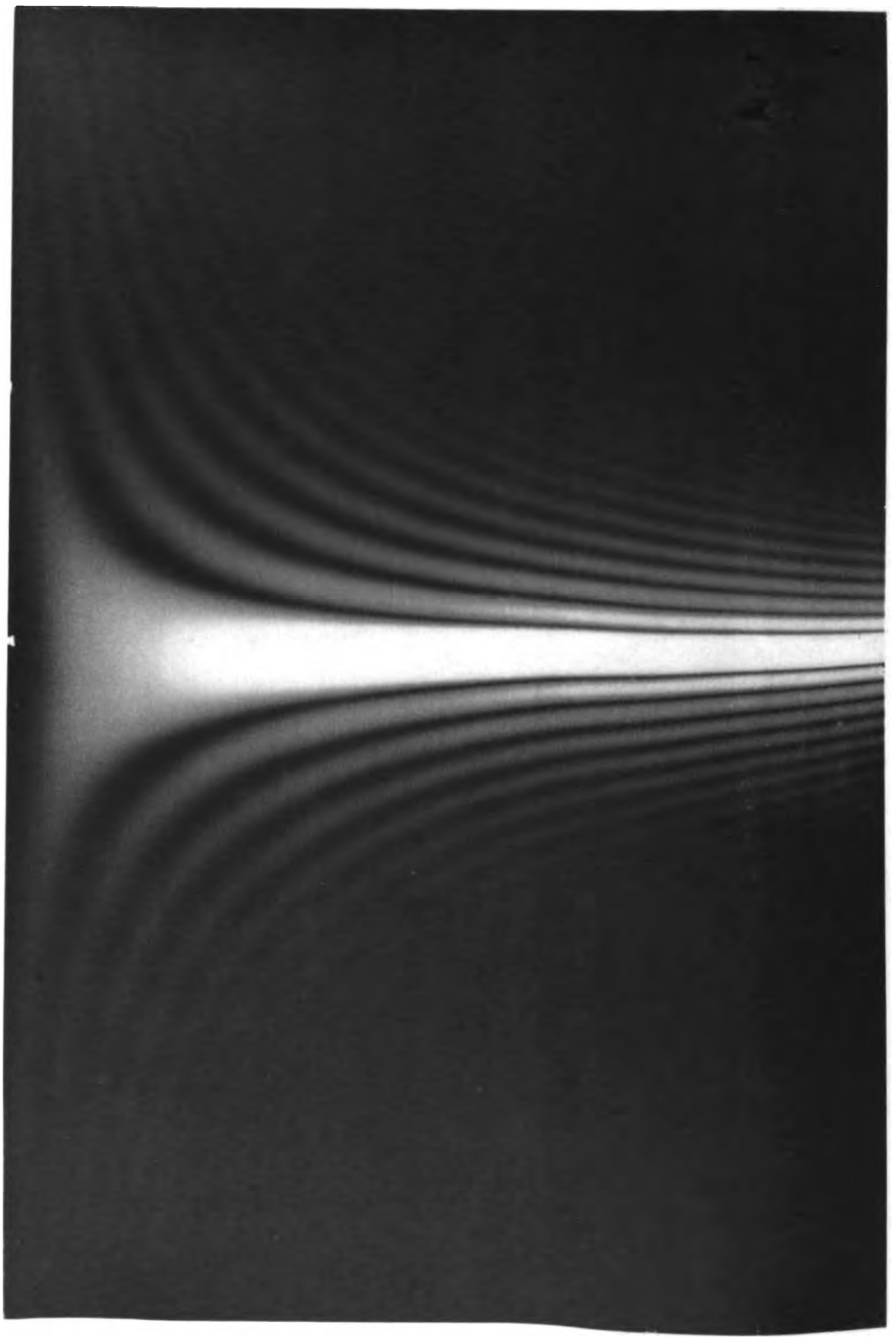
If the source of sound contains over-tones which are at all comparable in intensity to the fundamental, a construction similar to that of Fig. 1, will account for the division of the gas stream into three or more jets, a phenomenon which has been noticed.

In a number of cases the velocity of efflux of the gas from the orifice was measured by means of a gas meter, the pitch of the organ pipe was determined, and the angle *ajg*, Fig. 1, was measured on the photograph of the gas stream. From these data calculations of the amplitude of vibration of the air particles in the sound

wave were made, based on the geometrical construction given above. The results agreed very well with those obtained by other methods.

The Schlerin method was also employed in studying the vibrations of the tongue of air in the mouth of a speaking organ pipe. A stopped flute pipe of rectangular cross section was used. The two sides of the pipe were of plate glass and the air with which the pipe was blown passed over a surface of ether before entering the pipe, thus changing its optical density and making it readily visible in the Schlerin apparatus. A number of instantaneous photographs were taken showing different positions of the tongue of air in the mouth of the pipe. The direction of the vibration within the pipe was obtained by introducing a small jet of unignited illuminating gas in the pipe near its mouth and photographing it on the same plate with the tongue of air.

Plates V. and VI. are copies of two of the photographs showing the tongue of air in the two extreme positions of its swing. The only portions of the pipe shown in the photograph are the lip and a part of the butt containing the air slot. The curve of the gas stream serves to indicate the direction of the vibration within the pipe.



FRANKLIN: DIFFRACTION PHOTOGRAPHS.

SOME DIFFRACTION PHOTOGRAPHS.

By W S. Franklin.

THE accompanying plate is reproduced from one of a number of photographs of shadows cast upon photographic plates from a monochromatic point-source of light distant about sixteen meters from the plate. The objects casting the shadows were about three meters from the plate, and the photographic plate was in each case at right angles to the incident light.

Diffraction photographs were made in this way in the shadows of a number of different objects, with exposures ranging from twenty seconds to seven hours. Among the photographs obtained may be mentioned (1) the shadow of a broad straight-edged plate; (2) the shadow of a steel rod; (3) the shadow of five circular discs. These discs were suspended upon a network of fine wires, each 0.004 cm. in diameter. The shadows of the wires show in the photograph; (4) the shadow of a brass plate with several circular holes bored through it. These photographs, while well suited for lantern projection, are not sufficiently strong for reproduction.

Photograph No. 5, which is reproduced in the accompanying plate, is the shadow of a slit between very carefully worked straight edges of brass. The slit was about 15 centimeters in length, tapering from a width of 0.05 centimeter at the end *a*, to zero width at the end *p*. The original negative was 20×25 cm.

Lehigh University, South Bethlehem, Pa., April, 1901.

NOTE.

Rudolph Koenig.—This well-known authority in the science of acoustics died in Paris on the second of October, 1901. Koenig was the son of a teacher in mathematics at Koenigsberg, in which city he was born on the 26th of November, 1832. At the age of twenty he went to Paris and learned the trade of musical instrument maker under the celebrated maker of violins, Vuillaume. In 1859 he set up for himself as a manufacturer of acoustical instruments.

At the London exhibition of 1862 Koenig first made public his beautiful apparatus for the study of sound waves by means of the manometric flame. The applications of this method to various problems in the analysis of complex tones were subsequently published in three papers in Poggendorff's Annalen. The apparatus constructed in his workshop at Paris was largely of his own design and the experiments which he himself performed with it, and which he described in a series of papers, chiefly in the *Annalen* and in the *Comptes Rendus* of the French Academy, are among the most important contributions to the science of experimental acoustics. The results of his work in this field were in 1882 brought together in his well-known volume entitled "Quelques Expériences D'Acoustique."

Among the best known of Koenig's investigations may be mentioned his study of the fixed tones characteristic of the vowels (1870); his various papers on the synthesis of musical tones; his researches on the theory of the nature of summation and difference tones and his studies of the movement of air in organ pipes. Koenig was fortunate in being able to bring to bear upon his truly scientific work training of two sorts. His knowledge of acoustical theory enabled him to see definitely the nature of the problems demanding solution and his skill as a mechanician made it possible to produce apparatus perfectly adapted to the experimental solution of these problems. To his chosen field he devoted himself absolutely throughout his long career and even during the later part of the nineteenth century, when the interest of physicists was directed so strongly to electricity that the science of sound seemed likely to be altogether submerged and lost from sight, he continued to devote himself to the work which he had elected to do. Koenig's last investigation, completed in spite of failing health and of the growing disabilities of increasing age, was on the "Highest Audible Tones and upon the Inaudible Tones up to 180,000 Vibrations per Second." To the accurate adjust-

ment and tuning of forks and other instruments far beyond the limits of audibility, by means of their difference tones and by the observation of dust figures, he brought to bear the skill resulting from long years of practical experience, during which he had with his own hand made the final adjustment of every instrument constructed in his establishment.

Koenig died in his apartments over the workshops on the Quai d'Anjou, which had for many years been the scene of his scientific activity.

E. L. N.

NEW BOOKS.

Advanced Exercises in Practical Physics. By ARTHUR SCHUSTER, Ph.D., F.R.S., and CHARLES H. LEES, D.Sc. The Macmillan Co. $2.00. Pp. 368.

This book is a very welcome addition to our rapidly increasing list of laboratory manuals. Intended as a sequel to "Intermediate Physics" by the same authors, it is designed for a second year's course in physics.

It is more advanced than most of the manuals now in use in this country but whether it is advanced enough for a second year's course depends in each case on the amount and quality of the laboratory work of the first year. The experiments given had been worked through by several hundred students in Owens College before the book was published.

In general the methods used are accurate and thorough. One of the strongest features of the book is the careful explanation of errors of observation. The student is frequently shown the effects of errors of observation on a derived result, and his attention is continually directed towards measuring accurately those quantities whose errors would exert the greatest effect. Too many students, perhaps through lack of proper instruction, fail to form any idea of how accurate their result should be, even when they know how accurately each quantity has been measured.

Comparatively few errors are noticed. The first sentence in the paragraph beginning "The fractional error * * *" on page 6 needs a slight revision. On page 14 we find the authors saying, "the two first and the two last observations." The work on mechanics is so good that one is surprised in not finding more on the pendulum. Not a single experiment is given for determining g.

Some manuals seemed designed to teach the principles of Physics, while others are designed to teach laboratory methods. The present work is a happy combination of the two. However, a course based on this book should by all means be supplemented by lectures, problems, and quizzes. In the rapid development of laboratory work there has been a tendency to carry it too far. We find that in some of our schools and colleges practically all of the student's time that is devoted to Physics is spent in experimental work. In doing this the theoretical side is almost entirely neglected. The student learns manipulations, but does he learn Physics? Few students can get a working knowledge, for example, of Ohm's law and its various applications simply from the routine laboratory work and the instructions found in any manual. Laboratory courses should always be supplemented by other work.

O. M. STEWART.

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THE

PHYSICAL REVIEW.

THE VELOCITY OF IONS DRAWN FROM THE ELECTRIC ARC. SECOND ARTICLE.

BY C. D. CHILD.

AN investigation of the velocity of ions drawn from the electric arc has been described in a previous article. Further experiments on this subject have since been performed and the results may be of interest to others. Two methods have been employed to compare the velocity of the positive and negative ions, one similar to that used by Zeleny in his investigation of the velocity of ions produced by X-rays,[1] and one similar to that used by Rutherford in determining the velocity of ions produced by ultra-violet light.[2]

Former Method gave the Average Velocity.—First of all, it should be noticed that the method which has already been used gives the average velocity when there are ions present having different velocities. The fact that we need to make a distinction between the average velocity and the velocity of the more rapidly or more slowly moving ions has been emphasized by the work to be described in this article and in one to be published later on the discharge from hot wires.

The discharge carried by ions of one sign only between two infinite parallel planes has already been considered.[3] It was assumed in that discussion that all the positive ions had the same velocity. It was shown that if there are n_1 positive ions per unit volume having a velocity of k_1 cm. per unit potential gradient, $k_1 = \frac{8\pi ix}{X^2 - C^2}$

[1] Phil. Trans. Roy. Soc. Lon., 195, 193.
[2] Proc. Camb. Phil. Soc., 9, 401.
[3] PHYS. REV., XII., 65.

where X is the electric intensity at the point, i the current through unit cross section, x the distance from the plane at which the ions are produced, and C the value of X where x is zero. If there is an indefinitely large supply of ions at the plane where $x = 0$, $C = 0$ and $k_1 = \frac{32\pi i x^3}{9V^2}$, where V is the potential difference between the plane at which the ions are produced and the point considered.

If now the current is carried by two kinds of ions, n_1 ions having a velocity of k_1, and each carrying a charge e_1, and n_3 ions having a velocity of k_3 and each a charge of e_3, we would have

$$\frac{dX}{dx} = 4\pi(n_1e_1 + n_3e_3) \quad \text{and} \quad i = X(k_1n_1e_1 + k_3n_3e_3).$$

It is not possible to eliminate $(n_1e_1 + n_3e_3)$ between these equations. If, however, we assume $K_1 = \frac{k_1n_1e_1 + k_3n_3e_3}{n_1e_1 + n_3e_3}$ we have the same value for K_1 as we previously found for k_1. If there are several kinds of ions, we may assume $K_1 = \frac{\Sigma k_1n_1e_1}{\Sigma n_1e_1}$ and we have $K_1 = \frac{8\pi i x^3}{X^2 - C^2}$, or if $C = 0$, $K_1 = \frac{32\pi i x^3}{9V^2}$.

Similarly in the case of discharge between cylinders, if there are several kinds of ions, we may make the same assumptions and K_1 would appear in the formulæ wherever we now have k_1. We may call this K_1 the average velocity. This is evidently what has been found by the method already used. It is not necessarily the same as that which will be found by other methods.

First Modification of Zeleny's Method.—In Zeleny's method a stream of gas is blown between two cylinders and the velocity of the ions is determined by a comparison of the distance through which the ion is carried by the gas with that through which it is drawn by the electric field. His method must be modified because in the present case the ions are not produced uniformly in a region between the two cylinders, but are all produced at the arc and move from there outward. Two modifications are possible.

Let CC' and DD', Fig. 1, be two concentric cylinders insulated from each other at ee'. At the center of these two cylinders two

carbons are placed with the arc at *a*. An electrometer is connected to *CC'*. When the air is at rest the ions from *a* will move across to *DD'*. If a blast of air is sent in the direction of the arrow, some of the ions will be carried to *CC'*, provided the potential difference is not too great. If the potential difference is sufficiently large, the ions may be drawn to *DD'* before the current of air can carry them to *CC'*. It is evident that the greater the velocity of the ions for the same potential gradient, the smaller the potential difference necessary to draw them all to *DD'*. The electrometer connected to *CC'* will indicate whether any ions pass to *CC'* or not.

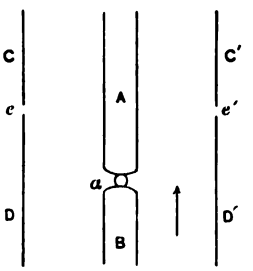


Fig. 1.

It is to be noticed that if there are different kinds of ions present, this method gives us information concerning the velocity of the ions moving most slowly. These will be carried by the current of air to *CC'* while the faster moving ones are drawn to *DD'*. This method will, therefore, only enable us to compare the slowest ions of the positive with the slowest of the negative discharge.

Second Modification of Zeleny's Method.—A second modification of Zeleny's method would be to connect *DD'* to the electrometer. Then no discharge to *DD'* should occur when the potential difference between *A* and *DD'* is small, since then all of the ions will be carried by the currents of air to *CC'*. If the potential difference be increased a point should be found where the discharge would begin to pass to *DD'*.

With such an arrangement a comparison would be made between the fastest moving ions, for the slower will at all times be carried to *CC'*.

Difficulties Met With.—There are some serious difficulties met with in attempting to apply the methods thus indicated. The first is due to the fact that the inner cylinder is not one having the same potential at all points as in the work of Zeleny, but it consists of two carbons with the arc between them. These carbons have quite different potentials, and the potential varies in an irregular manner. Experiments were made with the current passing through the arc in dif-

ferent directions, and in this way the endeavor was made to eliminate to some extent the effect of the variations of the potentials of the carbons.

The second difficulty was due to the fact that the field about the arc was greatly affected by the presence of the ions. In Zeleny's work it could be assumed that the ions were not sufficiently numerous to affect the field, but such an assumption can not be made here. The effect which the ions have on the field where there are no currents of air has been discussed in a preceding article,[1] but here the action is much more complicated. In the first place the ions all start from a small section of the inner cylinder instead of from all points upon it. If AB were a cylinder having the same potential at all points, if there were no currents of air, and if the field were not distorted by the presence of ions, the lines of force would be perpendicular to AB and all ions originating at A would move directly across to DD'. But let a large number of ions of one kind be drawn from a. Let these, for example, be positive. The region in the immediate neighborhood will contain a positive charge and will have a higher potential than the space either directly above or directly below it. Lines of force under these conditions will radiate in all directions, some of them even extending to CC', so that CC' may possibly receive a charge from a regardless of any currents of air. As the potential difference becomes larger the charge about a will become greater and more lines of force will radiate in all directions and more of the discharge will pass to CC'. So that the effect for which we are looking may be obscured by this action. The action is still more complicated because of the fact that the ions are not only drawn out from the arc but are also blown along between the cylinders.

Another difficulty scarcely less serious is due to the fluctuations of the arc. To determine exactly the conditions under which the discharge to one of the cylinders ceases, a comparison of rates of discharge must be made, and to do this successfully one needs a constant excitation of the ions. If between two sets of readings the arc itself materially changes the comparison is of little value. Variations may be due to convection currents from the arc, to changes in

[1] PHYS. REV., XII., 137.

the position and length of the arc, and to changes in the amount of current flowing through it. As a result it was found practically impossible to secure two sets of readings that were very much alike.

This method, therefore, is of little value as far as absolute values of the velocities of the ions are concerned, and yet, as we shall see, it gives us some information concerning the relative velocities of the positive and negative ions.

Arrangement of Apparatus.—In the experiment the outer cylinder was an iron pipe 4 cm. in diameter inside measure. The inner cylinder consisted of the carbons 1 cm. in diameter and the arc between them. In every case it was possible to have the current passing either from A to B or from B to A, and since the potential between the arc and the positive carbon is greater than that between the arc and negative one, the field between the carbons on the outer cylinder depends on the direction of the current through the arc.

The air was forced through the cylinders from a gasometer. A constant weight was placed upon the gasometer, but no correction was made for the different velocities of the air as the gasometer was filled to different heights. The air passed into the outer cylinder through four openings at the bottom. The cylinders were placed vertically and were insulated from the carbons. No attempt was made to dry the air.

The arc was grounded by placing a shunt about it and grounding this shunt $\frac{1}{4}$ of the way from the negative carbon. The cylinder DD' was maintained at the desired potential by means of a water battery. CC' was connected permanently to the electrometer. It was also connected at first to DD' and then insulated from it and allowed to discharge a given length of time. The amount of discharge was observed by the electrometer. It was found necessary to use the electrometer with different degrees of sensitiveness, but the values as given in the table are all reduced to volts. A correction was made for unavoidable leakage. The current was always passed through the arc so that A was charged oppositely to CC'.

Data Taken by the First Method.—It was found that the rate of discharge from CC' did not become less as the potential difference between B and DD' was made greater even when the velocity

of the air sent through the cylinder was small. At first it was thought that perhaps convection currents were carrying ions up to CC'. The apparatus was accordingly inverted and the air forced down against the convection currents. But this produced but little difference in the effect. The readings given in Table I. are those given with the apparatus thus arranged.

Column 1 gives the potential difference between the arc and the cylinder in volts, column 2 gives the change of CC' in volts in four seconds when the cylinder was charged negatively, and column 3 when charged positively. In the former case the discharge was carried by positive ions, in the latter by negative. The velocity of the air is 3.8 cm. per second. The distance from a to ee' was 9 cm.

TABLE I.

Pd Between a and DD'.	+ Discharge.	— Discharge.
40	2.4	1.5
70	4.1	4.1
100	4.7	5.1
150	5.4	6.3

While the discharge does not become smaller as the potential difference is increased, the positive discharge does become smaller than the negative. This would seem to indicate that even when we are dealing with the slower ions, the positive ones move the more rapidly. Unfortunately it was found impractical to carry the investigation to higher potentials. When such an attempt was made the discharge between a and DD' became so great that the water battery was not able to keep the potential difference constant and no other battery giving a potential sufficiently high was at my command. Moreover the ions at a distance from the arc have already been studied by Merritt and Stewart.[1] To carry this part of the investigation further would be simply to repeat their work.

In an investigation of which I hope soon to give an account I have compared the velocities of the positive and negative ions driven from hot platinum wires. I found that while some of the positive ions move more rapidly than any of the negative ions, another class

[1] PHYS. REV., VII., 141.

of positive ions appear to move as slowly or more slowly than any of the negative. Though there is no positive evidence in the data before us, it would not be surprising if the same were true of the discharge from the arc.

Data Taken by Second Method.—In this method the electrometer was connected to DD' and the endeavor was made to find the smallest potential difference at which the discharge passed to DD'. Here only small potential differences were used. It was accordingly possible to keep the outer cylinder at or near zero potential and to vary the potential of the arc by placing a shunt about the storage battery which produced the arc and grounding different points on the shunt. Under these conditions no leakage occurred for which correction should be made and the potential difference between the arc and the ground could be determined directly by a voltmeter. Here again a shunt was placed about the arc and a point one-fourth of the way from the negative carbon was assumed to have the potential of the arc.

But with the smaller potential differences the direction of the current through the arc came to be a matter of importance. Conse-

TABLE II.

Arc Positive.				Arc Negative.			
Current Flowing from B to A.		Current Flowing from A to B.		Current Flowing from B to A.		Current Flowing from A to B.	
Potential Difference.	Def. of Electrometer.	Potential Difference.	Def. of Electrometer.	Potential Difference.	Def. of Electrometer.	Potential Difference.	Def. of Electrometer.
43	17.7	38	12.2	43	16.3	40	2.5
32	13.7	28	9.2	33	6.3	28	0.
20	9.7	20	5.7	24	2.	15	
10	5.5	10	3.5	15	.5		

quently in Table II. data are given for the current flowing in both ways, both when the discharge was negative and when positive. The velocity of the air was the same as in the preceding experiment. The distance from a to cc' was 1.4 cm.

It is possible to construct some kind of an explanation for each of the peculiarities of this table, but probably such explanations would show more concerning one's ingenuity in adapting the ionic theory to all sorts of results than concerning the relative velocities

of the positive and negative ions. Aside, however, from these irregularities there is shown a decided tendency for the negative discharge to cease before the potential difference becomes zero, or before the positive discharge ceases. We are here comparing the most rapidly moving ions of the positive discharge with the most rapidly moving of the negative. The results indicate that there are no negative ions which move as rapidly as some of the positive.

The work given in Table II. was repeated many times, varying the length from a to the plane cc' and the velocity of the air currents between the cylinders. No results were found which were more regular or more consistent than those here given, but they were all similar in showing that the negative discharge ceases at higher potentials than those at which the positive discharge ceases.

Third Method.—Another method for comparing the velocity of the ions which is entirely different from the preceding is one used by Rutherford.[1] When the arc is produced by an alternating current an alternating potential difference will exist between the arc and the surrounding cylinder. In such a case the direction of motion of the ions will be periodically reversed. The potential difference can, of course, be varied by changing the position of a ground on the arc circuit. It can thus be regulated so that the slower moving ions will not reach the cylinder before the reversal of the potential difference takes place, while the more rapidly moving ones will reach it. In such a case the cylinder will acquire a potential similar to the charge on the more rapidly moving ion. Thus if the positive ions move the more rapidly the surrounding cylinder will be charged positively as compared with the arc.

One must remember, however, that the average potential of the arc itself is not zero. Thus if A in Fig. 1 be grounded, there will be a large fall of potential between A and a when the current is flowing from A to B, but a much smaller one when the current is flowing in the opposite direction. Thus for example, if A were grounded the minimum potential of the arc during the first half of the period might be -30, while during the second half the maximum potential would be only $+10$, so that the average potential would be negative. This statement is true whatever may be the correct theory of the arc.

[1] Proc. Camb. Phil. Soc., 9, 401.

Moreover the average potential of the carbons is zero. If the average potential of the carbons were the same as that of the arc, correction could be easily made, but with the conditions as they are we are not able to say what the combined effect of the arc and carbons will be.

However, it happens that the potential which the cylinder assumes is not only positive as compared with the average potential of the arc, but also as compared with that of the carbons. While the irregularities of the potential of the arc hinder us from making any quantitive determinations of the velocity of the ions by this method, we can make a comparison of the relative velocities. It is difficult to see why the cylinder about the arc should become thus positively charged, if it is not due to the greater velocity of the positive ions. The effect can not be due to ultra-violet light, since the cylinder was made of tin and no discharge takes place from tin under the action of such light.

It should be noticed that this method gives us information concerning the faster moving ions when there are different kinds of ions present. If the potential difference between the cylinders and the arc be made as small as possible and still have any ions pass to the cylinder, the slower ions will not reach the cylinder before the direction of motion is reversed.

The experiment was tried with an arc 2 mm. long through which a current was passing of approximately 5.5 ampères as measured by an alternating current ammeter. The average potential of the arc when one of the carbons was grounded was found to be — 4.5 volts. The cylinder about the arc was 9 cm. in diameter and 3 cm. in height. There were 260 alternations per second.

The cylinder became positively charged regardless of the point on circuit at which the ground connection was placed. Indeed a shunt was placed about the arc and different points on this were grounded and yet in every case the cylinder became positively charged. Some part of the arc or the adjacent carbons will always have a positive potential during at least half of the period and this is sufficient to drive positive ions to the cylinder.

With the ground connection at any point on the shunt the cylinder required a potential of about 3 volts. When the ground

was shifted to the resistance in series with the arc the potential increased. When the square root of the mean square of the potential difference between the ground and the arc was 85 volts, the cylinder acquired a potential of + 5 volts.

Cylinders 4.5 cm. and 12.5 cm. in diameter were also tried, and they both acquired in every case a positive charge, the smaller one acquiring it much more rapidly than the other. The highest potential which the smaller cylinder acquired was + 6 volts, the larger + 4 volts.

Summary.—We have thus compared the velocity of the positive and negative ions by four different methods. The first, the one described in the preceding article, enables us to compare the average velocity of the positive with the average velocity of the negative ions. Two of the methods described in this article enable us to compare the most rapidly moving of the positive ions with the most rapidly moving negative ions, while the fourth gives us a comparison of the slowest ions. In each of the comparisons of the most rapidly moving ions and also in the comparison of the average velocities, the positive ions were found to have the greater velocity. In the comparison of the slowest ions little difference was found between the two kinds of ions.

These results are not in any way surprising. One would have every reason to suppose that from anything as heterogenous as the ordinary electric arc many different kinds of ions would be produced, so that among the positive ions we might find the most rapidly moving and also the slowest. In this case the most rapidly moving ones appear to affect the average velocity so that it is greater than the average negative velocity. This no doubt is as true within the arc as about it, so that the potential of the arc may be explained in the manner already discussed.

COLGATE UNIVERSITY,
December, 1901.

ON THE DENSITY AND SURFACE TENSION OF LIQUID AIR.

By Chas. T. Knipp.

A SIMPLE lecture table experiment shows that the density of liquid air when allowed to boil freely increases rapidly. Therefore, determinations of its density by ordinary methods will give values too great. These values range from that of liquid air to approximately that of liquid oxygen, depending upon the activity of boiling and the time that has elapsed.

It was the first intention to confine this paper to the surface tension of liquid air, but it became evident that the surface tension too would change with age. By constructing a curve showing the variation of the surface tension with time for the liquid air contained in a given Dewar bulb exposed to constant radiation conditions we can approximate to its value when first made. However, in order to construct this curve, we must know the variation of the density with time under the same conditions. It is for this reason, therefore, that a few observations were made on the density and included in this paper.

The Density of Liquid Air.

Various methods have been employed by investigators for determining the density of liquids at low temperatures. One method,[1] for an approximate determination, is to select some substance that will just float in the liquid and then compare the density of this with that of water. This method can not be used advantageously in determining the variation with age.

Weighing a sinker in the liquid was also used. This method, too, is only approximate since little or nothing is known concerning the cubical contraction of the sinker at those extremely low temperatures. It is a very convenient method, however, for de-

[1] H. Moissan and J. Dewar, C. R., 125, pp. 505–511, 1897.

termining the variation of the density with time, and was therefore employed.

The liquid air was placed in a Dewar vacuum bulb of about three quarts capacity. The bulb was about half full to begin with. Two glass sinkers were used. The age of the liquid air at the time of weighing was noted. It was assumed that the coefficient of expansion of glass holds at the temperature of liquid air. Upon this assumption the formula[1] for the density becomes,

$$\text{Density} = \frac{V(W - W'')(\rho - \sigma) + (W - W')[V'\delta' - V(\delta - \sigma)]}{V''(W - W')}$$

in which,

V = volume at $t°$ C.
V'' = volume at $-191°$ C.
W = weight of sinker in air.
W' = weight of sinker in water at $t°$.
W'' = weight of sinker in liquid air.
ρ = density of water.
δ = density of sinker at $t°$.
δ' = density of sinker at $-191°$.
σ = density of air at $t°$.

For the sake of comparison all of the data are given in the following table:

TABLE I.

Sinker.	No. 1.	No. 1.	No. 2.	No 2.	No. 1.
July.	22d.	19th.	22d.	22d.	22d.
Age of Liquid Air.	30 min.	35 min.	40 min.	180 min.	190 min.
V	.724	.724	.5277	.5277	.724
V'	.7204	.7204	.5252	.5252	.7204
w	1.7986	1.7989	1.3036	1.3033	1.7986
w'	1.0769	1.0777	.7809	.7809	1.0769
w''	1.1068	1.0985	.7955	.7510	1.023
ρ	.998				
δ	2.4856	2.4856	2.4834	2.4839	2.4856
δ'	2.4908	2.4968	2.495	2.495	2.4968
σ	.0012				
Density.	.957	.9713	.9744	1.0594	1.075

[1] Stewart and Gee, Lab. Man., Vol. I., p. 139.

In Fig. 1 densities are plotted as ordinates, and time in minutes as abscissas. It is unfortunate that more values for the density were not obtained, though I think that those given will suffice. From the curve we infer that the density when first made is between .93 and .94. Olszewiski[1] obtained 1.124 and .885 as the

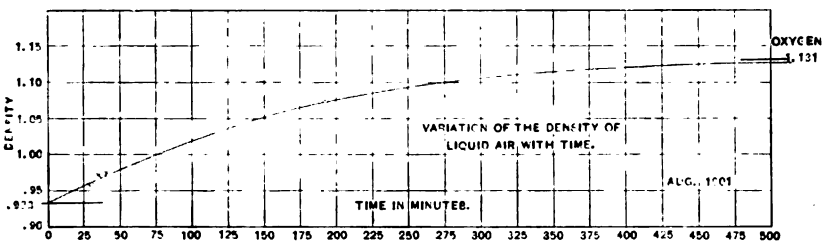


Fig. 1.

densities of liquid oxygen and liquid nitrogen respectively. These values give .932 as the density of liquid air. In 1895 Dewar[2] obtained for the above liquids 1.1375 and .85 respectively, and for liquid air .91. He weighed a large number of substances of known specific gravities in the liquids, making corrections for the contraction of the solids.

As time elapses the nitrogen boils away and the density approaches that of oxygen. The curve is asymptotic to an ordinate that represents the density of oxygen.[3]

The Surface Tension of Liquid Air.

In the work on the surface tension of liquid air the same Dewar bulb was employed. Capillary tubes were first used in the usual manner. The angle of contact was taken as zero. There are several objections to their use, namely:

1. It is almost impossible to read the liquid surface, because of the slight agitation due to boiling.

2. Distortion due to the walls of the bulb.

3. The meniscus was continually changing its position due to evaporation, because its position above the liquid surface unduly exposed it.

[1] Wied. Ann., 31, p. 58, 1886.

[2] Proc. Roy. Inst., 15, p. 133.

[3] J. Drugman and W. Ramsay, Chem. Soc., Journ., 77 and 78, pp. 1228–1233, Nov., 1900.

The following data were obtained with the five tubes employed:

TABLE II.

July 16th. Age of liquid air approx. 140 *min.* $\rho = 1.05$.

No. of Tube.	Diameter of Tube.	T in Dynes per cm.
1	.097 cm.	11.11
2	.066 "	12.18
3	.108 "	10.59
4	.0625 "	9.87
5	.1015 "	8.08
Average,		10.36

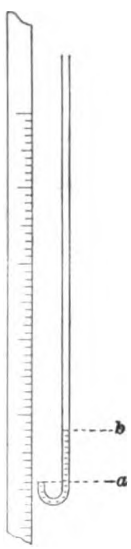


Fig. 2.

This result is somewhat less than that found by Forsch,[1] who gives for the surface tension by this method, after much of the nitrogen had evaporated, 12 dynes per cm.

In order to read the liquid level more accurately the capillary tube was slightly modified. The lower end of the tube was bent upward and cut off square at *a*, Fig. 2. The cathetometer was dispensed with and a thermometer stem used as a scale instead. This enabled the liquid level *a* to be accurately read (to within the errors of direct vision through the walls of the bulb). The meniscus at *b* was read three or four times and the mean taken. The values shown in table III were obtained:

One more modification was made, and that was to jacket the capillary tubes. This is shown in Fig. 3. The opening *a* in the jacket enabled the same to be

TABLE III.

July 17th. Age of liquid air approx. 260 *min.* $\rho = 1.1$.

No. of Tube.	Diameter of Tube.	T in Dynes per cm.
1	.0847 cm.	12.53
2	.0487 "	10.14
3	.0485 "	12.72
4	.05 "	10.34
5	.05 "	9.32
6	.061 "	12.67
Average,		11.28

[1] Phys. Zeitschr., I., p. 177, Jan. 13, 1900.

filled by merely lowering the system into the liquid. The meniscus remained steady. Three tubes were employed. It was a little more difficult to cut the capillary tubes directly at the meniscus; however, this was overcome by reading the position of a file mark on the capillary tube.

From the data contained in the above three tables we see that the surface tension increases as the liquid air becomes poorer in nitrogen. However, little confidence can be put in the results, because of the crude methods employed.

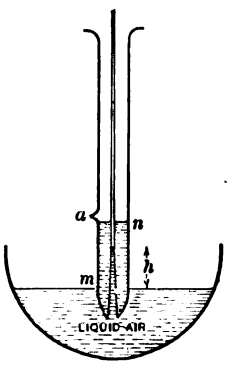


Fig. 3.

A much more satisfactory method, one free from the above troublesome features, is the maximum weight method first suggested by Hall,[1] and later improved by Foley.[2] Mica frames, varying in length rom 2 cm. to 6

TABLE IV.

July 22d. Age of liquid air approx. 75 *min.* $\rho = 1$.

No. of Tube.	Diameter of Tube.	T in Dynes per cm.
1	.079 cm.	9.90
2	.091 "	10.91
3	.081 "	9.93
Average,		10.24

cm., were used. These frames were supported by a single silk fiber at least 60 cm. long, thus guarding against any temperature change in the balance arms. Fig. 4 shows the manner in which the frames were used. By means of a copper wire stirrup s the frame mn was hung with its lower edge horizontal. The silk fiber that supported the stirrup and frame was so small that no appreciable amount of frozen moisture collected on it. The large Dewar bulb was used. The 6 cm. frames, because of their length, had to be lowered into the bulb endwise and hooked to the stirrup afterwards. The formula for these particular mica frames is,

[1] Phil. Mag., V., 36, p. 402, Nov., 1893.
[2] PHYS. REV., Vol. 3, No. 17, pp. 381–386, 1896.

$$T = g\left[\frac{W}{2(l-t)} + \frac{\rho l^2 t^2}{(l-t)^2} - \frac{lt}{4(l-t)^2}\sqrt{\rho^2 l^2 t^2 + 4w(l-t)\rho}\right],$$

in which

w = net maximum weight.
l = length of frame.
t = thickness of frame.
ρ = density of liquid.
g = force of gravity.

Observations with mica frames were made on three different days. The liquid air was run off in the morning. All measurements were made in the three quart bulb. This bulb had no mercury in the vacuum chamber. Carbonic acid snow and frozen moisture

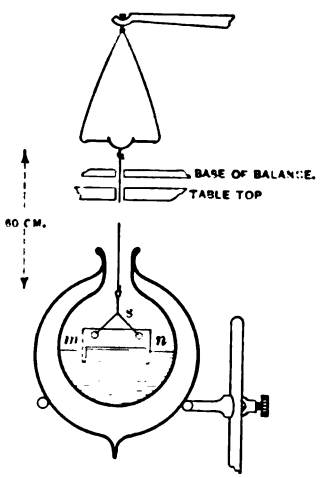


Fig. 4.

were removed by filtering the liquid through ordinary filter paper. The age of the liquid air includes the average age while it was being made, *i. e.*, if it took 30 minutes to run off a certain quantity of the liquid its age at the end of the 30 minutes was taken as 15 minutes.

Table V. contains the data obtained for one sample of liquid air for a number of mica frames. No difficulty at all was experienced in getting the values for w, in fact, by this method the data were as easily collected as when working with water.

TABLE V.

August 12th.

w in gms.	Length in cm.	l in cm.	ρ from Density Curve.	Age in Minutes.	T in Dynes per cm.
.0454	2.02	.00283	1.026	115	10.78
.04825	1.952	.00263	1.038	135	11.91
.05068	1.974	.00276	1.045	145	12.38
.04875	1.974	.00276	1.09	240	12.59
.05248	2.02	.00283	1.099	265	12.45
.0518	1.952	.00263	1.104	280	12.79
.05228	1.952	.00263	1.108	295	12.91
.06964	2.572	.0026	1.116	320	13.05
.18516	6.65	.0035	1.124	352	13.34
.18284	6.65	.0035	1.1245	370	13.17
.18376	6.65	.0035	1.127	385	13.23
.07502	2.732	.00336	1.13	410	13.18
.07534	2.714	.00303	1.131	450	13.35

Table VI. represents three observations on a sample of liquid air that was originally placed in a small Dewar bulb containing mercury in the vacuum chamber. Cotton waste was placed in the mouth of the bulb, and the whole wrapped in a cloth. About four hours afterwards the liquid was filtered into the large bulb, and the following data taken :

TABLE VI.

August 13th.

w	l	t	ρ	Age.	T
.04796	2.02	.00283	1.037	133	11.42
.0491	1.974	.00276	1.047	150	11.98
.0497	1.952	.00263	1.053	160	12.07

In Table VII. is contained, with one exception, the data for one frame. This run was made especially with reference to the variation of the surface tension with time, and in many respects it is the most reliable of all. One half gallon of the liquid was filtered into the large bulb. When the last measurement was made the liquid was nearly all gone—scarcely enough to wet the lower edge of the frame.

The data contained in the above three tables are represented graphically in Fig. 5. In order that the values given in Table VI. come on the curve as located by the data in Tables V. and VII. the age had to be halved. This shows that the mercury mirror and the

TABLE VII.

August 15th.

w	l	t	ρ	Age.	T
.1526	6.646	.00243	.995	67	11.07
.15684	6.63	.004	1.024	112	11.28
.1666	"	"	1.065	182	12.00
.1739	"	"	1.083	222	12.53
.1777	"	"	1.095	257	12.96
.1806	"	"	1.103	274	13.01
.1821	"	"	1.108	294	13.12
.1838	"	"	1.114	312	13.24
.1846	"	"	1.118	324	13.30
.1847	"	"	1.122	340	13.31

other precautions taken to prevent evaporation add much to the life of the liquid.

The curve also shows that, for the large bulb, the surface tension tends toward a constant at an age of from 300 to 350 minutes.

The fact that the surface tension becomes constant is a further

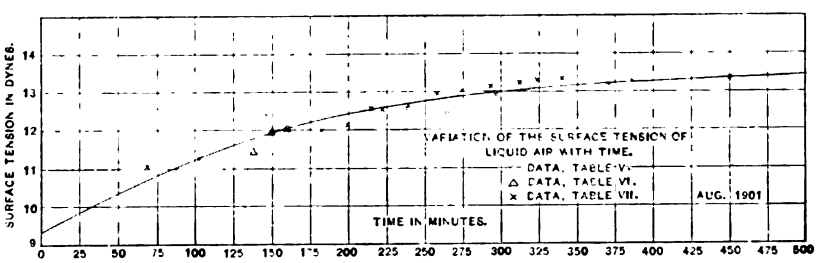


Fig. 5.

indication that the upper part of the density curve given in Fig. 1 is correct.

We would also infer from the curve that the surface tension of liquid air when first made is between 9 and 10 dynes per centimeter.

In conclusion I wish to thank Dr. E. L. Nichols for the courtesies of the department, also Dr. A. L. Foley for the die which he kindly sent me with which to cut the mica frames.

Since the above was put in type an article by L. Grunmach (Ann. d. Phys., No. 11, p. 559, 1901) on the "Surface Tension of Liquid Air" has come to my notice, in which he used the method of capillary ripples.

PHYSICAL LABORATORY, CORNELL UNIVERSITY,
August, 1901.

CRYSTALLIZATION UNDER ELECTROSTATIC STRESS.

By Paul R. Heyl.

THE thought underlying these experiments was that if crystals were allowed to form from solutions subjected to strong and intermittent electrostatic stress the crystals might, when large enough to be observed, bear some birthmarks which might be interesting or possibly valuable. A change in the interfacial angles seemed the most likely effect, and an alteration of density was deemed possible. Both these effects were looked for, and a salt sensitive to slight mechanical disturbances (HgI_2) was employed as an indicator; but no effect was found.

The method of procedure was as follows: a shallow glass crystallizing dish some 8 cm. in diameter, with a flat bottom, was placed upon a sheet of tinfoil. Within this, raised about 0.5 cm. from the bottom, was a similar but smaller dish. This second dish was supported in some cases by a wire twisted around its upper part, and in some cases by resting on a bent glass rod laid in the larger dish. The inside of the bottom of the smaller dish was coated with tinfoil. The two tinfoil sheets were connected to the knobs of an influence machine or to the secondary terminals of an induction coil. The solution under experiment was poured between the dishes after the electrostatic stress had been started.

Two solutions were used, viz., sulphur in carbon disulphide, allowed to evaporate spontaneously, and hot saturated solutions of HgI_2 in alcohol.

Sulphur in Carbon Disulphide.

Four lots of crystals are to be mentioned. Lots A and D were produced without electrostatic stress; lot B, under the stress produced by an induction coil giving a spark 0.2 cm. long; lot C, under the stress produced by an influence machine set for a spark 1 cm. long. With lot C, in the earlier stages of the crystallization,

sparks passed between the knobs of the machine from two to five times a second; but later, when the level of the solution had fallen so as not to touch the upper dish, sparks passed but rarely. The solution was violently agitated by electrification, and the crystals rose and fell constantly between the dishes. Owing to this agitation the crystals were much smaller than those of any of the other lots. In lot B the stress was so rapidly intermittent that the solution remained quiet.

The octahedron to which the sulphur crystals approximate has theoretically but three different dihedral angles, marked α, β, γ in Fig. 1. This notation is used in the following tables of observed angles. Each angle given in the tables is the mean of five or more observations, made by the reflection method, using a spectrometer reading to single minutes. The figure in parenthesis after each angle represents the number of minutes by which the given mean differs from the most widely divergent single observation; a number which, roughly speaking, varies inversely with the polish of the faces forming the angle. For our purposes this number will be more useful than the much smaller "probable error," which averages less than one minute for all the angles given.

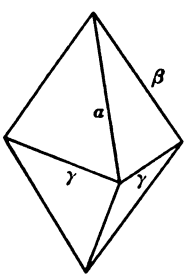


Fig. 1.

The two angles γ (Fig. 1) should, geometrically, be equal. It was not always possible to measure both angles in every crystal.

Crystal.	α	β	γ
1A	85° 8′ (2)	106°53′ (3)	
2A	85 0 (3)	106 18 (2)	143° 9′ (7) 143° 16′ (3)
3A	84 50 (12)	106 35 (4)	
4A			143 15 (3)
5A	85 8 (5)	106 39 (6)	
6A	85 11 (8)	105 53 (1)	143 9 (3) 143 2 (9)
7A		106 52 (4)	
8A	85 4 (5)	106 43 (4)	143 34 (3)
9A	85 15 (2)	106 12 (2)	142 56 (4)
10A	85 5 (7)	106 24 (4)	142 46 (1)
11A	85 4 (1)	106 33 (4)	
Mean	85 5	106 30	143 8
Average departure from mean.	5	15	10

Crystal.	α	β	γ
1B		106°38′ (7)	143°17′ (3)
2B	85° 1′ (4)	106 42 (5)	143 2 (6) 143°27′ (3)
3B	85 34 (3)	106 8 (4)	
4B	85 19 (6)	106 9 (2)	142 48 (5) 144 17 (3)
5B	85 22 (4)	106 11 (3)	
6B	84 26 (1)		142 6 (3) 143 27 (4)
7B	85 1 (4)	106 31 (2)	143 22 (6) 143 11 (3)
8B	84 48 (4)	106 24 (2)	142 58 (2)
9B		106 30 (2)	
10B	85 30 (2)	106 39 (1)	143 35 (3)
11B	85 8 (4)		143 42 (3)
12B	85 47 (4)	105 39 (4)	144 49 (3)
13B	85 25 (6)		143 57 (3)
Mean	85 13	106 21	143 26
Average departure.	18′	16′	28′
1C	83°19′ (1)	106°32′ (3)	
2C	85 20 (3)	106 32 (3)	
3C			143°20′ (2)
4C	85 23 (2)	106 17 (2)	
5C	85 16 (3)		
6C	85 16 (4)	106 43 (2)	142 33 (3)
7C		106 21 (6)	143 56 (5)
8C	85 27 (2)	106 26 (3)	143 22 (2) 143°15′ (2)
9C	84 56 (2)		143 26 (3)
Mean	85 0	106 28	143 19
Average departure.	30′	7′	16

but where possible it was done, and two values are given in the γ column. The "average departure from the mean" does not refer to the figures in parenthesis, but measures the variability of that angle among all the crystals.

The first point to be noticed is the variability of the angles. This cannot be ascribed to electrostatic stress, as it appears in lot A. That it is a fact, and is not due to experimental errors, is seen by considering the measures of precision. For instance, the angle β, crystal 6A, was the most perfect of all the angles measured. It is possible, though hardly probable, that its value may be as great as 105°54′ or as small as 105°52′. Especial care was taken with this angle to make sure that the number of degrees was correctly read. To insure this, readings were taken at different places on the

scale. Now compare this with the angle β of crystal 7A. It is possible, though not probable, that this latter angle may be as small as 106°48′. The discrepancy here is greater than experimental error can account for, and other instances of a similar kind are to be found in the tables. This variability of angle is doubtless to be ascribed to the rapidity with which the crystals were formed. The month was July, and the carbon disulphide evaporated spontaneously in from forty-five minutes to an hour and a half. Again, in lot C, the mechanical agitation of the solution, previously alluded to, may have had such an effect.

Nor does it certainly appear that this variability is either increased or diminished by the electrostatic stress. With the electric crystals the average departure from the mean is in some cases increased, in some diminished, as the stress is increased; and the change in variability is of the same order as the variations among individual crystals, so that it may be considered fortuitous.

A large number of small crystals formed under the stress of the influence machine were examined under a low power microscope, but no particular peculiarity of shape was evident. There was a rather frequent truncation of the upper and lower terminal points of the octahedron, but this is not unknown in crystals normally produced.

The density of the crystals was determined by the specific gravity bottle. The crystals were dried on a water-bath until they ceased to lose weight; the distilled water used was freshly boiled; the air pump was used to eliminate bubbles; and the temperature of the water was read to 0.1° as soon as equilibrium was attained, before even the weights were counted. The following values were obtained:

Lot A, Ordinary.		Lot D, Ordinary.	
2.036	24.5°	2.059	33.8°
2.030	25.2	2.059	33.4
2.035	25.7	2.058	33.0
2.034		2.059	

Lot B, Electric.		Lot C, Electric.	
2.048	27.6°	2.051	25.0°
2.052	27.7	2.039	27.9
2.050		2.041	28.0
		2.039	28.2
		2.043	

The usually accepted value for octahedral sulphur is 2.05. It will be noticed that one of the ordinary lots (A) departs more from this value than either of the electric lots. The inference is obvious; electrostatic stress produces no effect on the density. The volume-expansion of sulphur, as given by Kopp, is so nearly that of water that the temperature reductions could not affect the second decimal place, and may be neglected in comparison with the error introduced by the retention of air-bubbles; for to this source must be ascribed the above differences. Crystals that are smooth and well finished, as were those of lot D, yield up their air-bubbles readily in vacuo; but lots of a poorer quality, as was lot A, retain air in their crevices with the greatest tenacity, and the air pump is not adequate to remove the last traces. Frequently, after half an hour's pumping, small bubbles could be seen in the mass which no shaking could dislodge, and which did not shrink into invisibility on admitting the air. Several lots of crystals had to be discarded as worthless on this account. An attempt was made to use the crystals powdered, but such is the reluctance of powdered sulphur to being wet that the difficulty was increased many-fold.

If it be asked why the crystal angles were measured in lot A rather than in lot D, the answer is that lot A was formed under conditions of disturbance which more closely resembled those under which the electric crystals were formed. The solution was exposed to the air of a warm room, and the rapid evaporation kept the crystals continually in motion in their earlier stages; while solution D was shut up in a fume chamber, where the evaporation was much more prolonged and quiet.

Mercuric Iodide.

This salt was used simply as a delicate qualitative detector. Below 150° it forms red octrahedra; above 150°, yellow needles. From hot alcohol it crystallizes, though much below 150°, in the yellow form, which is so unstable at that temperature that on pressing the crystals under a glass rod, shaking the solution, or even allowing the crystals to stand quietly in the mother-liquor, they change into the red variety. It was hoped that electrostatic stress would shake the nascent crystals sufficiently to cause them to

assume the red form from the beginning, but under a 1 cm. spark from an influence machine the salt crystallized from the solvent mentioned in the yellow form.

As far as the examination has gone the conclusion is negative: that electrostatic stress does not affect crystallization. This conclusion may also be stated positively: any molecular forces called into play in a solution under electrostatic stress are not comparable with the forces of crystalline attraction.

BOYS' HIGH SCHOOL, READING, PA.

SOME EXPERIMENTS ON THE BEHAVIOR OF DIELECTRICS WHEN SUBJECTED TO HIGH POTENTIALS.

By John Sanford Shearer.

I.

ONE of the principal difficulties in the experimental study of the effects of high electrical potentials has been the erratic behavior of electrostatic machines. The transformer is available for the production of alternating electromotive forces; but in the production of unidirectional pressures we have hitherto been compelled to rely on various types of friction or induction machines. All of these are more or less fragile, and peculiarly subject to atmospheric influences, so that it is well-nigh impossible to carry on continuous work with them. The rise of potential after discharge is also more or less irregular, and one can never be quite sure what the actual conditions are likely to be. In general, no considerable amount of electric power is developed by such machines, and long sparks are lacking in volume even when working under favorable conditions.

The use of a great number of storage cells by Professor Trowbridge, by means of which a set of condensers were charged in multiple, and then discharged in series, marked a distinct advance in this line of work. The expense of installation of such a plant is, however, for most experimenters almost prohibitive. The care of such a set of cells must also be exceedingly vexatious; while the discharges produced per second are of necessity few.

An application of a similar principle was made by Professor Elihu Thomson in his dynamo-static machine. In this case the condensers were charged in multiple from a high tension transformer, contact being made for a portion of the wave only. The condensers were then discharged in series, thus multiplying the potential

of the transformer by the number of plates used. The charging and discharging device consisted of a rotating spindle carrying suitable projections passing near curved wires, the latter being attached to the plates. The spindle was driven by the generator that supplied current to the transformer. By this means the time of contact could be controlled; and whatever variations of speed might occur in the generator were synchronous with the changes in the speed of the spindle. Through the kindness of Professor Thomson, the writer was enabled to build a similar machine for use in the present investigation.

The generator used was a one-horse-power, 500-volt, direct-current motor, with a speed of 1,800 revolutions per minute. A pair of slip rings were attached to the armature shaft, and connected to opposite armature segments; the small motor then acted as a rotary converter and gave an effective E.M.F. of approximately 350 volts. It was found by trial that the self-induction of the armature was sufficiently low so that an alternating current of 1.8 ampères could be drawn from the slip rings. This alternating current passed through a transformer having 800 turns in the primary and 48,000 turns in the secondary, thus giving a maximum pressure of 30,000 volts. It was found expedient, however, to operate with a maximum pressure of 22,000 volts. As the secondary was wound in twelve separate coils, any E.M.F. from 2,500 to 30,000 could be readily obtained.

To one end of the armature shaft was attached a gear wheel arranged so as to give the spindle a speed of rotation equal to one-half that of the armature.

Eighteen condenser plates consisting of tin foil on glass were used as condensers. The dimensions of the coatings were 20 by 25 centimeters, the thickness of the glass about 2 millimeters. Brass blocks were soldered to the tin foil, and aluminum wires were inserted in the blocks and fastened by means of set screws. These wires, properly curved, were placed close to the path of the projecting wires on the rotating spindle, actual contact not being necessary because of the high potential of the transformer. The length of the wire arcs was chosen so that charging could take place during one-half of each alternate crest in the same direction.

The average charging potential in this case is nearly 90 per cent. of the maximum, and the time allowed for charging the condenser was .0139 second.

In order to protect the motor from dangerous sparks, the spindle was made somewhat longer than was otherwise necessary, and all parts were covered with several coats of shellac. Considerable trouble was experienced at first because of the tendency of charges to strike back to the terminals of the secondary, a slight puncture of the insulation being followed by the transformer current and thus injuring the coil. To avoid this, the transformer leads were carried, carefully insulated, into two deep grooves in a cylinder fitted to the end of the spindle; and as a further protection a lightning arrester was placed across the terminals of the secondary. A high tension discharge would then be forced by the self induction of the secondary across the arrester, and the arc produced by the transformer current would either break down by reason of convection, or sufficient current would be drawn through the motor to throw a small circuit breaker adjusted for two ampères. Most of these difficulties could be avoided by the use of an oil insulated transformer and a driving device which would serve to insulate the motor from the high tension portion of the apparatus. The details have since been worked out by Prof. Thomson, and embodied in the machine recently exhibited at the Conversazione of the American Institute of Electrical Engineers in New York.

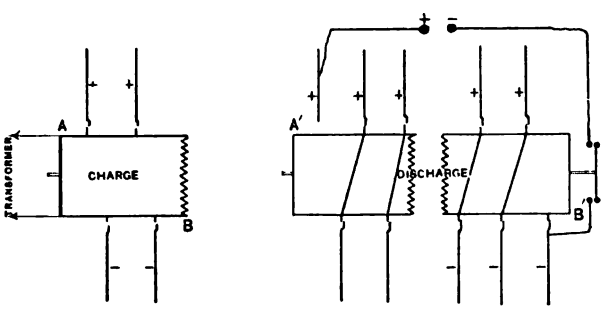


Fig. 1.

The machine may be readily used to charge Leyden jars, but it is necessary to avoid the back pressure during the multiple con-

nection, as otherwise the series E.M.F. of the condenser would be opposed to the E.M.F. used in charging, and might easily rupture the secondary. A spark length greater than the striking distance of the transformer may be inserted in series with the jars, or better, a rotating connector at the end of the spindle may be used so that connection with the batteries is only made when the condensers of the machine are connected in series.

Such a machine affords a source of high potential entirely independent of the weather or of any surrounding conditions. It requires only a source of direct current, and no operation other than turning a switch is necessary for starting. The generator may be driven by power as an alternating generator with the same results.

The length of spark in the present machine may vary from a fraction of a centimeter to 35 centimeters. A battery of 18 two-quart Leyden jars may be charged to overflowing in from 4 to 5 seconds. The discharge is extremely sharp, and resembles in many ways miniature lightning discharges.

Blocks of white pine placed between the discharging terminals, with the grain parallel to the lines of force could be readily lighted up by the discharge, while slivers of considerable length were often torn off. The variation of the resistance of wood along and across the grain was shown in a rather striking manner by using wooden terminals instead of metal. When the grain ran perpendicular to the lines of discharge the sparks came out of the wood along the grain and then across the air path.

An estimate of the potential actually developed may be reached by the use of discharges between needle points. It was found that the mean spark length, between bright steel needle points, was 3.7 centimeters for a single condenser plate. According to the ordinarily accepted values for spark potentials this means approximately 12,000 volts per plate, showing, as was to be expected, that the potential acquired by the plates was considerably below the average charging potential during contact. This is doubtless explained in part by the large capacity of the condensers, and by the fact that the resistance of the air gaps is considerable. Measurements were made with balls 2 centimeters in diameter with similar results.

The potential when the 18 plates are discharged in series would then be 216,000 volts, approximately. No method of conveniently measuring the potential, other than that just mentioned, was at hand. The agreement, however, between the measurements for the individual plates, and the fact that the striking distance directly across the transformer terminals was found to be almost exactly that computed from the ratio of transformation, indicate that the results are fairly accurate.

When connected so as to charge a large battery of Leyden jars, in shunt with which was placed a spark gap with bright needle points, a very steady glow discharge could be maintained. This suggested the possibility of measuring the current of such a discharge. A series of readings with a tangent galvanometer gave for the current the approximate value of 107.10^{-6} ampères for a four-inch air gap, and 200.10^{-6} for a two-inch air gap. The difficulty of measurement lay principally in the accumulation of static charges on the magnet; but by discharging after each reading, and protecting from such effect as far as possible, reasonably good readings were obtained.

The following summary of dimensions and data concerning the apparatus may be of interest:

Number of turns in primary, 800.
Number of turns in secondary, 4,000 per coil.
Number of coils in use on secondary, 9.
Number of turns in actual use, 3,600.
Ratio of transformation 45.
Maximum primary pressure, 500 volts.
Maximum secondary pressure 22,500 volts.
Primary current, 2 ampères, nearly.
Secondary current, .044 ampère.

The mean charging potential may be deduced from the alternating E.M.F. wave as follows, assuming the latter to be a sine function:

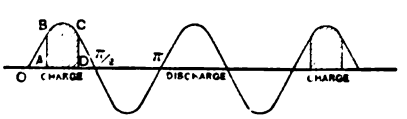


Fig. 2.

$$\text{Area } ABCD \text{ (Fig. 2)} = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin\theta d\theta = \surd 2$$

when maximum ordinate = 1 ;

$$\text{Mean value of ordinate} = \frac{\sqrt{2}}{\frac{\pi}{2}} = \frac{2\sqrt{2}}{\pi} \doteq .90$$

or the mean value of the ordinate between A and D taken symmetrically about the maximum $\doteq$ 90 per cent. of the maximum.

When $E = E_0 \sin pt$, mean value = .90 E_0 or mean charging potential is nearly 90 per cent. of maximum potential difference at the terminals of the transformer ; *i. e.*, 22,500 × .90 = 20,250 or about 20,000.

The total capacity of the 18 plates was approximately 127.10^{-10} farads. When charged to 12,000 volts the quantity would be 15.10^{-5} coulombs. The time allowed for charging was .0139 second, giving for a mean charging current .0108 ampère. The current output then was one-eighth of this or .0013 ampère. This mean output could be doubled by driving the spindle at the same rate as the armature, in which case the length of the charging arcs should be doubled.

Fig. 1 shows the general connections of the machine. AB is the rotating spindle at time of charging the condensers. $A'B'$ indicates the connections after the spindle has rotated 180°, when condensers are discharged.

II.

Among the unsettled questions in electrostatic theory is the relation between electrostatic strain and mechanical deformation of the dielectric. Ever since Faraday directed the attention of physicists to the influence of the medium separating two charged bodies upon the electric phenomena observed, the question of the real relation of the medium, whether visible or invisible, to the various phenomena involved has been a subject of widespread interest, and of elaborate theoretical and experimental investigations.

In the theoretical development, the equations derived by Maxwell for the energy per unit volume in the dielectric and its rate of change during redistributions of magnetic or electrical quantities, and the interpretation of electrostatic strain along and perpendicular to lines of force have been the subject of endless discussion.

Exactly what Maxwell meant by polarization, or what relation this polarization has, if it exists, to the various physical properties of the medium, is a question which may, perhaps, never be settled. Yet its suggestiveness in many ways, and its capability of interpretation in accordance with the various mechanical pictures which have been devised as aids to a clearer comprehension of the phenomena, make any attempt in this direction a matter of extreme interest.

Since mechanical force is brought into play by electric charges, and a non-conductor subjected to electrostatic stress may be ruptured, it seems at first sight natural to suppose that changes of volume and dimensions might be produced by such means. This question of the change of size and form of a solid or liquid separating two electrical charges has been investigated with considerable care by various observers, among whom may be mentioned Duter, Righi, Quincke, Korteweg, Julius, Cantone and others, with a great variety of results.

The work of many of these is undoubtedly subject to considerable criticism, partly because the precautions necessary to avoid serious error were underestimated, and partly because the means at hand for the measurement of small changes were sometimes inadequate.

In order to clearly understand the difficulties involved in an investigation of this kind, it may be well to call attention at once to the fact that the change predicted by the theoretical physicists is in all cases extremely small, and that unfortunately in the one case really open for experimental investigation, most of the errors due to various outside causes tend to produce a change in the direction expected. For example, according to the theory most generally accepted, there should be an elongation of the dielectric at right angles to the electrostatic strain, and this should be accompanied in most cases by a variation of volume. Any variation in the shape of the dielectric has a tendency to affect the measurements in the same way as a real change of length; and it is well known that variations of temperature so small as to be difficult to guard against, and which are perhaps altogether too small to measure, may produce an elongation of nearly the same order of magnitude as that which would be called for by these electrostatic theories. Also in

case conducting plates are used, with opposite charges in contact with the surfaces of the solid, the mechanical compression would tend to produce an elongation perpendicular to the lines of force. Where changes of volume have been used to deduce the various results, it is a significant fact that those receptacles which, in all probability, would be most likely to suffer a change of shape have been those giving the greatest variation in volume.

The history of the subject, together with a brief résumé of the principal experimental results, as well as a development of the theoretical side from a thermodynamical standpoint will be found in an excellent article by Sacerdote. It may be of interest, however, to give here a brief review of the previous experimental work, and to attempt to discover the real facts or assumptions upon which theoretical conclusions are based.

The first observations upon the expansion of dielectrics were made by Fontana in 1786. He believed that the volume of a Leyden jar was increased by charging. Govi in 1864 made the first observation with a thermometer tube and was concerned principally with the variation of volume of the cavity containing the conducting fluid.

Duter, in 1878, published the results of a series of experiments with a double bottle made by sealing one glass bulb within another. From each of these a capillary tube projected. Any change in the volume of either liquid due to electrification would be indicated by a rise or fall of the surface of the liquid in one or both of these tubes. The results obtained by this method can hardly be considered as quantitatively exact, but they appear to indicate an effect varying with the square of the difference of potential, and inversely as the thickness of the dielectric. The curve (*I*), Fig. 3, gives an idea of the magnitude of the observed effects. It is to be noted that the thickness of the glass was small, and in vessels of this kind it can hardly be supposed that the thickness could in any sense be considered as uniform.

In 1879, M. Righi published a paper concerning the dilatation of dielectrics used in cylindrical condensers. A long glass tube was rigidly fastened near the upper extremity, while the lower end pressed against a spring to which was attached a mirror. In this

way an extremely sensitive optical lever was produced, and variations in the condenser tube would produce deflections of a spot of light with considerable magnification. The condenser plates were attached to the conductors of a Holtz machine, and the potential difference required was measured directly by an electrometer of his own design. He used tubes of glass about 75 cm. in length, but of considerable thickness as compared with the bottles used by Duter, varying from 1.3 to 1.6 mm. He summarized his results as follows:

"We may conclude from the preceding results that a tube of glass 1 m. long and 1 mm. thick, charged to a difference of potential corresponding to a cm. spark length between brass balls 1.5 cm. in diameter, will increase in length 2 microns."

The first observations by Quincke were published in 1880, and contained an elaborate series of measurements with thermometer tubes. He worked principally with two classes of material; one an English flint glass, and the other Thuringian glass. In a second series of experiments Quincke investigated the change of length of cylindrical glass tubes, using for this purpose small tubes from 1 to 1.9 meters in length. These tubes were silvered on the inside and were usually immersed in a fluid conductor for the external dielectric Changes in length were measured by Oertling's lever; the length. for a range of 18 mm. being measured with an exactness of .0004 mm. The details of these measurements may be found in Wied. Ann., Vol. 246, pp. 334 to 378. After the discharge of the Leyden jars, both the flint and the Thuringian glass retained a slight elongation which in general was greater when the original stretch was greater. With the flint glass this amounted from one to two millionths of the original length. One point of interest in connection with Quincke's work relates to the puncture of glass by electrical tension. He attributes this not to the heat produced by the current through the dielectric, but rather to the unequal mechanical strain upon the glass produced by electrification. It should be observed that there was a considerable discrepancy in the results which he obtained under apparently similar conditions, and in those where the variation of thickness of the dielectric was found to be considerable, changes much more marked were found than in other cases. In fact, he constructed an electrometer by using a thin tube

of glass, the wall being much thinner on one side than on the other: when used as the dielectric of a condenser the tube was distorted upon charging, the displacement of the lower extremity being often several mm.

Quincke's measurements were carried out with great care, the capacity of condensers was measured, and a careful determination of the potentials was made, both by means of an electrometer and by the use of spark lengths. Some idea of his experiments may be

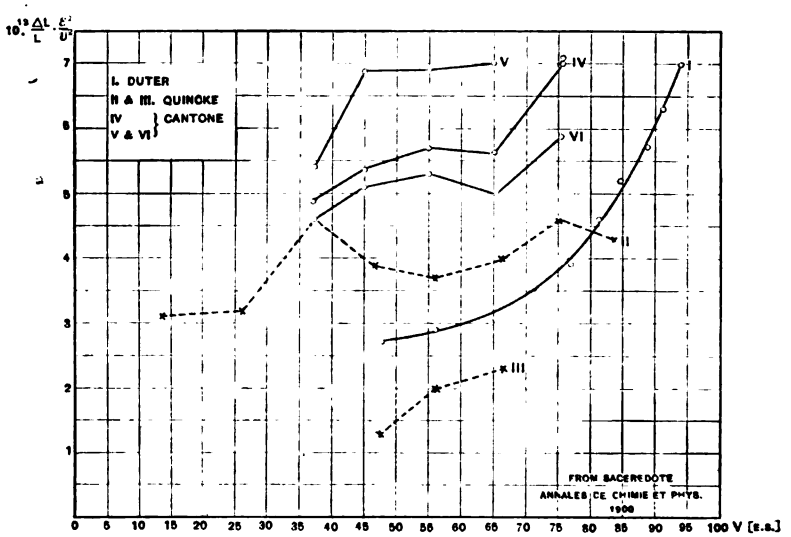


Fig. 3.

obtained from curve II and III (Fig. 3), representing the work upon condensers. Part of the conclusions drawn by Quincke from his experiments are as follows:

"1. Solid and viscous bodies change their volume when they are in any manner subjected to an electrical force; as in the case of glass used in a Leyden jar.

"2. This volume change is not to be considered as due to heat and is usually an expansion. Nevertheless, there may be a contraction as, for example, in the case of the fatty oils.

"3. In the case of gases, I have been unable to observe any change of volume through electrical force. If such a change exists, it must be smaller than $1\ 3.10^9$ of the original volume.

"4. The volume change follows immediately in the case of flint glass, and much more slowly in the case of the better conducting Thuringian glass, upon the application of the electrification. Upon the discharge of the spherical or cylindrical condensers the glass returned very nearly to its old volume; immediately in the case of the flint glass, more slowly in the case of the Thuringian glass.

"5. Simultaneously with the change of volume there occurs a change of length in cylindrical condensers.

"6. Volume and length changes are so much the greater the greater the electrical difference of potential maintained between the plates and the smaller the thickness of the insulating material of the condenser; in fact, very nearly, but not exactly proportional to the square of the ratio of the potential difference and thickness.

"8. After the discharge of the condenser plates there remains a remnant of the volume change in the same sense as the original, which is very small in the case of flint glass, and is much greater in the case of Thuringian glass, and appears to be dependent upon the electric polarization of the glass. The volume and length change do not depend upon the electric compression of the insulator. With flint glass the electric expansion takes place equally in all directions as in the case of temperature expansion and is independent of the nature and direction of the electric force.

"11. The electric volume change and change of length take place in glass in approximately the same manner as expansion by rise of temperature.

"12. Under the influence of electric force the elasticity of flint glass, Thuringian glass and caoutchouc rises and of mica and gutta-percha falls off.

"13. The electrical perforation of glass and other substances is a consequence of the unequal dilatation of the substance."

W. C. Roentgen was the first to object to these conclusions and in a paper published shortly after Quincke's, he maintained that the observed results could be readily explained in other ways.

Korteweg and Julius in 1881 measured the deformation of caoutchouc. They found the effect in this case to be immensely greater than the corresponding effect upon glass.

Quincke made a very elaborate series of experiments to determine the elastic constants of his tubes and also the specific inductive capacity of the glass used. In the latter paper he states the belief that the view of Roentgen and others was well taken, but maintains that direct pressure of the electrodes would not explain all the observed phenomena; citing, in particular, that in liquid dielectrics the volume diminished.

Further work in the same line has been done by Cantone, in a variety of experiments beginning in 1888. He also used cylindrical condensers, but instead of magnifying the movement by optical devices, he chose an interference method. He used a cylindrical vertical glass tube for his dielectric, to the upper end of which was attached a carefully calibrated capillary tube. The lower end of the condenser tube carried a small mirror, adjusted parallel to a fixed mirror. Measurements were made by observing the displacement of the interference fringes produced by the reflection of light between these mirrors. His potentials were expressed in terms of spark lengths between brass balls. For the internal coating, he used water, and the tube was silvered externally. It is to be observed that the tubes used were extremely thin, and also of small radius. Any variation of form due to charging would produce changes in the position of the fringes, which apparently would be very difficult to distinguish from those produced by changes in length. Curves IV., V., VI., Fig. 3, show some of his results, as reported by Sacerdote in the paper previously mentioned.

Dr. L. T. More, in August, 1900, described some experiments in which a negative result was reported, and the existence of such effects was called in question. Dr. More measured the variation in length of a cylindrical tube by means of an optical lever, using an influence machine as a source of potential difference. He called attention to the irregularities in previous work, especially as regards Quincke's results. His paper was criticised by Sacerdote, who asserted that the effect was too small to be distinguished from accidental movements in the arrangement used.

Dr. More in a later paper showed that his apparatus was certainly able to detect changes much smaller than those which should be expected if the work of Cantone is taken as a basis.

Several attempts have been made to develop a theory covering the subject of dielectric expansion, but in all cases the assumption is made that the phenomenon has a real existence. The principal causes which may be assigned to account for an observed expansion perpendicular to the lines of electrostatic strain are as follows:

1. Polarization of the dielectric. To use this, certain hypotheses must of necessity be made, since the exact nature of polarization is entirely unknown.

2. Mechanical pressure of charged metal electrodes, causing an expansion of a purely elastic nature.

3. Thermal expansion, caused either by partial conduction of the dielectric, or by surface discharge.

4. Apparent expansions due to change of form upon charging. The part played by this in such phenomena is often extremely difficult to estimate, and in case volume changes of the internal cavity are the subject of measurement, it is not probable that consistent results would be found for dielectric expansion.

Among the hypotheses made as a basis for analytical work may be mentioned that of Korteweg, who assumed the distribution of a great number of small partially conducting particles in the insulator, which alter their position when subjected to electrostatic strain. This view is in accordance with that commonly accepted in the theory of magnetism, and bears a striking resemblance to the modern electron theory.

Attempts to connect volume changes directly with the energy per unit volume were made by Moutier, whose equations ultimately reduced to a relation between an elastic constant of the dielectric and its energy per unit of volume. His expression for the variation of volume being

$$\frac{\gamma}{3}\,\frac{KV^2}{8\pi},$$

M. Vaschy has assumed that only a fractional part of the ether is concerned in movements or variations of condition of ponderable material. In his work, he finds that $K-1$ is a factor of volume changes upon charging, which would seem to agree with the fact that gases suffer no expansion under electrostatic strain, as in this case K may be assumed as 1.

In the developments along the idea of dielectric polarization, it has been customary to assume that K is a function of direction and elastic condition of the dielectric material, and elaborate formulæ connecting the various constants with the energy and strain equations has been found by Kirchhoff and Lorberg.

Duhem has applied the method of thermodynamic potentials to this subject and has assumed that dielectric expansion and mechanical pressure are uniform and independent of direction. His suggestion as to the method of attack has been followed up by several writers who have arrived at results more or less in accordance with the apparent indications derived from experimental researches.

Lippmann, applying his idea of the conservation of electricity, asserted that the change in dimensions, as found by Quincke and others, required that the dielectric constant should change with mechanical pressure. This assumption enables the reasoning of the Carnot cycle to be applied when the temperature of the condenser is maintained constant.

M. Curie endeavored to show by similar reasoning that in the case of quartz, which exhibits the phenomena of piezo electricity, the variations experienced on charging should vary with the potential difference and not with its square.

J. and P. Curie took up the question of dilatation of crystals, basing their theoretical work closely upon that of Lippmann, and carried the small element of volume through a series of changes in which the coördinates expressing conditions are potential and electrical quantity, instead of pressure and temperature. They deduce therefrom that the length would suffer a change of length proportional to the potential difference and not to the square of this quantity. They thus write $l' = l - av$. They predict that for this reason experiments undertaken with quartz would present a peculiar interest if it should prove that this theory was verified. Taking the case of a crystal of quartz in the form of a rectangular parallelopiped, the application of pressure along the electrical axis would produce an electrification given by the formula, $Q =$ piezo electric constant $(k) \times$ the force applied (F). If we take a kilogram weight as the applied force, it is found that a quantity of electricity is developed capable of charging a sphere of 16.6 cms. radius to a potential of 1 volt. Thus $k = 6.3 \cdot 10^{-8}$.

If when two surfaces are silvered and charged a reciprocal relation exists, then a measurable variation in length might result. The experimental methods of these observers were very ingenious, and results appear to bear out the theory; but the observed changes have even greater chances of other explanation than in the case of glass.

The most elaborate paper on the theory of this subject is that by M. P. Sacerdote, a brief review of which, as applied to long cylindrical condensers, is given here. Lippmann showed that if expansion occurred, a change of dielectric constant should result from mechanical stress. Sacerdote assumes the actual existence of this change and makes the following hypothesis concerning its nature.

Consider any rectangular portion of a dielectric and take the three edges as axes. Apply a mechanical stress dq per unit of area parallel to the zy plane, and define

$$\frac{\frac{d_1K}{dq}}{K} = k_1,$$

where k_1 is the specific inductive capacity of the substance. Also let

$$\frac{\frac{d_2K}{dq}}{K} = k_2$$

or a pressure normal to the zy plane. Assuming that these effects are independent and additive, we have, when pulls along z and y and pressures perpendicular to the zy plane are all present,

$$dK = (2k_1 + k_2)(Kdq).$$

When a pressure is exerted outward on a cylindrical surface a surface tension is produced. The relation between the pressure and tension is $dp = \frac{T}{R}$, when R is the radius of the cylinder. The variation of K due to such a pressure is then

$$dK = 2k_1 \frac{RK}{e} d = \frac{Rk_1}{e} Kdp$$

(e = thickness of cylinder wall).

Suppose U the volume of the dielectric of a condenser whose surfaces are S and A. Let the system be defined by the coördinates pressure and potential (p, V). When we have a variation dp, dV at constant temperature, work enough must be supplied to give the increase of electrical energy, less any work supplied otherwise.

Or

$$\delta\epsilon = VdM - pdU \quad (M = \text{charge on the condenser}).$$

But

$$M = f(p, V)$$

and

$$U = \varphi(p, V)$$

$$dM = \frac{\partial M}{\partial v} dV + \frac{\partial M}{\partial p} dp$$

and

$$dU = \frac{\partial u}{\partial v} dV + \frac{\partial u}{\partial p} dp.$$

But $\delta\epsilon$ is a perfect differential, since the process is a reversible one.

Or

$$\frac{\partial}{\partial p}\left(V\frac{\partial u}{\partial v} - p\frac{\partial u}{\partial v}\right) = \frac{\partial}{\partial v}\left[V\frac{\partial M}{\partial p} - p\frac{\partial u}{\partial p}\right].$$

Giving

$$\frac{\partial u}{\partial v} = -\frac{\partial M}{\partial p}.$$

But

$$M = cV$$

or

$$\frac{\partial M}{\partial p} = cV\partial.$$

If $\frac{\partial c}{\partial p}$ is very small, we may regard it as independent of V, and then integrate

$$\Delta u = \left[-\frac{\partial c}{\partial p}\right]_2^{V^2}.$$

If $W = 1/2cV^2 =$ energy stored in the dielectric, we have

$$\frac{V^2}{2} = \frac{W}{c}$$

and

$$\Delta u = \left[-\frac{1}{c}\frac{\partial c}{\partial p} \right] W.$$

But

$$W = \int \frac{KH^2}{8\pi}\, du$$

and when dielectric strain is uniform, KH is constant.

$$\therefore \frac{\Delta u}{u} = \left[-\frac{1}{c}\frac{\partial c}{\partial p} \right] \frac{KH^2}{8\pi}.$$

Consider the case of a long, thin-walled cylinder:

Length $= l$,

Wall thickness $= e$,

Mean radius $= R$,

Total tension $= Q$, parallel to the generating lines of the cylinder.

Then

$$\Delta l = \frac{\partial c}{\partial Q}\frac{V^2}{2}.$$

But

$$c = \frac{KRl}{2e}$$

for such a cylinder.

Then

$$\frac{1}{c}\frac{\partial c}{\partial Q} = \frac{1}{S}\left[\frac{1}{K}\frac{dK}{dq} + \frac{1}{R}\frac{dR}{dq} + \frac{1}{l}\frac{dl}{dq} - \frac{1}{e}\frac{de}{dq} \right]$$

where

$$\partial q = \frac{\partial Q}{S} =$$

tension per unit area. But

$$\frac{1}{k}\frac{dK}{dq} = k_1 \text{ [def.]}, \quad \frac{1}{l}\frac{dl}{dq} = a =$$

elastic modulus, and

$$\frac{1}{R}\frac{\partial R}{\partial q} = \frac{1}{e}\frac{\partial e}{\partial q}$$

or

$$\frac{1}{c}\frac{\partial c}{\partial Q} = \frac{a + k_1}{S}$$

$$\therefore M = \frac{\partial c}{\partial Q}\frac{V^2}{2} = \frac{\partial c}{\partial Q}\cdot\frac{1}{c}W$$

$$= \frac{a + k_1}{S}\int \frac{KH^2}{8\pi}\,du$$

[du = element of volume]

$$= \frac{a + k_1}{S}\frac{KH^2}{8\pi}\cdot \text{vol.}$$

[If KH^2 is constant throughout]

$$= \frac{a + k_1}{S}\,Sl\,\frac{KH^2}{8\pi}$$

or

$$\frac{M}{l} = [a + k_1]\frac{KV^2}{8\pi c^2}. \quad \left[\text{Since } H = \frac{V}{c}.\right]$$

It should be remarked that this derivation makes some assumptions in addition to those first mentioned. In particular, that

$$\frac{1}{c}\frac{\partial c}{\partial q} = \frac{1}{R}\frac{\partial R}{\partial q};$$

and further that c is constant throughout, so that all volume elements are equally strained, *i. e.*, K is taken as constant in the integration. These conditions are not fully realized in practice. When the inner cylinder is not exactly concentric with the outer, forces are brought into play not accounted for in such a discussion.

EXPERIMENTAL WORK.

Believing that the interference method was most desirable for work of this kind, a Michelson interferometer was arranged for the purpose. The instrument was placed on a pier in the basement of the laboratory, being brought to a convenient height by slabs of marble. One end of the piece to be tested was rigidly clamped to a metal plate which could be moved parallel to the length of the test

piece by a micrometer screw, one division on the micrometer indicating a movement of .0001 of an inch. The other end controlled the position of one mirror on the interferometer.

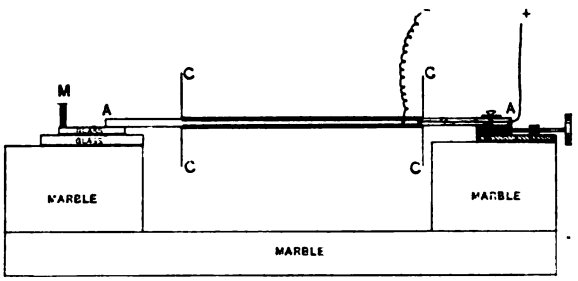


Fig. 4.

In a preliminary test it was found that a thin tube might buckle slightly if required to start the heavy brass block which carried the moving mirror. To avoid this, in the first test the tube was allowed to act against the small springs holding the adjustable mirror in position, and the micrometer screw was then used in the place of one of the adjusting screws of this mirror. This arrangement was extremely sensitive, but it was somewhat difficult to get the fringes in proper adjustment.

Later the brass block moving on the ways was removed and the mirror was fastened directly to the tube, being supported on two small pieces of plane plate glass. This arrangement gave a support responding to the slightest movement, and yet easy to adjust. By means of an auxiliary screw a very slow rotation could be given to the micrometer screw, and the number of fringes passing the cross-hair in a reading microscope could be counted. A series of such readings gave .0000275 cm. as equivalent to a displacement of one band width; that is, from center to center of adjacent black bands. The distance from center to center was measured by 1.95 turns of the screw on the microscope. This microscope was placed on an optical bench, so that the adjustments could be readily made. Considerable difficulty was experienced with vibration and it was necessary to confine work to those hours when no machinery was in operation in the immediate vicinity.

The first trial was made with a strip of hard rubber, 40 centimeters in length, 6 millimeters wide, and 3 millimeters thick. This rested on small glass rollers placed on a plate of glass, on the under side of which was pasted a strip of tin foil about 33 centimeters long and 5 centimeters wide. Another glass plate was supported so as not to touch the rubber, and having a similar strip of tin foil on the under side. The rubber thus formed a part of the dielectric of a condenser. The tin foil strips were connected to the inner and outer coatings of a battery of 9 Leyden jars placed in multiple, in order that the rise in potential might be sufficiently slow to secure observations. This rate of rise could also be governed by varying the speed of the machine, or the number of transformer coils in use. In this particular case no direct measurement of potential was made, but it is sufficient, perhaps, to say that it was carried high enough to puncture the lower glass plate, which was 3 millimeters in thickness.

Under these circumstances a displacement of approximately one-fourth of a band width was observed, increasing gradually, and returning to the original position after each discharge. After a few such movements, this effect could not be produced even on succeeding days. The percentage change of length in this case was approximately 25.10^{-7} and might easily have been due to other causes. However, when a spark gap was placed across the leads from the machine, a considerable displacement of the bands, increasing greatly with time, was observed. Upon stopping the machine without discharging the condensers, this motion was reversed and the bands came slowly back to their original position. It is evident in such a case that the condenser may be considered as subject to an alternating E.M.F. as the jars doubtless tended to discharge across the air gap between two successive series connections of the machine. On looking between the condenser plates, it was observed that the whole region was filled with a glow discharge, and occasional small, bright streamers were seen. It is certainly not unreasonable to assume in such a case that the motion was due entirely to a gradual heating of the hard rubber. Whether the slight motion observed at first was due to elongation of the dielectric under electrostatic strain, or to a slight twisting of the material by reason of

such a field, would be extremely difficult to say. It is readily seen that a very slight twist might easily give such an effect. The potential reached here was high enough to cause frequent discharges over the tops of the jars.

Several more trials were made with hard rubber and in no case was the displacement more than one-fourth of a band; and it was observed that after the first few chargings, this effect disappeared.

A glass rod with a capillary bore was next tried. The diameter of the rod was 5 mm.; its length, 45 cm. It was placed between glass plates in the same way as the rubber rod previously mentioned, and the condensers were subjected to potentials varying from 14 to 80 kilovolts. No movement whatever was observed, except when the connections were made in such a way as to insure an alternating discharge between the condenser plates.

Some small glass tubes with mercury for the inner electrode and tin foil for the outer, were next used. It was found that with very thin tubes the vibration of the building was so great as to make the readings uncertain at times. It is certain in all these cases, however, that no movement as great as one-half a band width was observed when the conditions were such as to avoid a direct heating effect, although in each case the potential was carried high enough to rupture the tube. These tubes were approximately 75 cm. in length, and the thickness of the walls varied from .08 to .12 cm. It was observed that a slight lateral motion, such as that produced by hanging a string over the middle of the tube, gave a rotation of the fringes; while a movement parallel to the length of the tube, such as could be given by the micrometer screw, gave a movement perpendicular to their length. In one case a very slight movement synchronous with charge and discharge of the jars was observed. It ceased after a few discharges, and was too small to measure with certainty, since the bands were then vibrating with an amplitude of approximately one-fourth of a band width.

It is of interest to note also that the passage of a spark over the tops of the jars, when these were near the refractometer, was accompanied by a sudden movement of the fringes, often causing a displacement of three or four bands when the tube was not in the circuit; evidently this was an air wave effect. It was therefore

found necessary to place the battery of jars in such a position as to diminish this effect. The trouble was avoided by placing them near the ceiling, and above the refractometer.

The next trial was made with a glass tube, silvered inside and out, the coatings being joined to those of the battery of nine Leyden jars in multiple. This tube seemed to show a decided effect upon continued application of the charge. The following readings will indicate the behavior:

Duration of Connection.	Total Movement to Left.
20 seconds,	1.25
40 seconds,	2.25
60 seconds,	3.5
After Discharge.	**Total Movement to Right.**
30 seconds,	1.5
60 seconds,	2.
120 seconds,	3.
150 seconds,	3.5–

These observations were repeated several times with almost identical results. It was found that the direction of motion on charging was the same as that obtained by moving a lighted match beside the tube at a distance of three or four inches from it, so as to produce a very slight heating.

Six of the jars were removed and the discharge over the surface of the remaining three was accompanied by a rupture of the tube. This tube, like the others used in the same manner, was covered with a layer of rubber cement on the part not silvered, which served to prevent surface discharge along the glass. When this was not done, it was found that with fairly high potential differences, discharges occurred along the surface of the glass to the refractometer or to the marble surface on which the instrument rested. The expansion observed in this case showed every indication of having its origin in thermal effects. Whether these were produced by the passage of the charge along the extremely thin layer of silver forming the electrodes, or whether the glass was a partial conductor, it is hardly possible to determine.

Another tube was silvered, care being taken to secure a heavy deposit, and the resistance of 75 cms. of the layer was found to be

about 9 ohms. The dimensions of this tube were carefully determined by immersion in mercury.

Length of the tube,	95 cms.
Length of the silvered portion,	73
External diameter,	.84
Interior diameter,	.36
Thickness of the walls,	.24

We were unable to secure a tube of uniform external and internal diameter, and having thinner walls. According to the theory of the subject developed by Sacerdote, the increase of length varies inversely as the square of the wall-thickness, and if uniformity and stability could be secured, it would undoubtedly be desirable to use thin walls, thus bringing the charges close together. Inasmuch, however, as the potential difference at hand was sufficient to rupture the tube, it was thought better to make the trial with a tube as nearly uniform as possible.

In the first observations 18 Leyden jars were used in multiple with each other and with the coatings of the tube. A spark gap 2 cm. long was placed in shunt with the coatings, brass balls 2 cm. in diameter being used for electrodes. During the first 6 or 8 discharges, a movement slightly more than one-fourth and considerably less than one-half of a band width was noted; this increased as the potential rose and diminished after the discharge between the balls. A slow creep of the fringes across the field, in a direction such as would indicate heating, was also observed.

Nine of the jars were then removed and the same effect was observed, the creeping being slightly faster and the springing movement less certain. With one centimeter spark length between the discharge balls, the effect was almost identical with that observed before. After discharging the jars and waiting some hours, another trial gave a slow movement of 3 bands, and the backward jerk of about one-quarter of a band upon discharge.

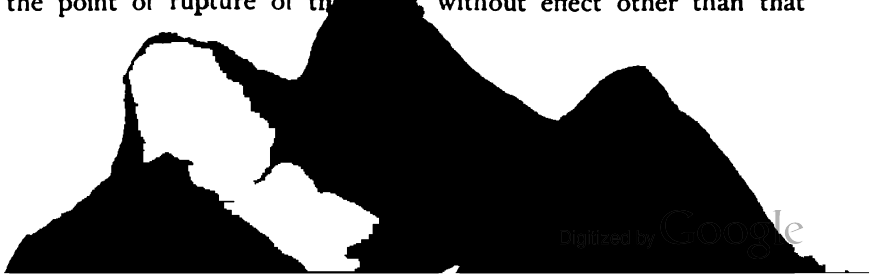
Going back to the 18 condensers in multiple again, and 2 cm. spark, nothing but a slow creeping of the fringes could be observed. On the following day the same series of trials was made with an entire absence of the motion of rising and falling with charge and discharge; the creeping effect varying with the time of charge was,

however, still present, and the return was much slower than the original change.

In some cases the variation in the temperature of the room was sufficient to cause an extremely slight creep of the bands, which could easily be distinguished from other effects. Due allowance could be made for this. In all cases the movement was in the direction indicating expansion during charging. The spark length wasgradually increased up to 4 or 5 cm., and then the jars were removed and the tube connected directly to the terminals of the machine with the spark gap in shunt. After a few sparks had passed between the discharge terminals, a rapid motion of the fringes was observed and the tube was ruptured.

Taking data deduced from Cantone's work as reported by Sacerdote in the *Journal de Physique* of April, 1901, and his approximate value of "a" given in the March number, and assuming the dielectric constant of the glass used as considerably below the average, we find that the observed expansion should have been .000024 for a difference of potential of 130 electrostatic units. If the effect increases with the square of the potential a considerably greater expansion should have been observed in this series of experiments, since a 4 cm. spark length between the balls would demand a breaking potential much larger than assumed above. I have been unable to find data for explosive potential difference for balls of this diameter above 1 cm. spark distance. From a comparison of the curves of spark lengths drawn from data given in the Smithsonian tables, page 244, it is probable that potentials above 150 C.G.S. units were used before rupture of the glass. If K for this particular glass were the average, namely, about 7.5, this expansion would be increased in the ratio of 1.8 to 1 or an amount of .000043, which is certainly far within the limits of possible readings upon the instrument. A displacement of one-fourth of one band width was unquestionably in excess of any observed except where heating was fairly certain, and this corresponds to an increase of length of .000007 cm.

Another trial was made with a tube 86 cm. long, 3.5 cm. external diameter, and 1 mm. thick. This was silvered on the inside and covered with tin foil outside. Various potentials were used up to the point of rupture of th[illegible] without effect other than that



plainly due to temperature change. Not the least motion of the bands could be detected when connection was made to a set of highly-charged jars.

The results given by Kortweg and Julius for hard rubber indicated an expansion greatly in excess of any observed in the case of glass. [Of the order 10^{-8} instead of 10^{-13}.] Several tubes were accordingly secured for a trial on this material. These tubes were about 100 cm. in length, 2 cm. external diameter, with walls approximately 2.5 mm. in thickness. An attempt was made to copper-plate one of these, but it was found difficult to get a coating on the inside. A tube of tin foil could be readily inserted and made smooth, and such a tube 85 cm. long was used. The outside was covered over the same length with light foil. One end was entirely plugged with hard rubber cement, and mica collars were cemented on the ends of the foil to prevent leakage along the tube. The first trial was made with a Wimshurst machine, as the use of the dynamo static machine had been subject to some criticism. A slow and steady movement of the bands was observed, followed by a slow and complete return after discharging. From 8 to 18 bands crossed the field in from 30 seconds to one minute. About ten minutes were required for complete return.

This was repeated several times and several observers noted the movements. All the conditions were favorable for accurate readings, as the bands were broad, distinct and absolutely steady. The large machine was then used and much more marked movements of exactly similar nature were produced. When the spark length between 2 cm. balls was gradually increased from 2 cm. to 4 cm. the motion increased very rapidly and the tube ruptured.

Another tube was then arranged as follows: A long piece of very fine copper wire was passed between metal rollers giving a thin flat ribbon of copper. This was wound spirally around the tube and the ends soldered to two heavy copper rings. A thin layer of shellac served to hold the wire in place and to insulate it from the outer tin foil tube. Heavy copper leads with well amalgamated ends joined the rings to mercury cups from which connections were made to a Wheatstone bridge. One copper ring was connected to the tin foil to avoid puncture.

When the machine was operated with only part of the coils active only a slight movement was noted, except that when the Leyden jars were discharged a sudden backward jerk was seen. This was shown to be due to an air wave by firing a blank cartridge in an adjacent hall, which reproduced the jerk in direction and amount when the proper distance was secured.

It was observed that a slight *decrease* in the resistance of the spiral resulted when the machine was operated even when the condenser tube was entirely disconnected. Doubtless due to a slight coherer action at the various junctions in the circuit. When the connections were arranged so as to give a 2 cm. spark between balls of that diameter the same slow motion of the fringes was seen. As soon as possible the tube was discharged, connections were made with the bridge and a distinct *increase* in resistance was noted.

Sample readings are as follows: Temperature of room very constant. Bands clear and steady. Resistance of spiral 14.80 ohms before starting machine; after running machine a few minutes before it was connected to the tube a deflection of the bridge galvanometer to the left .5 div. indicated a reduction of resistance. Later runs gave no change in this. On connecting to the plates of the cylindrical condenser until 20 bands passed the cross wire a deflection of 8 divisions to the right was noted. When 25 bands more went by a farther deflection of 12 scale divisions in the same direction was observed. It was found that a change of .01 ohm gave a deflection of 24 scale divisions so that the resistance increased .005 ohm while 25 bands crossed the field. A lighted match held two or three inches from the tube for two seconds reproduced the movement of the bands in amount and direction and an increase of resistance very nearly the same as before.

Assuming the temperature coefficient for copper as .004

$14.8 \times .004 \times t^\circ = .005$ [for 25 bands displaced],
Giving $t = .086^\circ$ approximately,
Effective length of dielectic 85 cm.,
Coefficient of expansion for hard rubber .00007,
$85 \times .00007 \times .086 = .000512$,
$25 \times .000029 = .000725$,

so that the displacement observed was of the same order as that computed from the change of temperature. It could hardly be expected that extremely close agreement should be found, as some time elapsed while discharging the condenser and connecting to the bridge.

The general conclusion from the foregoing experiments would seem to be that all the observed changes may be readily accounted for without reference to anything other than heat and the slight distortions due to an unsymmetrical distribution of charge. The phenomena of residual charge and of variations in dielectric resistances are entirely consistent with the idea of the passage of initially weak conduction currents into or through the dielectric. That these would cause a slight heating could hardly be denied. When this current attains any appreciable value in a small region, owing to accidental irregularities in potential difference, or to slightly higher local conductivity, a sudden rise of current with a rapid development of heat would result, causing unequal thermal expansion and possible vaporization of material, thus rupturing the dielectric.

In most of the above experiments a movement of $\frac{1}{20}$ of a band width could have been noted with certainty so that movements as large as often reported could hardly have escaped detection.

In view of the great variation in the results of different observers, and of the fact that all work on the theoretical side has been based on experiments more or less in dispute, it would seem that the production of changes of form and dimensions in matter by purely electrostatic strain is extremely improbable.

In conclusion, the writer wishes to express thanks to Mr. Ernest Blaker, Instructor in Physics at Cornell University, for assistance in construction of the machine; to Mr. Frank Allen, Fellow in Physics, for aid in making readings; and especially to Professor E. L. Nichols for his kind interest and assistance in the work.

REFERENCES.

Cantone, M.

1888 Rendiconti della R. Accademia dei Lincei, t. IV.

1888 Nuovo metodo per la determinazione delle due constanti di elasticita, C. R. della Accademia del Lincei, vol. IV.

Cantone, et Sozzani.

1900 R. C. del R. Instuto Lombardo, Serie 2, vol. XXXIII.

1900 Il nuovo Cimento, Serie 4, t. XII, p. 150, Oct.

Boltzmann.
1880 Wien. Ber. (2), vol. 82, pp. 1–14.

Curie, M.
1888 Lumiere electrique, t. XXX, p. 423.
1888 Lumiere electrique, t. XXX, p. 465.

Desau.
Rendiconti della R. Acc. dei Lincei, 5 serie, t. III, pp. 488–494.

Duhem.
1886 Lecons sur l'Elect. et le Mag., t. II, liv. XII.
1895 Le Potentiel Thermodynamique, p. 210 a 220.
1856 Mem. des Savants etrangers a l'Academie, t. XIV, pp. 288–393.

Duter.
1878 Comptes rendus, t. LXXXVII, p. 828.
1878 Comptes rendus, t. LXXXVII, p. 1036.
1878 Comptes rendus, t. LXXXVII, p. 960.
1879 Comptes rendus, t. LXXXVII, p. 1260.

Ercolini.
1891 Rendiconti della R. Acc. dei Licei, 5 serie, t. VII, pp. 172–177 et 183–189.
1898 Nuovo Cimento, 4 serie, t. VIII.
1902 Jour. de Physique, p. 40.

Gorbino, O. M.
1897 Rivista scientifica e industriale, 29 annee, n 8–9, septembre.
1899 Rendiconti della R. Acc. d. Lincei, vol. VIII, 2 sem. 5 novembre.

Gorbino et Gannizzo.
1898 Rendiconti della R. Acc. dei Lincei, 5 serie, t. VII.
Il nuovo Cimento, 4 serie, t. VIII.

Holtz.
1883 Wied. Ann., XIX, pp. 559–566.

Kirchhoff.
1885 Wied. Ann., XXIV, p. 52, etc.

Korteweg.
1880 Wied. Ann., IX, p. 48, etc.
1879 Comptes rendus, t. LXXXVIII, p. 338.

Julius et Korteweg.
1881 Wied. Ann., XII, pp. 647–665.

Lippmann.
1881 Principe de la conservation de l'electricite, Ann. de Chem. et de Phys., 5 series, t. XXIV, p. 159.

Lorberg.
1884 Wied. Ann., XXI.
1884 Wied. Ann., XXI, p. 300, etc.

More.
1900 (2) Phil. Mag., p. 198, Aug.
1901 Phil. Mag., Nov.

Moutier.
1878–'79 Bulletin de la Societe philomathique de Paris, 7 serie, t. III, p. 88.
1879–'80 Bulletin de la Societe philomathique de Paris, 7 serie, t. IV, p. 182.

Panighi, Ugo.
1898 Il nuovo Cimento, 4 serie, t. VIII.

Pellat, M.
1895 Annales de Physique et Chimie, 7e serié, t. IV.
1895 Journal de Physique, 3e serié, t. V, p. 525.

Quincke.
1883 Electrische Untersuchungen, Wied. Ann., XIX, p. 569.
1883 Wied. Ann., XIX, p. 573.
1880 Wied. Ann., X, pp. 161–202.
1880 Wied. Ann., X, pp. 513–553.
1880 Wied. Ann., X, pp. 374–414.

Righi.
1879 Sulla dilatazione dei coibenti armati par a fetta della carica (Bologna). Extrait—Comptes rendus, t. LXXXVIII, p. 1262.

Roentgen.
1880 Wied. Ann., X, pp. 771–786.

Sacerdote.
1900 (1) Ann. de Chim. et de Phys., 7, t. XX, p. 289, Juillet.
1901 Journal de Phys., 3e, t. X, p. 281, Avril.
1901 Journal de Phys., 3e, t. X, pp. 196–200, Mars.
1901 (3) Phil. Mag., March.

Vaschy.
Theorie de l'electricite, p. 71.
1887 Comptes rendus, t. CIV, p. 51.
Traite d'Electricitie et de Magnetism, t. I, pp. 107–108.
1887 Comptes rendus, t. CIII, p. 1186.
Traite d'Elect. et de Mag., t. I, p. 119, etc.

A CARBON ELECTROLYTIC INTERRUPTER.

By A. H. Taylor.

A WEHNELT interrupter using a platinum point in a solution of sulphuric acid has the great disadvantage that when carrying large currents the platinum wears away. This suggested the substitution of a cheaper metal for the anode.

An interrupter can be made by using a lead plate as cathode in a solution of sulphuric acid, and as anode a heavily insulated copper wire. The tip of the wire is laid bare for about 1.0 to 1.5 mm. However, this interrupter works poorly and requires constant adjustment on account of the wearing away of the copper and the tearing of the insulation. Other experiments were tried with indifferent success. Under suitable conditions lead, iron, copper, brass or carbon may be used as anodes, and, as has been pointed out by others, many different solutions may be used as electrolytes. An acid seems to give a little the best results, but is open to the objection that obnoxious fumes are then emitted by the interrupter.

The critical voltage below which an interrupter fails to work satisfactorily seems to vary with different metals.

Although the critical voltage for carbon is rather high, it gave by far the most satisfactory results. The first form of carbon anode used was a 3 mm. carbon rod in a close fitting hard rubber jacket, covering it to within a few millimeters of the tip. However, this overheated with a current of four to six ampères, and fused the rubber jacket. The type shown in Fig. 1 was then devised: C is a carbon rod 3 mm. in diameter, and copper-plated to within a few millimeters of the tip. This rod fits loosely in a thistle-tube, T, which in turn fits the cork K of the lead jar L and dips into the electrolyte, in this case a solution of KOH.

The end of the tube T has a little stirrup S drawn out on one side of it. Upon the point of this stirrup rests the carbon rod C.

The rod has a binding post at B and the jar one at B'. The copper plating, by increasing the conductivity of the rod prevents excessive development of heat in it. As the rod wears away it slides down the tube and the copper wears off. The cork K has a small opening to allow the escape of any gases generated.

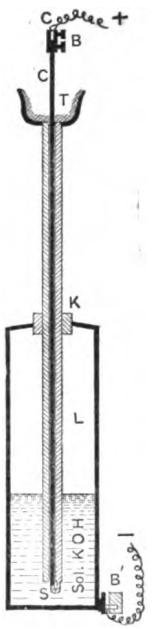


Fig. 1.

Such a size of interrupter as this one, where the rod is 3 mm. in diameter and projects below the tube T 1.5 mm., is adapted for use with a 6-inch spark coil directly on a 100–110-volt direct or alternating current circuit without the use of a resistance in series. The size of the lead jar and the quantity of solution vary with the facilities for disposing of the heat developed in the interrupter. If some simple cooling device is used, such as immersing the jar in cold water, it may be of the dimensions shown in the sketch, *e. g.*, about 2.7 cm. in diameter and 8.0 cm. high.

To allow automatic adjustment, the carbon rod fits the tube T rather loosely and during operation the liquid is forced up into the bulb of the thistle tube until equilibrium is attained. The period of the interrupter may be varied as with other interrupters, by varying the depth of the solution. The concentration of the solution affects the current strength only by a limited amount after a certain point is reached, beyond which the current strength may be regulated by the length of the projecting stirrup S and the size of the carbon rod. With a solution of one part concentrated KOH to four of water, the above dimensions give a current of about 5 to 6 ampères on 100 volts with a 6-inch spark coil intended for use on an 8-volt circuit. This easily gives the required spark length and shows some peculiarities in the spark which will be spoken of later.

By adjusting the stirrup S or varying the size of the rod, an interrupter may be made to give almost any spark length up to the maximum spark length which the secondary of the coil will stand. Similarly, interrupters may be adjusted for different coils.

For use with X-ray tubes the direct current is preferable, but for

many other purposes, as for wireless telegraphy, etc., the alternating current is preferable, because with it the interrupter works more evenly. This interrupter requires a minimum D. C. voltage of about 65–70 volts to give satisfactory results and works best between 85 and 110 volts. With the alternating current the minimum voltage is lower, just how much has not been determined.

For any given interrupter on a constant voltage there seems to be a critical spark length at which the interrupter works best.

When this spark length is attained the ordinary ragged, bright discharge is accompanied by a smooth, curved, red or violet discharge, which rapidly oscillates from point to point of the discharge knobs. This part of the discharge is more easily deflected by a blast of air than the bright ragged part. By shortening the spark gap and introducing a capacity such as a Leyden jar into the secondary circuit, an intensely bright and noisy spark of great energy is produced, while the Leyden jar, if suddenly disconnected, is found to be charged so as to give a spark of 2 to 5 millimeters.

Most of these peculiarities of the spark are characteristic of electrolytic interrupters, and are only mentioned here to show that this interrupter, which is cheap and simple in construction, can be used in almost any case where the platinum interrupter can.

One striking peculiarity is noticed when the interrupter is used on an alternating current circuit, with an alternate and a direct current ammeter in series. On several occasions when the A. C. ammeter read from 5–7 ampères the D. C. ammeter read 0.3 to 0.4 ampère.

This was not accidental, but could be repeated, not with exactly the same current readings, but the D. C. instrument invariably gave a reading showing a slight excess of current from the carbon to the lead. A copper voltameter in the circuit gave a result corresponding to 0.38 ampère while the total current as measured by an electro-dynamometer was about 4.0 ampères. This voltameter also showed the excess current to be from carbon to lead.

These results suggested that possibly there might be a rather high E.M.F. of polarization and a D. C. or A. C. voltmeter was used giving on a 100-volt circuit, 108 volts fall across the interrupter, and 55 volts fall across the coil. To determine whether

these results were altogether due to phase differences, a D. C. voltmeter was used. This showed about 0.1 volt on the whole circuit, about 0.2 across the coil, and about 0.7–0.8 across the interrupter.

These results are necessarily only approximate since the action of the interrupter is to a slight degree intermittent, and causes unavoidable quivering and oscillating in the voltmeter needle. Such as they are, however, they do not justify the assumption of any very large E.M.F. of polarization in the interrupter. The readings of the D.C. voltameter are only what would be expected on account of the 0.3 or 0.4 ampère excess current from the carbon to the lead.

This excess current is probably due to the fact that the current is interrupted a less number of times in passing from carbon to lead than when passing from lead to carbon. For in the first case the break is probably caused mostly by the accumulation of oxygen on the anode, and in the second case mostly by the accumulation of hydrogen, although analysis shows that both gases appear at the anode when a direct current is used. But since the hydrogen accumulates the faster it is not unfair to assume that the interruptions would be more frequent in one direction than in the other with the alternating current.

A wattmeter indicated a consumption of 62.5 watts for a six-inch spark, but the spark was plainly of much greater continuity and energy than that produced by any mechanical interrupter.

In conclusion, this interrupter may be recommended as a cheap, simple, self-adjusting instrument well suited for continuous service.

PHYSICAL LABORATORY, MICHIGAN AGRICULTURAL COLLEGE,
November 6, 1901.

NEW BOOKS.

The Experimental Study of Gases. By MORRIS W. TRAVERS. 8vo, pp. xiii, 323. London, Macmillan & Co., 1901.

The field covered by the work of Dr. Travers is well indicated by its title—The Experimental Study of Gases. But the title alone gives no indication of how comprehensive the book is in its discussion of experimental methods and in the description of important investigations. The great advances that have been made in recent years-in our knowledge of gases, and especially of the gases of the atmosphere, make such a book peculiarly appropriate at the present time. It is a matter for congratulation that the work has been prepared by one who, on account of his experience as an investigator in this field, is unusually well prepared to undertake the task.

After two pages devoted to a statement of the fundamental laws of Boyle and Gay-Lussac and Avogadro's hypothesis, the author begins at once an account of experimental methods and apparatus, the first subject considered being that of mercury pumps. A somewhat extended elementary knowledge of the general properties of gases is presupposed. No time is devoted to a duplication of work of this character that has been well presented elsewhere. This is especially true as regards the theoretical side. In fact, the author has confined himself so closely to the experimental side of the subject that theoretical discussions occur only in cases where experiment and theory are so closely related that it is almost impossible to separate them.

It will be impossible here to do more than touch upon the many topics treated. The following headings of chapters will give an idea of the scope of the book in dealing with apparatus and methods: Mercury Pumps; Gas Manipulation, Stopcocks, Collecting and Storing Gases, etc.; the Preparation of Pure Gases; Gas Analysis; Determination of Density; Relationship of Temperature, Pressure and Volume; the Liquefaction of Gases, and Manipulation of Liquefied Gases; Vapor Pressure and Critical Constants; Specific Heats; Spectrum Analysis.

Among the portions of the book that are devoted to investigations of the properties of gases, rather than to apparatus and methods, may be mentioned the chapters on the Composition of Atmospheric Air, and the Gases of the Helium Group. An interesting account is here given of the dis-

covery of argon and helium, as well as of the more recently discovered members of this remarkable group of gases, krypton, neon, and xenon. When it is remembered that Dr. Travers is one of the discoverers of the three gases last named, and that he has been associated with Professor Ramsay and others in a large part of the work done in this field since the discovery of argon, it is natural to expect that the book before us should be especially strong in its treatment of the helium group. And this is in fact the case. The methods used in obtaining these gases, especially the very generally applicable method of fractional distillation, are described in detail; while numerous tables are given of data regarding their properties and constants. Probably nowhere except in the original papers of Rayleigh and Ramsay, Ramsey, Ramsay and Travers, and the other investigators in this field, can so complete an account of the recent work be found.

As the author says in the preface, the breadth of the subject does not permit of its complete treatment within the limits of a single volume. In many cases, therefore, it has not been possible to go into such detail as the importance of the topic would make desirable. This is true, for example, in the chapters dealing with the liquefaction of gases, gas analysis, and spectrum analysis. But each of these subjects deserves a volume for itself if its treatment is to approach completeness. The author has been forced to restrict his consideration to the most essential points; the choice of those points that are to be regarded as essential appears to me to be an excellent one. In the chapter on the liquefaction of gases a brief historical account is first given. Reference is then made to the apparatus of Linde and Hampson for liquefying air, the latest form of the Hampson apparatus being described in detail. The dependence of such methods of liquefaction on the Joule-Thomson effect is clearly brought out, as is also the failure of the method when applied to hydrogen, under ordinary conditions of pressure and temperature, on account of the fact that the Joule-Thomson effect is negative for this gas. The method may be used, however, if the hydrogen is first cooled to about — 200° C. since the Joule-Thomson effect changes sign somewhere between — 80° C. and — 200° C. Apparatus for liquefying hydrogen is described in detail.

A very satisfactory feature of the book is the frequent citation of original articles. In only a few cases do such references seem insufficient. Mention of the numerous tables of important constants connected with the gases, which appear throughout the book, should also not be omitted. Dr. Travers' work can only be regarded as an important addition to the literature of physics.

ERNEST MERRITT.

Leichtfassliche Vorlesungen über Elektrizität und Licht. By DR. G. JAUMANN. Leipzig, Barth, 1902. Pp. xii + 375.

This volume has as its basis a series of university extension lectures; its special object as expressed in the preface being to introduce to students in the high schools the Maxwellian theories of electricity and light and to assist teachers in the lower schools in their endeavors to modernize their instruction. The author places a certain limit upon the modernity of his own treatise, however, in that he expressly omits from consideratien all theories involving ions, electrons, or other corpuscular concepts and he is consistent in this matter even to the point of regarding Röntgen Rays as a variety of light of extremely short wave-length, cathode rays as electro-magnetic rays produced by longitudinal vibrations accompanied by circular magnetic alternating currents, etc. With these exceptions the treatise is for the most part in accordance with the accepted theories concerning the nature of light and its relations to the phenomena of electricity.

The standpoint of the whole treatise is that of flux. The phenomena of the magnetism of iron are treated in lecture I., under the title of *The Magnetic Flux*. Electrostatics to which the second lecture of the course is devoted has the heading, *The Electric Flux*. The voltaic current is designated as a new form of electric flux brought about by the action of the voltaic element or of some analogous generator in which the lines of flux are closed curves, forming electric whirls or vortices. In pursuance of the same system we find the sixth lecture devoted to the magnetic whirl of a voltaic current, in other words to the magnetic effects of the current, while electromagnetic induction is treated as an electric vortex motion.

To introduce concepts of this sort to the general public and to beginners in the study of physics, such as those for which this book is intended, without producing utter perplexity of mind, is a task of the greatest difficulty. To prepare the reader for the conceptions of electric and magnetic flux Professor Jaumann has written an introductory chapter on the *flow of water*. By the use of stream lines a number of very interesting diagrams are worked up and discussed and the consideration of these which are selected on account of their close analogy to the most important kinds of electromagnetic and electrostatic fields the reader is undoubtedly assisted in the consideration of electric and magnetic phenomena. This introductory chapter is of great interest and the student who succeeds in working through it consistently will doubtless be in position to comprehend more readily the corresponding cases in the electric and magnetic portions of the book. It is, however, by no means easy, although the treatment is elementary throughout in the sense of not demanding extensive mathematical preparation, and it is to be feared

that a comparatively small proportion of readers who are without previous scientific training will be able to cope with it.

Professor Jaumann's treatment of the phenomena of electricity and magnetism by the method of electric and magnetic flux is worked out with all the completeness possible and with as high a degree of consistency as this purely artificial hypothesis will permit. The author errs on the side of giving his system too great a semblance of reality. Why rescue the student of electricity from the old idea of the electric current and from the still older one of the two electric fluids only to imbue him with equally artificial conceptions of electric and magnetic flux? The drawing of lines of force pure and simple as originated by Faraday and developed by Maxwell and his followers affords a system capable of yielding all that can be obtained from this newer modification and it has the great advantage of being frankly geometrical in its character. The treatment of electric and magnetic phenomena as though they were fundamentally phenomena of flux instead of utilizing flux simply as a convenient but avowedly artificial means of computing magnetic and electric field is surely open to objections no less serious than were the methods of our forefathers. E. L. N.

A Laboratory Manual of Electro-therapeutics. By W. J. HERDMAN and F. W. NAGLER. 8vo., pp. 128. Ann Arbor, Mich., George Wahr.

It is hardly to be expected that many readers of the PHYSICAL REVIEW will be in a position to take an appreciative interest in this manual from the standpoint of the physician. Nor is the book one that would attract attention from the physicist as a laboratory manual of electricity for general use. But as indicating the character and extent of the legitimate applications of electricity in medicine I think the text-book of Doctors Herdman and Nagler will be found of much interest.

Probably no department of medicine has offered such opportunities to the deliberate or ignorant "quack" as the therapeutic use of electricity. That the current and the electric spark produce well marked physiological effects has long been known. That the physiological action of electricity, if properly directed, might be of important assistance in the treatment of disease seemed obvious. But strangely enough this very fact resulted in retarding the scientific development of the subject. Popular expectation in this branch of medicine went far in advance of systematic knowledge. In the early days there was a general impression abroad that electricity could accomplish anything; and the physician who used this new curative agent only in those few cases where its efficacy had already been well established was looked upon as not up-to-date. The unscrupulous were not slow in taking advantage of this condition of

affairs. The most extravagant claims were made regarding the applications of electricity, and it was used in absurd and reckless ways, with results more often harmful than beneficial. Finally public credulity could stand it no longer, and a natural reaction came. Among intelligent people the whole subject fell into disrepute, and there was a tendency to regard the employment of electrical methods with suspicion. We have not entirely outgrown this feeling yet. It has certainly deterred many physicians from proper and legitimate experiments in this field, with the result that this branch of electricity has progressed less rapidly than would otherwise have been the case. Those who have continued the scientific and conservative study of electro-therapeutics in spite of the prejudice against the whole subject are all the more deserving of credit.

From the manual before us the reader has a chance to learn the character of laboratory instruction in electricity and electro-therapeutics in a medical school of recognized standing. Probably there is no better way of finding what are regarded by those competent to judge as the present legitimate and established medical applications of electricity.

Among these applications are many in which the physiological effects of the current play no part; for example the use of incandescent lamps for illuminating the nasal and other cavities of the body, the electric cautery, the use of Röntgen rays, etc. In order to employ electricity for these purposes the physician needs an elementary but practical knowledge of fundamental electrical laws. Such knowledge the authors aim to impart by means of simple experiments, often performed directly upon apparatus intended for the physician's use. In this respect the plan of the book seems to me excellent, while the experiments are well suited to give the student a good practical grounding in those parts of the subject of electricity with which he needs to be familiar.

In the portions of the book that deal with the physiological effects of the current the same plan is followed of using direct experiments whenever this is possible. Reference to a few such experiments will indicate their character: For example, the measurement of the current that is just sufficient to produce muscular contraction on make or break under specified conditions; the effects of electrolysis when current is sent through fresh meat; the resistance of the body under various conditions; the introduction of cocaine into the tissues through the skin by means of electric osmosis. In all the experiments of this kind stress is laid on the desirability of actually *measuring* the current used. This feature of the book can scarcely be too highly commended.

In many parts of the subject it is of course impossible to illustrate the therapeutic principles involved by experiments suited to the laboratory. In such cases the effects are described and the method of procedure in using the current is briefly indicated. The mode of treatment resembles

in some respects the account of the use of some drug in a book on materia medica. Chapters are devoted to the therapeutic uses of the induction coil discharge, the high potential sinusoidal current, the Tesla high frequency discharge, the influence machine, etc. It is interesting to learn from these chapters the results that are accomplished by the use of the current, as well as the differences between the physiological effects produced by the different forms of current. For example, a short spark discharge from a static machine is used to produce local irritation and a stimulating effect on the skin; while the "spray" or brush discharge is soothing in its effects and is used to relieve nervous excitement, headache, and insomnia. The sinusoidal current (the ordinary alternating current) is regarded as the current *par excellence* "for exciting to vigorous action muscular tissue, whether it be of the voluntary or involuntary variety," and is used for general muscular weakness, local paralysis, etc. The discussion of the effects and uses of the high frequency currents is especially interesting. In general these currents are found "to impart an extraordinary activity to nutritive changes and to cellular life." The method of treatment is to enclose the patient, without contact, in a large solenoid traversed by the current, so that the currents that are effective are those produced by induction in the body itself. It will be recalled that remarkable results were obtained by d'Arsonval a few years ago by this method. It appears that these results have since been in the main confirmed. Experiments are described indicating that the same procedure with currents of ordinary frequency produces similar but less marked results. ERNEST MERRITT.

A Manual of Laboratory Physics. By H. M. TORY and F. H. PITCHER. First Edition. New York, John Wiley & Sons, 1901. Pp. 284. Price, $2.00.

This book is intended for instruction in elementary laboratory work in sound, light, heat, magnetism and electricity. No experiments are given in mechanics. The manual is based on the experience of the authors as demonstrators in actual laboratory work with elementary students, and is arranged so as to present those experiments and subjects first which appeal most readily to the beginner.

Under each experiment some notes are given on the theory involved which are more or less complete according to the capacity of the student. The formulas to be used are given and in some cases are worked out very fully. Especially is this true in the part of the work devoted to heat, magnetism and electricity. With each experiment is also given a complete list of references to standard text-books which will be found of much value in supplementing the theory given in the manual. Following the theory there are found complete directions regarding

the order and methods of experimentation and precautions to be taken; and a list of necessary apparatus is also given.

The beginner in the laboratory has little idea of how and what data to take and how to arrange them in most convenient forms. There is given, with each experiment, an example of tabulation for a complete set of data and results, which in itself teaches accuracy, order and neatness. There are also blank forms given, directly below the examples, to be filled out by the student. They will be found too small for the data that should be taken, but will serve as guides in working up the data to be embodied in the student's report.

The manual will be found very useful in the field for which it is intended, and especially in a laboratory course to be given in conjunction with text-book work in the class room, for the range of experiments given is sufficiently large and varied to emphasize the facts brought out in the text. ERNEST BLAKER.

Schweizerische Bergbahnen: Die Industrielle und Kommerzielle Schweiz, No. 3 and 4, June, 1901. Polygraphisches Institut A. G., Zürich.

Nowhere has the mountain railway received such development as in Switzerland and few are the systems which are not there represented. Indeed many are to be found in Switzerland alone, or have there been originated. To layman and engineer alike this volume of 178 pages will be of great interest, and to those who seek information about mountain railways and to those concerned with traction of any kind it will prove of exceeding value.

Comprising as it does only mountain railways, some twenty-three of which are treated in great detail, we find in the volume no description of any tram lines, not even of the historic tram at Lugano.

A feature of the book is its magnificent illustrations, more then three hundred in all, many of them from photographs showing characteristic views in the vicinity of the railways, and these will prove of interest to the non-technical reader. It is however for the technical data that the book is particularly valuable and for this it should be consulted as a most convenient and authoritative source. F. BEDELL.

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THE

PHYSICAL REVIEW.

THE ELASTIC PROPERTIES OF HELICAL SPRINGS.

By Joseph Warren Miller, Jr.

1. *Historical Note.*—In 1814 M. J. Binet[1] noticed the fundamental principle that helical springs act chiefly by torsion. Apparently nothing was done in this line between that time and 1848, when James Thomson[2] showed that a helical spring, of infinitely small slope, acts exactly as a torsion balance using the same wire straightened. He verified his theory by experiments. The fact that a spring acts as a torsion balance means it follows Hooke's law. It is this close accord with Hooke's law that is utilized in the ordinary spring balance. Apparently a complete statement of the theory of helical springs was first given by Kelvin and Tait in their "Natural Philosophy,"[3] 1867. Certain aspects of the theory were also given by Clebsch[4] and Kirchhoff.[5] Some of the most recent contributions upon this subject are those of Prof. A. E. H. Love, who devotes several pages of his "Theory of Elasticity,"[6] to helical springs, and who also published a paper on "The Propagation of Waves of Elastic Displacement along a Helical Wire," which appeared in the volume of "Memoirs on the occasion of the jubilee of Sir George G. Stokes."[7]

[1] Journal de L'École Royal Polytechnique, Tome X., p. 419.
[2] Camb. and Dub. Math. Journal, Vol. III., p. 258.
[3] Vol. II., Arts. 604–607.
[4] Vorlesungen über Mathematische Physik, Mechanik, 1876.
[5] Theorie der Elasticität fester Körper, 1864.
[6] Vol. II., Chapters XIV.–XVI., 1892.
[7] p. 346, 1900.

The purpose of the present paper is to set forth the results of a research conducted in the Department of Mechanics in Columbia University. The original object of this enquiry was to learn whether helical springs could be used to determine the acceleration due to gravity. Early experiments showed the need of a more extended enquiry into the behavior of springs under static conditions. Thus the following investigation refers chiefly to the static constants and properties of a set of helical springs of different dimensions and metals.

2. *General Theory.*—To define the bending and twisting essential in a wire whose elastic central line assumes the form of a helix of slope α on a cylinder of radius r, consider the relations shown in the accompanying diagram.

At any point O on the elastic central line draw in the plane tangent to the cylinder the tangent to the helix OX', the element OZ, the horizontal line OX, and the line OZ' perpendicular to OX'. Let ds be an element of the elastic central line measured from O towards X'. The angular rotation about the axis of the cylinder corresponding to this element, or what is the same thing, the angle between two meridian planes through the ends of ds, is $ds \cos \alpha/r$. This rotation is equivalent to its components about OZ' and OX' that is to $ds \cos \alpha \cos \alpha/r$ and $ds \cos \alpha \sin \alpha/r$. The first of these is a pure bending and the second is a pure twisting when applied to the wire. Thus for a right cylindrical helix the amounts of bending and twisting per unit length of the wire are $\cos^2 \alpha/r$ and $\cos \alpha \sin \alpha/r$ respectively.[1] The potential energy of a wire of length s, measured along its elastic central line, bent into a helix of slope α and radius r is hence easily deduced. For the part due to bending is proportional to $(\cos^2 \alpha/r)^2$ per unit length, and the part due to twisting is proportional to $(\cos \alpha \sin \alpha/r)^2$ per unit length. Therefore the work done in producing the helix is

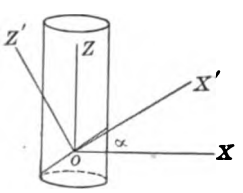

Fig. 1.

$$\tfrac{1}{2} s \left\{ A \frac{\cos^2 \alpha \sin^2 \alpha}{r^2} + B \frac{\cos^4 \alpha}{r^2} \right\},$$

[1] Compare Thompson and Tait's Natural Philosophy, Part II., Art. 602, p. 137.

where A is the nodulus of twist, and B is the nodulus of bending.

If we write

$$\begin{aligned} z &= s \sin \alpha, & \varphi r &= s \cos \alpha, \\ z_0 &= s \sin \alpha_0, & \varphi_0 r_0 &= s \cos \alpha_0; \end{aligned} \qquad (1)$$

where z_0 is the axial length of the helix, r_0 is the radius of the cylinder containing the elastic central line of the wire, and φ_0 the angle between two planes through the ends of the helix and its axis in the unstrained condition; and, similarly, z, r, and φ the corresponding quantities for the strained condition, then the curvature

$$\cos^2 \alpha / r = \varphi \cos \alpha / s$$

and the change in curvature, per unit length, is

$$(\varphi \cos \alpha - \varphi_0 \cos \alpha_0)/s.$$

The potential energy per unit length, due to the bending, is proportional to $(\varphi \cos \alpha - \varphi_0 \cos \alpha_0)^2/s^2$, and hence this part of the energy for the whole wire is

$$\tfrac{1}{2} \frac{B}{s} (\varphi \cos \alpha - \varphi_0 \cos \alpha_0)^2.$$

Similarly the change in twist per unit length is

$$\frac{\cos \alpha \sin \alpha}{r} - \frac{\cos \alpha_0 \sin \alpha_0}{r_0} = (\varphi \sin \alpha - \varphi_0 \sin \alpha_0)/s;$$

whence the potential energy due to twist alone is

$$\tfrac{1}{2} \frac{A}{s} (\varphi \sin \alpha - \varphi_0 \sin \alpha_0)^2.$$

Calling the entire potential energy V, it follows that

$$V = \tfrac{1}{2} \frac{A}{s} (\varphi \sin \alpha - \varphi_0 \sin \alpha_0)^2 + \tfrac{1}{2} \frac{B}{s} (\varphi \cos \alpha - \varphi_0 \cos \alpha_0)^2. \qquad (2)$$

Let

P = longitudinal pull

and

L = torsional moment.

Then we have

$$P = \frac{dV}{dz} = \frac{dV}{da}\frac{da}{dz} = \frac{1}{s\cos a}\frac{dV}{da},$$

or

$$P = \frac{A}{s^2}(\varphi \sin a - \varphi_0 \sin a_0)\varphi + \frac{B}{s^2}(\varphi_0 \cos a_0 - \varphi \cos a)\varphi \tan a, \quad (3)$$

and

$$L = \frac{dV}{d\varphi} = \frac{A}{s}(\varphi \sin a - \varphi_0 \sin a_0)\sin a + \frac{B}{s}(\varphi \cos a - \varphi_0 \cos a_0)\cos a. \quad (4)$$

If the applied couple L is zero equation (4) gives

$$\frac{A}{B} = \frac{\cos a}{\sin a}\cdot\frac{\varphi_0 \cos a_0 - \varphi \cos a}{\varphi \sin a - \varphi_0 \sin a_0}. \quad (5)$$

If we introduce (5) in (3) there results

$$\frac{Ps^2}{A} = (\varphi \sin a - \varphi_0 \sin a_0)\varphi \sec^2 a. \quad (6)$$

Expanding the second member of this by Maclaurin's series and solving for P we get

$$P = \frac{A\varphi_0^2}{s^2}\sec^2 a\left\{\sin a - \sin a_0 + (2\sin a - \sin a_0)\frac{\Delta\varphi}{\varphi_0} + \sin a\left(\frac{\Delta\varphi}{\varphi_0}\right)^2\right\}, \quad (7)$$

$$= \frac{A\varphi_0^2}{s^2}\sec^2 a\left\{z - z_0 + (2z - z_0)\frac{\Delta\varphi}{\varphi_0} + z\left(\frac{\Delta\varphi}{\varphi_0}\right)^2\right\}. \quad (8)$$

From (8) we see that P varies nearly as $(z - z_0)\sec^2 a$, since $\frac{\Delta\varphi}{\varphi_0}$ is generally small. If we divide (3) by B and then introduce (5) we get

$$\frac{Ps^2}{B} = (\varphi_0 \cos a_0 - \varphi \cos a)\varphi(\tan a + \cot a), \quad (9)$$

$$= (\varphi_0 \cos a_0 - \varphi \cos a)2\varphi/\sin 2a.$$

As before, expanding the second member by Maclaurin's series,

$$P = \frac{2B\varphi_0^2}{s^2 \sin 2\alpha}\left\{\cos\alpha - \cos\alpha_0 + (\cos\alpha_0 - 2\cos\alpha)\frac{\Delta\varphi}{\varphi_0} - \cos\alpha\left(\frac{\Delta\varphi}{\varphi_0}\right)^2\right\}. \tag{10}$$

From (7) and (10) we have

$$\frac{1}{A} = \frac{\varphi_0^2 \sec^2\alpha}{Ps^2}\left\{\sin\alpha - \sin\alpha_0 + (2\sin\alpha - \sin\alpha_0)\frac{\Delta\varphi}{\varphi_0} + \sin\alpha\left(\frac{\Delta\varphi}{\varphi_0}\right)^2\right\}, \tag{11}$$

$$\frac{1}{B} = \frac{2\varphi_0^2}{Ps^2 \sin 2\alpha}\left\{\cos\alpha_0 - \cos\alpha - (2\cos\alpha - \cos\alpha_0)\frac{\Delta\varphi}{\varphi_0} - \cos\alpha\left(\frac{\Delta\varphi}{\varphi_0}\right)^2\right\}. \tag{12}$$

In case $\Delta\varphi$ is negligible these formulas give A and B independently. In general, however, terms in $\Delta\varphi$ may not be neglected in (12). For springs of small slope (11) will usually give a close approximation to A. Either formula may be used for exact computations when $\Delta\varphi$ or A/B is known, as shown below.

From (5) we get

$$\frac{\varphi}{\varphi_0} = \frac{B\cos\alpha_0 + A\sin\alpha_0\tan\alpha}{B\cos\alpha + A\sin\alpha\tan\alpha} = \frac{B\cos\alpha\cos\alpha_0 + A\sin\alpha\sin\alpha_0}{B\cos^2\alpha + A\sin^2\alpha} = \frac{\cos(\alpha - \alpha_0) + x\cos(\alpha + \alpha_0)}{1 + x\cos 2\alpha}, \tag{13}$$

wherein

$$x = \frac{B - A}{B + A}.$$

To compute the change in φ the following formula is best

$$\frac{\varphi}{\varphi_0} - 1 = \frac{\varphi - \varphi_0}{\varphi_0} = \frac{\Delta\varphi}{\varphi_0}$$
$$= 2 \sin \tfrac{1}{2}(\alpha - \alpha_0) \frac{x \sin \tfrac{1}{2}(3\alpha + \alpha_0) - \sin \tfrac{1}{2}(\alpha - \alpha_0)}{1 + x \cos 2\alpha}. \tag{14}$$

From (14) we see that $\Delta\varphi = 0$ for two values of α, namely, for $\alpha = \alpha_0$ and for the value obtained from

$$x \sin \tfrac{1}{2}(3\alpha + \alpha_0) = \sin \tfrac{1}{2}(\alpha - \alpha_0),$$

and, approximately, from

$$\alpha - \alpha_0 = x \sin 2\alpha_0 + \tfrac{3}{2} x^2 \sin 4\alpha_0 + \cdots.$$

From (14) we also see that as the spring elongates $\varphi - \varphi_0$ increases up to a maximum and then decreases. If we write $\xi = A/B$, then $x = (1 - \xi)/(1 + \xi)$, and the maximum of φ or $\Delta\varphi$ is given by the following cubic equation in $\tan \alpha$

$$\tan \alpha = \frac{\xi}{2\xi - 1} \tan \alpha_0 \frac{1 + (2 - \xi) \tan^2 \alpha}{1 + \frac{\xi}{2\xi - 1} \tan^2 \alpha}. \tag{15}$$

Since, for the case here considered, $\tfrac{2}{3} < \xi < 1$, as shown below by equation (21),

$$2 - \xi < \frac{\xi}{2\xi - 1},$$

a real root of the above cubic lies between

$$(2 - \xi) \tan \alpha_0 \quad \text{and} \quad \frac{\xi}{2\xi - 1} \tan \alpha.$$

From the graph of (15) we see that there is but one real root.[1]

If the elongation $z - z_0$ of maximum twist be measured ξ comes readily by successive approximations from

$$\frac{2\xi - 1}{\xi} = \frac{\tan \alpha_0}{\tan \alpha} + (2 - \xi) \tan \alpha \tan \alpha_0 - \tan^2 \alpha_0. \tag{16}$$

[1] Supposing $\xi = \frac{8}{9}$, its value when Poisson's ratio is $\frac{1}{4}$, the real root lies between $\frac{8}{9} \tan \alpha_0$ and $\frac{8}{7} \tan \alpha_0$.

Also from (5), if $\varphi = \varphi_0$,

$$\frac{A}{B} = \xi = \frac{\cos\alpha}{\sin\alpha} \cdot \frac{\cos\alpha_0 - \cos\alpha}{\sin\alpha - \sin\alpha_0} = \frac{\tan\frac{1}{2}(\alpha + \alpha_0)}{\tan\alpha}. \qquad (17)$$

From A, B and $\frac{A}{B}$ we get

E = Young's modulus,

n = rigidity, or slide modulus,

and

η = Poisson's ratio = ratio of lateral contraction to longitudinal extension.

Thus, if a is the radius of the cross section of the wire,[1]

$$A = \tfrac{1}{2}\pi a^4 n, \quad \text{whence} \quad n = \frac{2A}{\pi a^4}; \qquad (18)$$

$$B = \tfrac{1}{4}\pi a^4 E, \quad \text{whence} \quad E = \frac{4B}{\pi a^4}; \qquad (19)$$

$$2n(1 + \eta) = E, \quad \text{whence} \quad \eta = \frac{B}{A} - 1; \qquad (20)$$

and

$$0 < \eta < \tfrac{1}{2}, \quad \text{whence} \quad \tfrac{2}{3} < \xi < 1. \qquad (21)$$

3. *Methods of Observing and Apparatus.*—Most of the commercial helical springs are "closed"; that is, the coils of wire are in close contact and sometimes require considerable force to separate them. Before "opening," that is, before giving the spring a permanent set which leaves the coils open, or apart, when hanging under no load, the radius of the cross section of the wire, a, can be found by measuring, with an ordinary standard scale, the lengths of several different sets (in hundreds) of coils. The radius of the cylinder, r_0, which envelopes the elastic central line of the wire, when hanging under no load, can be found if we know the external diam-

[1] The modulus of twist is equal to the modulus of rigidity times the polar moment of inertia of the circular cross section of the wire; and the modulus of bending is equal to Young's modulus (or the engineer's modulus) times the moment of inertia of the same cross section with respect to its diameter. For the relation between E, n and η see, for example, section 3 of Saint-Venant's edition of Clebsch, Théorie de l'Élasticité des Corps Solides, Paris, 1883.

eter of the helix and the radius of the wire. The external diameter of the helix was measured with a Brown and Sharpe micrometer screw-gauge. Thus

$$r_0 = \tfrac{1}{2} \text{ (external diameter of helix} - \text{diameter of wire).}$$

In order to avoid the uncertainty as to where the helix began and ended small dots of india ink were placed on the wire nearly in the same vertical plane, one several turns from the top and one several turns from the bottom. The number of turns from one dot to the other multiplied by 2π gives φ_0. The ends of all the springs were annealed and drawn out straight, thus bringing these ends into the axis of the cylinder of the helix. The method of suspension was to clamp the straight part of the wire in a vise which was fastened to a stable wall. By means of the micrometers of the telescopes of a Geneva cathetometer of precision used in the work it was possible to determine the exact positions of the dots with respect to one another, and thus obtain a correction to the value of φ_0 mentioned above due to the fact that the dots were not usually quite in the same vertical plane.

The vertical distance between the dots was determined by setting the movable parallel wires of the cathetometer telescopes on the upper and lower dots respectively and then turning the shaft, upon which the telescopes are mounted, until a standard meter of white bronze, which was mounted beside the spring, came into view. This meter is divided into millimeters and was made vertical by means of a plumb line. Although we do not know the absolute length of this meter an examination has shown that there are no graduation errors large enough to affect the work here considered.

A hollow brass cylinder with a three-jaw chuck enabled me to apply any load P up to six kilograms. Shot of small size were used to secure loads of the desired amounts.

The method of finding the position of maximum twist was as follows: On the straightened part of the lower end of the helix a small cork was slipped. Through the cork a straight piece of fine wire was thrust, thus forming an index. The cork was twisted until the index pointed to a plumb line suspended nearby. The empty cylinder was then clamped on the wire protruding through

the cork and in every case the index indicated a twist in accord with the law defined by equation (14). The index was then moved back until it again pointed to the plumb-line. The cylinder was then gradually loaded with shot, the index meanwhile being turned back successively to the plumb line. The rate of twist is quite rapid at first but as the position of maximum twist is approached the rate becomes less and less. The amount of twist was small in every instance, in no case did it exceed 40°. However, a considerable error in the elongation of maximum twist produces only a slight error in the value of ξ as shown by equation (16).

To find the position of no twist the index is not moved from its original position, in which it points to the plumb line, but shot are added until the index again points to the plumb line.

Thus, knowing φ_0, r_0, z_0, and z, we find a_0, s, and a from the following formulas:

$$\tan a_0 = z_0/\varphi_0 r_0, \quad s = z_0/\sin a_0,$$
$$\sin a = z/s, \quad s = \varphi_0 r_0/\cos a_0. \tag{22}$$

4. *Specimen of Computations of Constants of a Spring.*—For a copper-plated steel spring the following data were observed: diameter of wire 0.1223 cm., external diameter of helix 0.8515 cm., number of turns 295, mean axial length of helix under no load 39.615 cm., axial length at position of maximum twist 54.250 cm., axial length at position of zero twist 68.955 cm., produced by load $P = 5495.9$ grams.

(*a*) Computation of r_0, a_0 and s, with check for s.

$r_0 = \frac{1}{2}(0.8515 - 0.1223) = 0.3646$ cm.

Log 295	= 2.46982	
Log 2π	= 0.79818	
Log φ_0	= 3.26800	$\varphi_0 = 1853.5$
Log r_0	= 9.56182	
Log $\varphi_0 r_0$	= 2.82982	
Log z_0	= 1.59786	
Log tan a_0	= 8.76804	$a_0 = 3°21'17.3''$
Log sin a_0	= 8.76730	
Log z_0	= 1.59786	

$$\begin{aligned}
\text{Log } s &= 2.83056 \quad s = 676.96 \\
\text{Log cos } \alpha_0 &= 9.99926 \\
\text{Log } \varphi_0 r_0 &= 2.82982 \\
\text{Log } s &= 2.83056 \quad s = 676.96
\end{aligned}$$

(b) Computation of α, $\xi = A/B$ and η from position of zero twist and maximum twist.

$$\begin{aligned}
\text{Log } 68.955 &= 1.83857 \\
\text{Log } s &= 2.83056 \\
\text{Log sin } \alpha &= 9.00801
\end{aligned}$$

$$\begin{aligned}
\alpha &= 5^\circ 50' 47.0'' \\
\alpha_0 &= 3\ 21\ 17.3 \\
\alpha + \alpha_0 &= 9\ 12\ 04.3 \\
\alpha - \alpha_0 &= 2\ 29\ 29.7 \\
\tfrac{1}{2}(\alpha + \alpha_0) &= 4\ 36\ 02.2 \\
\tfrac{1}{2}(\alpha - \alpha_0) &= 1\ 14\ 44.8
\end{aligned}$$

$$\begin{aligned}
\text{Log tan } \tfrac{1}{2}(\alpha + \alpha_0) &= 8.90563 \\
\text{Log tan } \alpha &= 9.01028 \\
\text{Log } \xi = A/B &= 9.89535 \quad A/B = 0.7859
\end{aligned}$$

$$\begin{aligned}
\text{Log } 54.250 &= 1.73440 \\
\text{Log } s &= 2.83056 \\
\text{Log sin } \alpha &= 8.90384 \quad \alpha = 4^\circ 35' 47.9'' \\
\text{Log tan } \alpha &= 8.90524 \\
\text{Log tan}^2 \alpha &= 7.81048 \quad \tan^2 \alpha = 0.00646
\end{aligned}$$

$$\begin{aligned}
\text{Log tan } \alpha_0 &= 8.76804 \\
\text{Log tan } \alpha &= 8.90524 \\
\text{Log tan } \alpha_0/\tan \alpha &= 9.86280 \quad \tan \alpha_0/\tan \alpha = 0.72912.
\end{aligned}$$

$$\begin{aligned}
\text{Log tan } \alpha \tan \alpha_0 &= 7.67328 \\
\text{Log } (2 - \xi) &= 0.08408 \\
\text{Log } (2 - \xi) \tan \alpha \tan \alpha_0 &= 7.75736 \quad (2 - \xi) \tan \alpha \tan \alpha_0 = 0.00572.
\end{aligned}$$

$$0.72912 - 0.00646 + 0.00572 = 0.72838.$$

Therefore

$$2\xi - 1 = 0.72838\,\xi,$$

or

$$1.2716\,\xi = 1.$$

Now

$$1/\xi - 1 = B/A - 1 = \eta = 0.2716$$

and

$$\xi = A/B = 0.7864.$$

(*c*) Computation of A.

Log 2	= 0.30103	
Log $\sin \frac{1}{2}(\alpha - \alpha_0)$	= 8.33728	Log $(\sin \alpha - \sin \alpha_0)$.
Log $\cos \frac{1}{2}(\alpha + \alpha_0)$	= 9.99860	
Log $\sec^2 \alpha$	= 0.00453	
Log φ_0^2/s^2	= 0.87488	
colog P	= 6.25996	
Log $1/A$	= 5.77628	$1/A = 0.000059742$.
Log A	= 4.22372	$A = 16739$.

(*d*) Computation of B, E and n.

Log $1/A$ = 5.77634 (Mean of five measures.)
Log A/B = 9.89570
Log $1/B$ = 5.67204

Log B	= 4.32796	$B = 21280$.
Log 4	= 0.60206	
Log $4B$	= 4.93002	
Log πa^4	= 5.64274	
Log E	= 9.28728	$E = 1937700000$.
Log E	= 9.28728	$n = \frac{1}{2}\xi E$.
Log ξ	= 9.89570	
Log ξE	= 9.18298	
Log 2	= 0.30103	
Log n	= 8.88195	$n = 762000000$.

(*e*) Computation of $\frac{\Delta\varphi}{\varphi_0}$ at position of maximum twist.

$$x = (1-\xi)/(1+\xi) \qquad \xi = 0.7865.$$

$$\begin{aligned} \text{Log } (1-\xi) &= 9.32940 \\ \text{Log } (1+\xi) &= \underline{0.25300} \\ \text{Log } x &= 9.07740 \end{aligned}$$

$$\tfrac{1}{2}(3\alpha + \alpha_0) = 8°34'20''.5$$

$$\tfrac{1}{2}(\alpha - \alpha_0) = 0°37'15''.3$$

$$2\alpha = 9°11'35''.8$$

$$\begin{aligned} \text{Log } x &= 9.07740 \\ \text{Log sin } \tfrac{1}{2}(3\alpha+\alpha_0) &= \underline{9.17336} \\ \text{Log } x \text{ sin } \tfrac{1}{2}(3\alpha+\alpha_0) &= 8.25076 \qquad x \text{ sin } \tfrac{1}{2}(3\alpha+\alpha_0) = 0.017814 \\ \text{Log sin } \tfrac{1}{2}(\alpha-\alpha_0) &= 8.03490 \qquad \text{sin } \tfrac{1}{2}(\alpha-\alpha_0) = 0.010836 \end{aligned}$$

$$x \text{ sin } \tfrac{1}{2}(3\alpha+\alpha_0) - \text{sin } \tfrac{1}{2}(\alpha-\alpha_0) = 0.006978$$

$$\begin{aligned} \text{Log sin } \tfrac{1}{2}(\alpha-\alpha_0) &= 8.03490 \\ \text{Log } 2 &= 0.30103 \\ \text{Log } 0.006978 &= \underline{7.84373} \\ \text{Log numerator} &= 6.17966 \end{aligned}$$

$$\begin{aligned} \text{Log } x &= 9.07740 \\ \text{Log cos } 2\alpha &= \underline{9.99438} \\ \text{Log } x \text{ cos } 2\alpha &= 9.07178 \end{aligned}$$

$$1 + x \text{ cos } 2\alpha = 1.1180$$

$$\text{Log } (1 + x \text{ cos } 2\alpha) = \text{Log denominator} = 0.04844$$

$$\text{Log } \frac{\Delta\varphi}{\varphi_0} = 6.13122 \qquad \frac{\Delta\varphi}{\varphi_0} = 0.00013528.$$

(f) Table of values for $\Delta\varphi/\varphi_0$ and $\Delta\varphi$ for different values of α.

α	$\Delta\phi/\phi_0$	$\Delta\phi$
$\alpha_0 = 3°21'17.3''$	+ 0.00000000	+ 0.0°
3 59 17.3	+ 0.00008635	+ 9.2
4 17 17.3	+ 0.00012701	+ 13.5
4 35 47.9	+ 0.00013528	+ 14.4
5 00 17.3	+ 0.00012047	+ 12.8
5 25 17.3	+ 0.00005711	+ 6.1
5 50 47.3	— 0.00002802	— 3.0
7 00 17.3	— 0.00037801	— 40.2

5. *Tables of Observed and Derived Results.*—The following tables give the numerical results appertaining to seven steel and three brass springs subjected to examination.

Table I. gives the observed data for these springs. The quantities measured are all linear. They are expressed by five significant figures; and it is believed that the precision of the measurements is correctly indicated by that number of figures.

Table II. gives the computed constants φ_0, α_0 and s for the springs. As explained above (section 3), the value of φ_0 is 2π times the number of turns of the helix used, plus a small correction for the excess or defect of that number of turns from a round number. The precision of the data here given is also supposed to be represented without exaggeration by the figures used.

Table III. gives the observed and mean values of the modulus of twist A, and the ratio of the latter to the modulus of bending B or A/B. The probable errors attached to the mean values are derived from the discrepancies between the individual values and their mean for any spring, equal weights being assigned to the individual values. The values of A/B were derived mostly from the observed elongations of maximum twist ($\Delta\varphi$ = a max.). A few values were derived from the observed elongations of zero twist ($\Delta\varphi = 0$). Such values are designated by the letter b in parenthesis following them. Apparently, the results by these two methods are of about equal weight, and hence no distinction is made among them.

Table IV. gives the computed values of the modulus of bending B, the modulus of rigidity n, Young's modulus E, and Poisson's

ratio η. The values of E and n are given in dynes per square centimeter, the value of the normal acceleration at the place of observation, latitude 40°48′28″, being $g = 980.2^{cm}/(sec.)^2$.

TABLE I.

Observed Data of Springs.

Number and Metal of Springs.	Radius of Cross-Section of Wire. a	Radius of Unstrained Helix. r_0	Unstrained Length of Helix. z_0	Strained Length of Helix. z	Attached Mass. m
	Centimeter.	Centimeter.	Centimeter.	Centimeter.	Grams.
1. Copper-plated steel.	0.05680	0.32305	41.328	72.586	5408.8
				72.502	5390.2
				72.529	5392.9
				72.727	5427.4
				72.606	5403.3
				72.500	5385.9
2. Steel.	0.05605	0.32580	39.032	69.407	5756.2
				69.524	5773.6
				69.675	5802.0
				69.784	5818.5
				69.780	5816.2
3. Copper-plated steel.	0.06092	0.36430	38.153	44.273	1248.0
				46.731	1748.0
				49.174	2248.0
				51.612	2748.0
				53.593	3154.0
				56.030	3654.0
				56.049	3654.0
				60 949	4654.0
				65.850	5654.0
				66.605	5804.0
4. Copper-plated steel.	0.06115	0.36460	39.615	68.642	5444.8
				68.955	5495.9
				69.009	5504.2
				68.672	5438.9
				68.704	5440.9
5. Steel.	0.09325	1.09975	33.327	58.650	2378.1
				59.515	2459.0
				59.575	2466.0
				59.352	2453.5
				59.483	2456.8
				59.143	2441.0

TABLE I.—*Continued.*

Observed Data of Springs.

Number and Metal of Springs.	Radius of Cross-Section of Wire. a	Radius of Unstrained Helix. r_0	Unstrained Length of Helix. z_0	Strained Length of Helix. z	Attached Mass. m
	Centimeter.	Centimeter.	Centimeter.	Centimeter.	Grams.
6. Steel.	0.10328	1.10480	23.689	43.529	3384.3
				43.820	3402.2
				43.496	3348.3
				43.406	3332.0
				43.407	3333.3
7. Steel.	0.15315	1.8395	30.621	35.895	2070.0
				36.478	2298.0
				41.855	4368.1
				42.343	4452.0
8. Brass.	0.06526	0.4174	57.135	63.650	341.2
				80.890	1246.2
				82.753	1338.5
				88.940	1660.9
9. Brass.	0.08239	0.5716	58.050	63.000	341.2
				76.211	1246.2
				89.380	2135.5
				92.587	2347.8
10. Brass.	0.10280	0.8494	52.860	57.444	341.2
				70.030	1246.2
				71.106	1338.5
				84.245	2163.9

TABLE II.

Computed Constants of Springs.

Number of Spring.	Angle Between Ends of Spring. ϕ_0	Slope of Unstrained Spring. α_0	Length of Spring Along Elastic Central Line s.
	Radians.	° ′ ″	Centimeter.
1	2054.6	3 33 46.5	665.03
2	1847.3	3 42 38.2	603.12
3	1690.2	3 32 43.0	616.99
4	1853.5	3 21 17.3	676.96
5	955.04	1 58 43.2	964.92
6	596.90	2 03 26.5	659.90
7	307.88	3 05 41.5	1191.10
8	2600.9	3 00 45.0	1087.20
9	1947.8	2 59 05.0	1114.90
10	1400.9	2 32 36.7	1191.10

TABLE III.

Observed and Mean Values of A and A/B.

Number of Spring.	Individual Values of *A*.	Mean Value of *A*.	Individual Values of *A/B*.	Mean Value of *A/B*.
1	11910		.7781	
	11900		.7776	
	11900		.7751	
	11890		.7761	
	11900	11900 ± 2.1	.7770	.7768 ± .00036
2	12020		.7794	
	12010		.7789	
	12010		.7783	
	12000	12008 ± 2.5	.7778	
			.7779	
			.7786 (*b*)	.7785 ± .00017
3	16660		.7798	
	16640		.7792	
	16650		.7789	
	16650		.7799	
	16660		.7794	
	16650		.7793	
	16640		.7814	
	16620		.7781	
	16590		.7776	
	16570	16633 ± 6.5	.7806	.7794 ± .00024
4	16760		.7872	
	16740		.7859	
	16730		.7856	
	16730		.7871	
	16720		.7869	
		16736 ± 4.5	.7864 (*b*)	.7865 ± .00018
5	92120		.7835	
	92090		.7794	
	92150		.7791	
	92473		.7802	
	92130		.7795	
	91830	92130 ± 51	.7799 (*b*)	.7803 ± .00045
6	137000		.7715	
	135700		.7716	
	136400		.7697	
	135700		.7724	
	135700		.7723	
		136120 ± 260	.7723 (*b*)	.7716 ± .00028

TABLE III.—*Continued.*

Observed and Mean Values of A and A/B.

Number of Spring.	Individual Values of A.	Mean Value of A.	Individual Values of A/B.	Mean Value of A/B.
7	752000		.7845	
	751500		.7832	
	743900		.7814	
	743000	747600 ± 1600	.7754	.7811 ± .0013
8	9914		.7489	
	9873		.7437	
	9855		.7510	
	9854		.7432	
			.7496	
			.7471	
			.7427	
		9874 ± 9	.7442	.7463 ± .0007
9	25100		.7448	
	24970		.7462	
	24760		.7509	
	24670		.7486	
			.7420	
		24875 ± 66	.7433	.7460 ± .0009
10	65410		.7500	
	62270		.7434	
	62860		.7414	
	61430		.7468	
			.7471	
			.7449	
			.7498	
			.7419	
			.7424	
		61742 ± 350	.7437	.7452 ± .0006

TABLE IV.

Computed Values of B, E, n and η.

Number and Metal of Spring.	Modulus of Bending. B	Young's Modulus. E	Modulus of Rigidity. n	Poisson's Ratio. η
		Dynes per Square Centimeter.	Dynes per Square Centimeter.	
1. Steel.	15320± 7.6	(1837±0.9)×10⁹	(7134± 1.3)×10⁸	0.2873±6
2. "	15420± 4.6	(1951±0.6)×10⁹	(7592± 2.0)×10⁸	.2845±3
3. "	21340±10.6	(1933±0.9)×10⁹	(7533± 2.9)×10⁸	.2830±4
4. "	21280± 7.5	(1899±0.7)×10⁹	(7469± 2.0)×10⁸	.2715±3
5. "	118100±94	(1949±1.6)×10⁹	(7604± 4.2)×10⁸	.2816±7
6. "	176400±343	(1939±3.8)×10⁹	(7481±14.3)×10⁸	.2960±5
7. "	957100±2594	(2171±5.8)×10⁹	(8479±18.1)×10⁸	.2802±22
8. Brass.	13230±17	(911±1.2)×10⁹	(340± 0.3)×10⁹	.3400±13
9. "	33340±97	(904±2.6)×10⁹	(337± 0.9)×10⁹	.3405±16
10. "	82850±474	(938±5.4)×10⁹	(345± 2.0)×10⁹	.3419±10

6. *Precision of the Constants Derived.*—From equation (8), in view of the small value of $\Delta\varphi/\varphi_0$, it is seen that so far as errors of observation are concerned the modulus of twist may be defined by

$$A = \frac{Ps^3 \cos^2 a}{(z - z_0)\varphi_0^2}.$$

Since $\cos^2 a = 1 - (z/s)^2$, and since z/s is small for each of the springs, it appears that the relative errors of A from different determinations for the same spring depend essentially on the errors of P and $(z - z_0)$ only. It may be safely assumed, however, that the errors of P, and likewise those of φ_0 and s, are negligible in comparison with those of $(z - z_0)$. Thus we conclude that

$$\frac{\Delta A}{A} = -\frac{\Delta(z - z_0)}{z - z_0},$$

or that the probable error of A is the same fraction of A as the probable error of $(z - z_0)$ is of it. We believe, therefore, that the probable errors of the mean values of A are essentially correct as given in Table III.

From similar reasoning we conclude that the probable errors of the ratios A/B given in Table III. afford trustworthy indications of the precision of the mean values of those quantities.

For computing the probable errors of B, E, n, and η from those of the independently observed quantities A and A/B, it is essential to note the relations expressed by equations (18) to (20) in connection with the two following equations. Thus let

$$A = u, \quad \text{and} \quad \frac{A}{B} = \xi,$$

wherein u and ξ are supposed to be observed quantities. Then

$$B = \frac{u}{\xi},$$

and if δB, δu, and $\delta\xi$ denote the probable errors of B, u and ξ, respectively,

$$\left(\frac{\delta B}{B}\right)^2 = \left(\frac{\delta u}{u}\right)^2 + \left(\frac{\delta\xi}{\xi}\right)^2.$$

Hence from the values of δu and $\delta\xi$ given in Table III. the probable errors of B in Table IV. are derived.

From equation (18) it is seen that the probable error of n is the same fraction of n as the probable error of A is of it; and similarly from (19) the probable error of E is the same fraction of E as the probable error of B is of it.

Finally, from equation (20) it appears that the probable error of η is equal to the probable error of the reciprocal of A/B; and this is the same as $(B/A)^2$ times the probable error of A/B.

It will be observed from Table IV. that the precision is highest, and of about the same order, for the first four steel springs; the probable errors of A and n being about 1/3500 part, those of B and E about 1/2000 part, and those of η about 1/700 part. This higher precision is most likely due to the fact that the springs in question were the best as to uniformity, temper and finish of all the springs used. Next in order of precision are the constants for the steel springs Nos. 5 and 6. These are inferior in quality to Nos. 1–4 as regards uniformity and finish. Steel spring No. 7 is markedly different from the others in being "open-wound," of large diameter, of heavier wire than the others, and relatively short in axial length z_0. The means at hand did not permit a sufficient elongation of this spring to attain results of higher precision for A, and the ratio A/B was also less definitely measurable than in the cases of the other springs.

The precision is relatively low, as would be expected, for the constants of the brass springs, although they are surprisingly uniform as to slope and diameter for commercial products. The well-known instability of brass as compared with steel is undoubtedly the source of this lower order of precision.

7. *Summary of Conclusions.*—From the details of the investigation given above the following conclusions appear to be established:

(*a*) The static theory of helical springs as expressed by equations (1) to (4), and given for the first time by Kelvin and Tait in the first edition of their Natural Philosophy, 1867, has been experimentally verified. Starting from this theory the above paper shows that:

(*b*) To a high order of approximation the force required to elongate a helical spring varies directly as the product of the elongation of the spring and the square of the secant of its slope. The exact law for finite elongations of any such spring is expressed in a simple form by equations (7), (8) and (14).

(*c*) When such springs are elongated by an axial pull, with no applied torsional couple, they undergo a twist about the axis of the helix. This twist increases as the pull increases up to a maximum, and thereafter decreases continuously with increasing pull.

(*d*) The law connecting the elongation of a spring with the applied force furnishes a ready method of finding the modulus of twist and hence the modulus of rigidity of the wire of the spring.

(*e*) Observations of the elongations of maximum twist and zero twist furnish independent methods of deriving the ratio of the modulus of twist to the modulus of bending.

(*f*) By a combination of the results furnished by (*d*) and (*e*) the modulus of bending, Young's modulus, and Poisson's ratio are readily derived.

(*g*) By the methods explained it is easily practicable to attain, in the case of a well-made steel spring, a precision indicated in round numbers by a probable error of one part in three thousand for the modulus of twist, of one part in two thousand for Young's modulus and of one part in five hundred for Poisson's ratio.

8. *Acknowledgments.*—The author desires to express his obligations to the officers of the Department of Physics as well as to those of the Department of Mechanics of Columbia University for aid at all times cordially rendered during the progress of this investigation. He is especially indebted to Professor R. S. Woodward for assistance in developing the mathematical theory of the springs and for suggestions with reference to the reduction and discussion of the observations.

COLUMBIA UNIVERSITY,
May 1, 1901.

ON THE BOUNDARY CONDITIONS OF THE ELECTRICAL FIELD.

By William Duane.

1. In the method of presenting the modern theory of electricity suggested first by Heaviside,[1] and several years later by Hertz,[2] the laws and definitions represented by the following equations are taken as a basis, and from them are deduced many of the laws and facts that are suitable to direct experimental verification. The fundamental equations are:

$$-\operatorname{curl} P_e = 4\pi i_m \tag{1}$$

$$\operatorname{curl} P_m = 4\pi i_e \tag{2}$$

$$E_e = \frac{K}{8\pi} P_e^2 \tag{3}$$

$$E_m = \frac{\mu}{8\pi} P_m^2 \tag{4}$$

$$i_m = \frac{1}{4\pi} \frac{d\mu P_m}{dt} \tag{5}$$

$$i_e = \frac{1}{4\pi} \frac{dKP_e}{dt} + CP_e \tag{6}$$

in which the letters represent the quantities printed opposite to them in the following table.

P_e = electric force.
P_m = magnetic force.
i_e = electric current density.
i_m = magnetic current density.
E_e = electric energy density.
E_m = magnetic energy density.

[1] Electrician, 1885.
[2] Wied. Ann., 36, p. 1, 1889.

K = electric inductivity.

μ = magnetic inductivity.

C = electric conductance.

For points where there is impressed electric or magnetic force equations 3, 4, 5 and 6 must be modified slightly. The modifications, however, do not affect the following conclusions.

2. In reasoning from these equations it is customary to deduce the usual expressions for the electric, magnetic and vector potentials, and from them the general laws and the phenomena. The author has found some difficulty in following the reasoning without making special assumptions as to the character of the electromagnetic field at the boundary of the electrical system, or at infinity if the system is not bounded. In fact it would appear impossible to prove the relations between the potentials and the densities of electricity, magnetism and current without some boundary condition or its equivalent.

3. In the electrostatic case the line of reasoning is somewhat as follows: Since the currents are zero, curl $P_e = 0$, and hence the line integral of the electric force taken from any point P to a fixed point O is independent of the path along which the integration is performed, *i. e.*, is a function of the coördinates of P only. The value of the potential V at P is defined to be this line integral, the potential at O being chosen as the zero of potential. From this it follows that the electric force at any point is minus the space derivative of the potential at the point taken in the direction of the force, and if X, Y and Z are the rectilinear components of P_e,

$$X = -\frac{\partial V}{\partial x}, \quad Y = -\frac{\partial V}{\partial y}, \quad Z = -\frac{\partial V}{\partial z}.$$

The density e of free electricity is defined to be the divergence of the electric force divided by 4π or in symbols

$$e = \frac{1}{4\pi} \text{div. } P_e = -\frac{1}{4\pi} \nabla^2 V.$$

In order to deduce an expression for V in terms of e, it is first shown that if e is given at every point in space the potential can have but one value at each point, the potential at O being zero, as follows:

Assume there are two values V_1 and V_2 of the potential at every point, and let $\varphi = V_1 - V_2$. In Green's Theorem substitute φ for both the functions, and apply the theorem to the space inside a sphere of radius ρ, the center of which is in or near the electrical field. Thus:

$$\int \left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2 \right] d\tau$$

$$= \int \varphi \frac{\partial \varphi}{\partial n} dS - \int \varphi \nabla^2 \varphi \, d\tau. \qquad (7)$$

The last volume integral vanishes because

$$-4\pi e = \nabla^2 V_1 = \nabla^2 V_2$$

and therefore

$$\nabla^2 \varphi = \nabla^2 V_1 - \nabla^2 V_2 = 0$$

everywhere. If the surface integral approaches zero as the spherical bounding surface is taken larger and larger, the first volume integral taken throughout all space is zero. Hence since it is a sum of terms each of which is positive, each term must be zero itself, and

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial y} = \frac{\partial \varphi}{\partial z} = 0.$$

φ is therefore independent of x, y and z, and since at the point of zero potential $\varphi = 0$, φ must be zero everywhere, and $V_1 = V_2$. Hence if the above surface integral vanishes for $\rho = \infty$, the potential can have but one value at each point in space, the density of free electricity being given everywhere.

4. There is nothing, however, in the fundamental equations (1)–(6) to indicate that this surface integral vanishes. In order to prove the solution unique it is necessary that some boundary condition or the equivalent should be given. In fact the potential is not completely determined by the value of

$$e = \frac{-1}{4\pi} \nabla^2 V,$$

and the potential at any point of reference O; for any function of the form

$$f(xyz) = Ax + By + Cz + D$$

in which A, B, C and D are constants, can be added to V, and, if the constants are determined so that f is zero at the point O, $V+f$ satisfies the equation

$$e = \frac{-1}{4\pi} \nabla^2(V+f)$$

and the conditions at O.

5. The gap in the reasoning may be filled in some such way as the following: Assume that the electrical system has a finite quantity of energy; an assumption that evidently is true in all actual cases. The quantity of electrical energy in a spherical shell of thickness $d\rho$ and radius ρ is

$$dE_e = \frac{d\rho}{8\pi} \int KP_e^2 dS,$$

the integration extending over the surface of the sphere. The whole energy is

$$E_e = \frac{1}{8\pi} \int_0^\infty d\rho \int KP_e^2 dS.$$

Since the surface integral is always positive, and E_e is finite, the surface integral expressed as a function of ρ must vanish for $\rho = \infty$. The surface integral may be expressed in the form

$$\int KP_e^2 \rho^2 d\omega,$$

in which $d\omega$ is the solid angle subtended at the center of the sphere by dS. K is always positive (for energy is never negative), and becomes unity for points in free space. Hence, as the integral is the sum of positive terms, each term must vanish for $\rho = \infty$, and the electric force P_e must vanish at infinity in such a way that $\rho P_e = 0$, for $\rho = \infty$. This then is the condition equivalent to a boundary condition that the electric forces must fulfill at infinity.

6. From equations (1)–(6) we cannot tell whether the potential has the same or different values at points on the infinite sphere; but a condition for the potential may be obtained from the finiteness of the energy as follows: The difference of potential between any two points P and P' on the sphere of radius ρ is

$$V_{P'} - V_P = \int_{P'}^{P} P_e \cos \varepsilon \, dl,$$

dl being the element of length of the great circle passing through P and P', and ε, the angle between dl and P_e. If $d\varphi$ is the angle subtended at the center of the sphere by dl, $dl = \rho d\varphi$ and

$$V_{P'} - V_P = \int_{P'}^{P} P_e \cos \varepsilon \, \rho d\varphi.$$

The integral is numerically less than or equal to the integral

$$\int_{P'}^{P} \overline{P}_e \, \rho d\varphi$$

in which $\overline{P}_e$ is the maximum value of P_e on the great circle. Hence

$$V_P - V_P \leqq \int_{P'}^{P} \overline{P}_e \, \rho d\varphi$$

or

$$V_{P'} - V_P \leqq P_e \rho\varphi,$$

φ being the angle subtended by the radii drawn from the center of the sphere to P and P' respectively. Since this angle cannot be greater than 2π, and $\overline{P}_e \rho$ approaches zero as ρ increases, we have

$$V_{P'} - V_P = 0 \quad \text{for} \quad \rho = \infty,$$

i. e., the potential at all points on the infinite sphere has the same value.

7. In a similar manner it can be proved that the potential at infinity is finite, the potential at any point O being zero. If we integrate along a straight line from O to infinity the potential at infinity is

$$V_\infty = - \int_0^\infty P_e \cos \varepsilon \, d\rho.$$

Since $\cos \varepsilon$ is never numerically greater than one, V_∞ is numerically less than or equal to

$$\int_0^\infty P_e d\rho.$$

P_e is never infinite, and vanishes for $\rho = \infty$ more rapidly than $\frac{A}{\rho}$, A being a constant. Hence the integral is finite and V_∞ is finite.

8. It follows from these conditions for the potential that as ρ increases the surface integral in equation (7) approaches the value

$$\varphi_\infty \int \frac{\partial \varphi}{\partial n} dS$$

in which

$$\varphi_\infty = V_{1\infty} - V_{2\infty},$$

and is finite and constant. By a well-known theorem (a particular case of Green's theorem)

$$\int \frac{\partial \varphi}{\partial n} dS = \int \nabla^2 \varphi d\tau.$$

These integrals are zero, since $\nabla^2 \varphi = 0$ everywhere. Hence as ρ increases the surface integral approaches zero, and V is completely determined by the values of e, and the conditions at infinity.

9. Heaviside's proof[1] that the electric field is uniquely determined by the values of e is expressed in terms of vector analysis. If the two vectors R and R' have no curl and the same divergence, their geometric difference $R - R'$ has zero curl and zero divergence everywhere. It does not follow from this, however, that the vector $R'' = R - R'$ is zero. Any vector, for instance, that is constant in magnitude and direction has zero curl and zero divergence and is not zero itself. Nor is the simple statement of the condition "if R'' vanishes at infinity" sufficient, for, in the first place, no proof is given that R'' vanishes at infinity, and in the second, it does not follow from the vanishing of R'' at infinity that its lines must form closed curves. The truth of the latter is evident in the simple case of the electric field of force due to a single charge.

10. That R'' is zero, however, follows immediately from the conditions that the potential has the same value at all points on the infinite sphere. As Heaviside says, "the lines of R'' can neither form closed curves nor end at any point." Further, they cannot run off in both directions to infinity, for if they did the line integral of

[1] "Electrical Papers," Vol. I., p. 217.

R'' taken along any one of them would not be zero, and the above-stated conditions for the potential would not be fulfilled. Hence R'' must be zero.

11. We know from the theory of the Newtonian potential function that the expression

$$\varphi = \int \frac{e d\tau}{r}$$

satisfies the equation

$$\nabla^2 \varphi = -4\pi e$$

and that it vanishes at infinity. Hence $\varphi + k$ approaches the constant value k at infinity, and is the only function that can represent the potential. The value of k depends upon the point chosen as the zero of potential.

12. In the general case of an electro-magnetic field in which the electric current and the free magnetism are not necessarily zero, the magnetic force P_m satisfies the equations

$$\text{curl } P_m = 4\pi i_e, \tag{2}$$

$$\text{div. } P_m = 4\pi m; \tag{8}$$

equation (8) being the definition of the density of free magnetism m. If i_e and m are given these two equations do not determine P_m completely, as any vector whose curl and divergence are zero (for instance one constant in magnitude and direction) can be added to P_m and the resulting vector satisfies equations (2) and (9). We can find, however, a third condition that must be satisfied by the magnetic force by reasoning from the finiteness of the magnetic energy. The condition is as before, that P_m must vanish at infinity in such a way that $\rho P_m = 0$ if $\rho = \infty$. This holds even in variable states. In these there may be radiation of energy in any or all directions, but it can be proved from the fundamental equations that the energy travels with finite velocity, and hence the radiation does not affect the conditions at infinity.

13. The condition $\rho P_m = 0$, if $\rho = \infty$, together with equations (2) and (8), if i_e and m are given, can be shown to determine P_m completely as follows. We know from the Newtonian potential theory that the vector P_m', the rectilinear components of which are

$$\alpha' = -\frac{\partial}{\partial x}\int \frac{m d\tau}{r}, \quad \beta' = -\frac{\partial}{\partial y}\int \frac{m d\tau}{r}, \quad \gamma' = -\frac{\partial}{\partial z}\int \frac{m d\tau}{r} \qquad (9)$$

satisfies the equations (8) and

$$\text{curl } P_m' = 0,$$

and that P_m' vanishes at infinity in such a way that $\rho P_m' = 0$, if $\rho = \infty$. From the vector potential theory we know that P_m'', the components of which are

$$\alpha'' = \frac{\partial H}{\partial y} - \frac{\partial G}{\partial z}, \quad \beta'' = \frac{\partial F}{\partial z} - \frac{\partial H}{\partial x}, \quad \gamma'' = \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y} \qquad (10)$$

in which F, G and H are the integrals

$$F = \int \frac{u d\tau}{r}, \quad G = \int \frac{v d\tau}{r}, \quad H = \int \frac{w d\tau}{r} \qquad (11)$$

u, v and w being the rectilinear components of i_e, satisfies the equations (2) and

$$\text{div. } P_m'' = 0,$$

and that P_m'' vanishes at infinity in such a way that $\rho P_m'' = 0$, if $\rho = \infty$. Hence the sum of the two vectors

$$P_{m_1} = P_m' + P_m''$$

must fulfill the conditions $\rho P_{m_1} = 0$, if $\rho = \infty$, and must satisfy the equations (2) and (8). P_{m_1} can be shown to be equal to P_m; for their difference

$$P_{m_2} = P_m - P_{m_1}$$

must satisfy the equations (2) and (8) and the above-stated condition at infinity. From equation (2) it appears that P_{m_2}, if not zero everywhere, must have a potential φ. By using Green's theorem as before, it follows that φ must be constant, and therefore $P_{m_2} = 0$. Hence $P_m = P_{m_1}$, and the magnetic force is determined completely by the values of m, i_e and the condition at infinity, P_m being given by equations (9), (10), (11) and

$$P_m = P_m' + P_m''.$$

14. The same reason is applicable to the electric force in the variable states, and it, therefore, is determined completely by the values of i_m, e and the finiteness of the energy.

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NOTE ON KIRCHOFF'S THEORY OF MAGNETOSTRICTION.

BY S. SANO.

THE medium treated by Kirchoff[1] in his theory of magnetostriction is supposed initially to be an isotropic elastic solid, which in the magnetized condition is completely free from hysteresis and time-lag, and to be such that the components of the intensity of magnetization in the medium are not only dependent upon the magnetizing force, but are also linear functions for the strain components. The object of the present investigation is to make a slight extension of Kirchoff's theory to the case where the susceptibility k and two strain coefficients k' and k'' are functions of the strength of the magnetizing force, the other assumptions made to the elastic medium being retained. I shall first find the expressions for the components of the intensity of magnetization in the medium, and then discuss the mechanical forces acting on the elastic solid.

Let H be the strength of the magnetizing force, and A, B, C the components of the intensity of magnetization, and u, v, w those of the displacement at a point (x, y, z) in the elastic solid, referred to rectangular coördinate axes. At first take the axis of z parallel to the direction of the magnetic force at the point (x, y, z); then A, B, C take the following forms:

$$\left.\begin{aligned} A = {} & a_1\frac{\partial u}{\partial x} + a_2\frac{\partial v}{\partial y} + a_3\frac{\partial w}{\partial z} + a_4\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ & + a_5\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) + a_6\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \\ B = {} & b_1\frac{\partial u}{\partial x} + b_2\frac{\partial v}{\partial y} + b_3\frac{\partial w}{\partial z} + b_4\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ & + b_5\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) + b_6\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \end{aligned}\right\} \quad (1)$$

[1] Sitzber. d. k. Acad. d. Wiss. zu Berlin, p. 137 (1884); Wied. Ann., Vol. XXIV. p. 52 (1885); Gesammelte Abhandlungen, Nachtrag, p. 91, Leipzig (1891).

$$C = c_0 + c_1\frac{\partial u}{\partial x} + c_2\frac{\partial v}{\partial y} + c_3\frac{\partial w}{\partial z} + c_4\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) + c_5\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) + c_6\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),$$

where the a's, b's and c's are dependent on H but not on the strain components. Next introduce a new set of rectangular coördinate axes x', y', z', the axis of z' coinciding with the old axis of z. Let (A', B', C') and (u', v', w') be the intensity of magnetization and the displacement referred to new axes, then we have

$$A' = a_1'\frac{\partial u'}{\partial x'} + a_2'\frac{\partial v'}{\partial y'} + a_3'\frac{\partial w'}{\partial z'} + a_4'\left(\frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'}\right) + a_5'\left(\frac{\partial w'}{\partial x'} + \frac{\partial u'}{\partial z'}\right) + a_6'\left(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right),$$

$$B' = b_1'\frac{\partial u'}{\partial x'} + b_2'\frac{\partial v'}{\partial y'} + b_3'\frac{\partial w'}{\partial z'} + b_4'\left(\frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'}\right) + b_5'\left(\frac{\partial w'}{\partial x'} + \frac{\partial u'}{\partial z'}\right) + b_6'\left(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right),$$

$$C' = c_0' + c_1'\frac{\partial u'}{\partial x'} + c_2'\frac{\partial v'}{\partial y'} + c_3'\frac{\partial w'}{\partial z'} + c_4'\left(\frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'}\right) + c_5'\left(\frac{\partial w'}{\partial x'} + \frac{\partial u'}{\partial z'}\right) + c_6'\left(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right),$$

where the a''s, b''s and c''s have significations similar to those of the a's, b's and c's. Denoting the angle between the axes x and x' by θ, which we shall suppose to be positive when the angle between the axes y and y' diminishes by increasing θ, the new coefficients, a_1', a_2', a_3' ··· are related to the old coefficients a_1, a_2 ···, c_0, c_1 ··· by the following equations:

$$\begin{aligned}
a_1' &= a_1\cos^3\theta + (b_1 + 2a_6)\cos^2\theta\sin\theta + (a_2 + 2b_6)\cos\theta\sin^2\theta + b_2\sin^3\theta,\\
a_2' &= a_2\cos^3\theta + (b_2 - 2a_6)\cos^2\theta\sin\theta + (a_1 - 2b_6)\cos\theta\sin^2\theta + b_1\sin^3\theta,\\
a_3' &= a_3\cos\theta + b_3\sin\theta,\\
a_4' &= a_4\cos^2\theta + (-a_5 + b_4)\cos\theta\sin\theta - b_5\sin^2\theta,
\end{aligned}$$

$$\left.\begin{aligned}
a_5' &= a_5\cos^2\theta + (a_4+b_5)\cos\theta\sin\theta + b_4\sin^2\theta,\\
a_6' &= a_6\cos^3\theta + (-a_1+a_2+b_6)\cos^2\theta\sin\theta +\\
&\qquad (-a_6-b_1+b_2)\cos\theta\sin^2\theta - b_6\sin^3\theta,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
c_0' &= c_0,\\
c_1' &= c_1\cos^2\theta + 2c_6\cos\theta\sin\theta + c_2\sin^2\theta,\\
c_2' &= c_2\cos^2\theta - 2c_6\cos\theta\sin\theta + c_1\sin^2\theta,\\
c_3' &= c_3,\\
c_4' &= c_4\cos\theta - c_5\sin\theta,\\
c_5' &= c_5\cos\theta + c_4\sin\theta,\\
c_6' &= c_6\cos^2\theta + (-c_1+c_2)\cos\theta\sin\theta - c_6\sin^2\theta.
\end{aligned}\right\} \qquad (2)$$

The supposition that the medium is originally isotropic leads to the following relations among the coefficients:

$$\begin{aligned}
a_1' &= a_1,\\
a_2' &= a_2,\\
&\cdots\cdots\\
c_6' &= c_6,
\end{aligned}$$

whence by (2),

$$\begin{aligned}
&a_1 = a_2 = a_3 = a_6 = b_1 = b_2 = b_3 = b_6 = c_4 = c_5 = c_6 = 0,\\
&a_5 = b_4,\\
&b_5 = -a_4,\\
&c_2 = c_1,
\end{aligned}$$

Thus (1) may be written

$$\left.\begin{aligned}
A &= a_4\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) + b_4\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right),\\
B &= b_4\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) - a_4\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right),\\
C &= c_0 + c_1\frac{\partial u}{\partial x} + c_1\frac{\partial v}{\partial y} + c_3\frac{\partial w}{\partial z}.
\end{aligned}\right\} \qquad (3)$$

Putting $\quad c_0 = kH, \quad c_1 = -k'H, \quad 2b_4 = -k''H,$
$\qquad c_3 - c_1 - 2b_4 = -k'''H, \quad 2a_4 = -k''''H,$

and changing the coördinate axes, we have from (3)

$$\left.\begin{aligned} A = lkH - lk'H\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) \\ - k''H\left\{l\frac{\partial u}{\partial x}+\frac{n}{2}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)+\frac{m}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\} \\ - lk'''H\left\{l^2\frac{\partial u}{\partial x}+m^2\frac{\partial v}{\partial y}+n^2\frac{\partial w}{\partial z}+mn\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right. \\ \left. + nl\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)+lm\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\} \\ - k''''H\left\{mn\left(\frac{\partial v}{\partial y}-\frac{\partial w}{\partial z}\right)-\frac{m^2-n^2}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right. \\ \left. - \frac{lm}{2}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)+\frac{nl}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\}, \end{aligned}\right\} \quad (4)$$

with analogous expressions for B and C, where l, m, n are the direction cosines of the magnetic force at the point (x, y, z).

Let α, β, γ be the components of the magnetic force, so that $\alpha = lH, \beta = mH, \gamma = nH$. The absence of hysteresis in the medium gives the following equations:

$$\frac{\partial B}{\partial \gamma}-\frac{\partial C}{\partial \beta}=\frac{\partial C}{\partial \alpha}-\frac{\partial A}{\partial \gamma}=\frac{\partial A}{\partial \beta}-\frac{\partial B}{\partial \alpha}=0. \quad (5)$$

The necessary and sufficient conditions that (4) may satisfy (5) are easily found to be

$$k'' = \frac{H}{2}\frac{dk'}{dH},$$
$$k'''' = 0.$$

Hence we have finally

$$\left.\begin{aligned} A = k\alpha - k'\alpha\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) \\ - k''\left\{\alpha\frac{\partial u}{\partial x}+\frac{\gamma}{2}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)+\frac{\beta}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\} \\ - \frac{\alpha}{2H}\frac{dk''}{dH}\left\{\alpha^2\frac{\partial u}{\partial x}+\beta^2\frac{\partial v}{\partial y}+\gamma^2\frac{\partial w}{\partial z}+\beta\gamma\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right. \\ \left. + \gamma\alpha\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)+\alpha\beta\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\}, \\ B = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad (6)$$

When k'' is independent of H (6) takes the forms identical with Kirchoff's corresponding equations, though k and k' may depend on H. The essential difference between the above expressions for the components of the intensity of magnetization and those of Kirchhoff is due to the presence of the terms involving the differential coefficient of k'' with respect to H. These terms represent a magnetization in the direction of the magnetic force whose intensity is

$$-\tfrac{1}{2}\frac{dk''}{dH}\left\{\alpha^2\frac{\partial u}{\partial x}+\beta^2\frac{\partial v}{\partial y}+\gamma^2\frac{\partial w}{\partial z}+\beta\gamma\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right.$$
$$\left.+\gamma\alpha\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)+\alpha\beta\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\},$$

being proportional to the linear dilatation

$$\frac{1}{H^2}\left\{\alpha^2\frac{\partial u}{\partial x}+\beta^2\frac{\partial v}{\partial y}+\gamma^2\frac{\partial w}{\partial z}+\beta\gamma\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right.$$
$$\left.+\gamma\alpha\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)+\alpha\beta\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\}$$

of the solid in the direction of the magnetic force. Thus (6) may be considered to represent a magnetization which is obtained by superposing the magnetization given by Kirchhoff's equations upon that whose direction is parallel to that of the magnetic force and whose intensity is proportional to dk''/dH and the linear dilatation in the direction of the magnetic force, with the due coefficient of proportionality. As has been already stated, the extension of Kirchhoff's theory to cases in which k' depends on H but k'' does not, requires no change of the forms in the components of the intensity of magnetization. But this is not the case for the expressions of forces acting on the solid, as will be seen later.

The expressions for the mechanical forces acting on the elastic solid can be obtained by proceeding in a similar way as Helmholtz[1] and Kirchhoff,[2] or we may proceed in the following manner.

Let us consider a system at rest, consisting of a permanent magnet and the elastic magnetic solid separated by air. The whole

[1] Helmholtz, Wied. Ann., Vol. XIII., p. 385; Monatsber. d. Berl. Akad, v. 17, Feb, 1881; Wissenschaftliche Abhandlungen, Vol. I., p. 798, Leipzig (1882).

[2] Kirchhoff, l. c.

system is supposed to be at an uniform temperature. It is convenient to assume the continuous transition in the boundary of the different media, as is usually done in similar discussions. Considering the thin stratum of transition from the solid to the air to be a portion of the solid, we have $k = k' = k'' = 0$ at its boundary. Let a, b, c be the components of the magnetic induction at the point (x, y, z), and let $d\tau_m$, $d\tau_a$, $d\tau$ be the volume elements of the permanent magnet, the air, and the solid respectively.

Now the work done by external forces to produce the magnetic field reckoned per unit volume is

$$\frac{1}{4\pi}\int (\alpha da + \beta db + \gamma dc),$$

where the integral is to be taken from the state $H = 0$. Hence the total work done by external forces to produce the magnetic field in the system under consideration is

$$W = \frac{1}{4\pi}\int R_m d\tau_m + \frac{1}{4\pi}\int R_a d\tau_a + \frac{1}{4\pi}\int R d\tau,$$

where R_m, R_a, R denote the values of $\int(\alpha da + \beta db + \gamma dc)$ in the permanent magnet, the air, and the solid, respectively.

Let the element of the elastic solid at (x, y, z) make an infinitesimal virtual displacement $(\delta u, \delta v, \delta w)$, while the permanent magnet is completely fixed. The virtual displacement is supposed to take place so slowly that the temperature of the system remains unaffected. The variation of W which the system acquires during the virtual displacement is

$$\begin{aligned}\delta W &= \frac{1}{4\pi}\int \delta R_m d\tau_m + \frac{1}{4\pi}\int \delta R_a d\tau_a + \frac{1}{4\pi}\int \delta R d\tau. \\ &= \frac{1}{4\pi}\int (\alpha\delta a + \beta\delta b + \gamma\delta c) d\tau_m \\ &\quad + \frac{1}{4\pi}\int (\alpha\delta a + \beta\delta b + \gamma\delta c) d\tau_a + \frac{1}{4\pi}\int \delta R d\tau, \qquad (7)\end{aligned}$$

since

$$\delta R_m = \alpha\delta a + \beta\delta b + \gamma\delta c.$$
$$\delta R_a = \alpha\delta a + \beta\delta b + \gamma\delta c.$$

δR can not be expressed in a simple form as δR_m or δR_a.

The absence of electric current in the system warrants the existence of the magnetic potential V at the point (x, y, z). If we multiply

$$\frac{\partial \delta a}{\partial x} + \frac{\partial \delta b}{\partial y} + \frac{\partial \delta c}{\partial z} = 0$$

by V and integrate the result over the whole space we have

$$\int V\left(\frac{\partial \delta a}{\partial x} + \frac{\partial \delta b}{\partial y} + \frac{\partial \delta c}{\partial z}\right) d\tau_m + \int V\left(\frac{\partial \delta a}{\partial x} + \frac{\partial \delta b}{\partial y} + \frac{\partial \delta c}{\partial z}\right) d\tau_a + \int V\left(\frac{\partial \delta a}{\partial x} + \frac{\partial \delta b}{\partial y} + \frac{\partial \delta c}{\partial z}\right) d\tau$$
$$= \int (a\delta a + \beta \delta b + \gamma \delta c) d\tau_m + \int (a\delta a + \beta \delta b + \gamma \delta c) d\tau_a + \int (a\delta a + \beta \delta b + \gamma \delta c) d\tau = 0. \tag{8}$$

With the aid of (8), (7) may be written

$$\delta W = \frac{1}{4\pi} \int \{\delta R - (a\delta a + \beta \delta b + \gamma \delta c)\} d\tau. \tag{9}$$

Now a, b, c and R are explicit functions of α, β, γ, k, k', k'', (and dk''/dH), and the strain components $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y} \cdots \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$ only, while k, k', k'' are again functions of x, y, z, and H. Hence the variation δ may conveniently be considered as taking place in three different stages: During the first stage there is neither deformation nor change of position of elementary portions, so that the only varying quantities are α, β, γ, and k, k', k'', since the latter are dependent on H. During the second stage of the variation the material point at $(x - \delta u, y - \delta v, z - \delta w)$ arrives at (x, y, z) after the virtual displacement, the magnetic field and the strain components being supposed invariable. Thus it corresponds to the mere effect of the change of position without deformation. The variation of the strain components in constant field and without the change of position forms the third stage. Let these three kinds of the variations be respectively donated by δ_1, δ_2, and δ_3; then we may write

$$\delta R = a\delta_1 a + \beta \delta_1 b + \gamma \delta_1 c + \delta_2 R + \delta_3 R,$$

$$a\delta a + \beta\delta b + \gamma\delta c = a\delta_1 a + \beta\delta_1 b + \gamma\delta_1 c + \delta_2(aa + b\beta + c\gamma) + \delta_3(aa + b\beta + c\gamma).$$

Hence (9) becomes

$$\delta W = \frac{1}{4\pi}\int [\delta_2\{R - (aa + b\beta + c\gamma)\} + \delta_3\{R - (aa + b\beta + c\gamma\})d\tau \tag{10}$$

$$= -\int \delta_2 S d\tau - \int \delta_3 S d\tau,$$

where

$$S = \frac{1}{4\pi}\{(aa + b\beta + c\gamma) - R\} \tag{11}$$

$$= \frac{1}{4\pi}\int (ada + bd\beta + cd\gamma).$$

No by virtue of equations (6) (11), and the well known relations

$$a = \alpha + 4\pi A,$$

$$b = \beta + 4\pi B,$$

$$c = \gamma + 4\pi C,$$

we get at once

$$S = \frac{H^2}{8\pi} + \int_0^H kHdH - \int_0^H k' HdH\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{k'}{2}\left\{\alpha^2\frac{\partial u}{\partial x} + \beta^2\frac{\partial v}{\partial y} + \gamma^2\frac{\partial w}{\partial z} + \beta\gamma\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) + \gamma\alpha\left(\frac{\partial w}{\partial z} + \frac{\partial u}{\partial z}\right) + \alpha\beta\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right\}.^1$$

We have immediately

$$\int \delta_2 S d\tau = -\int\left\{\left(\frac{\partial S}{\partial x}\right)_H \delta u + \left(\frac{\partial S}{\partial y}\right)_H \delta v + \left(\frac{\partial S}{\partial x}\right)_H \delta w\right\} d\tau,$$

where $\left(\frac{\partial S}{\partial x}\right)_H$, $\left(\frac{\partial S}{\partial y}\right)_H$, $\left(\frac{\partial S}{\partial z}\right)_H$ represent the differential coefficients of S with respect to x, y, z, respectively, keeping α, β, γ constant during the differentiation. The suffix H implies in the following a similar signification. Also

[1] In the above-cited paper Kirchhoff denotes by G a certain expression which requires a modification, when the susceptibility k is dependent on the strength of the magnetizing force.

$$\int \delta_3 S d\tau = -\int \Big[\int_0^H k' H dH \Big(\frac{\partial \delta u}{\partial x} + \frac{\partial \delta v}{\partial y} + \frac{\partial \delta w}{\partial z} \Big) + \frac{k''}{2} \Big\{ \alpha^2 \frac{\partial \delta u}{\partial x} + \beta^2 \frac{\partial \delta v}{\partial y} + \gamma^2 \frac{\partial \delta w}{\partial z} + \beta\gamma \Big(\frac{\partial \delta v}{\partial z} + \frac{\partial \delta w}{\partial y} \Big) + \gamma\alpha \Big(\frac{\partial \delta w}{\partial x} + \frac{\partial \delta v}{\partial z} \Big) + \alpha\beta \Big(\frac{\partial \delta u}{\partial y} + \frac{\partial \delta v}{\partial x} \Big) \Big\} \Big] d\tau$$

$$= \int \Big[\Big\{ \frac{\partial}{\partial x} \int_0^H k' H dH + \tfrac{1}{2} \frac{\partial}{\partial x} (k'\alpha^2) + \tfrac{1}{2} \frac{\partial}{\partial y} (k'\alpha\beta) + \tfrac{1}{2} \frac{\partial}{\partial z} (k'\gamma\alpha) \Big\} \delta u + \Big\{ \frac{\partial}{\partial y} \int_0^H k' H dH + \tfrac{1}{2} \frac{\partial}{\partial x} (k'\alpha\beta) + \tfrac{1}{2} \frac{\partial}{\partial y} (k'\beta^2) + \tfrac{1}{2} \frac{\partial}{\partial z} (k'\beta\gamma) \Big\} \delta v + \Big\{ \frac{\partial}{\partial z} \int_0^H k' H dH + \tfrac{1}{2} \frac{\partial}{\partial x} (k'\gamma\alpha) + \tfrac{1}{2} \frac{\partial}{\partial y} (k'\beta\gamma) + \tfrac{1}{2} \frac{\partial}{\partial z} (k'\gamma^2) \Big\} \delta w \Big] d\tau,$$

since $k = k' = k'' = 0$ at the surface of the solid. Hence (10) becomes

$$\delta W = -\int \Big[\Big\{ -\Big(\frac{\partial S}{\partial x} \Big)_H + \frac{\partial}{\partial x} \int_0^H k' H dH + \tfrac{1}{2} \frac{\partial}{\partial x} (k'\alpha^2) + \tfrac{1}{2} \frac{\partial}{\partial y} (k'\alpha\beta) + \tfrac{1}{2} \frac{\partial}{\partial z} (k'\gamma\alpha) \Big\} \delta u + \cdots \Big] d\tau. \tag{12}$$

If we denote the components of the mechanical force per unit of volume due to magnetism by X, Y, Z, then we have, by the principle of conservation of energy,

$$\delta W = -\int (X\delta u + Y\delta v + Z\delta w) d\tau. \tag{13}$$

Comparing the coefficients of δu, δv, δw in (12) and (13), we obtain

$$X = -\Big(\frac{\partial S}{\partial x} \Big)_H + \frac{\partial}{\partial x} \int_0^H k' H dH + \tfrac{1}{2} \frac{\partial}{\partial x} (k'\alpha^2) + \tfrac{1}{2} \frac{\partial}{\partial y} (k'\alpha\beta) + \tfrac{1}{2} \frac{\partial}{\partial z} (k'\gamma\alpha), \tag{14}$$

and analogous expressions for Y and Z.

Assume u, v, w to be infinitely small. Then (14) becomes

$$\left.\begin{aligned} X = -\int_0^H \left(\frac{\partial k}{\partial x}\right)_H H dH + \frac{\partial}{\partial x}\int_0^H k' H dH + \tfrac{1}{2}\frac{\partial}{\partial x}(k'\alpha^2) \\ + \tfrac{1}{2}\frac{\partial}{\partial y}(k'\alpha\beta) + \tfrac{1}{2}\frac{\partial}{\partial z}(k'\gamma\alpha), \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}\right\} \quad (15)$$

These expressions show that we must slightly modify Kirchhoff's expressions for the components of the mechanical force per unit volume, when we consider k and k' to depend upon the strength of the magnetizing force. When we return to the case discussed by Kirchhoff, we see that (15) agrees with his result, excepting differences in notations. The above expressions are not the only forms in which we can write the component forces; they may sometimes be put in the following forms. When $u = v = w = 0$, we have

$$\frac{\partial S}{\partial x} = \left(\frac{\partial S}{\partial x}\right)_H + \frac{1}{4\pi}\left(a\frac{\partial \alpha}{\partial x} + b\frac{\partial \beta}{\partial x} + c\frac{\partial \gamma}{\partial x}\right)$$

$$= \left(\frac{\partial S}{\partial x}\right)_H + \frac{1}{4\pi}\left\{\frac{\partial(a\alpha)}{\partial x} + \frac{\partial(b\alpha)}{\partial y} + \frac{\partial(c\alpha)}{\partial z}\right\},$$

making use of the equations

$$\frac{\partial \beta}{\partial x} = \frac{\partial \alpha}{\partial y},$$

$$\frac{\partial \gamma}{\partial x} = \frac{\partial \alpha}{\partial z},$$

$$\frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} + \frac{\partial c}{\partial z} = 0.$$

Consequently (14) becomes

$$\left.\begin{aligned} X = \frac{\partial}{\partial x}\left\{-S + \frac{a\alpha}{4\pi} + \int_0^H k' H dH + \frac{k'\alpha^2}{2}\right\} \\ + \frac{\partial}{\partial y}\left\{\frac{b\alpha}{4\pi} + \frac{k'\alpha\beta}{2}\right\} + \frac{\partial}{\partial z}\left\{\frac{c\alpha}{4\pi} + \frac{k'\gamma\alpha}{2}\right\} \\ = \frac{\partial}{\partial x}\left\{\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)\alpha^2 - \frac{H^2}{8\pi} - \int_0^H (k - k')H dH\right\} \end{aligned}\right. \quad (16)$$

$$+\frac{\partial}{\partial y}\left\{\left(\frac{1}{4\pi}+k+\frac{k'}{2}\right)\alpha\beta\right\}+\frac{\partial}{\partial z}\left\{\left(\frac{1}{4\pi}+k+\frac{k'}{2}\right)\gamma\alpha\right\}.$$

The corresponding expression for Y and Z can be written down from (16) by symmetry.

The surface-traction $(\bar{X}, \bar{Y}, \bar{Z})$ referred to unit of area can be easily found by integrating the bodily force (X, Y, Z reckoned per unit volume) along the normal to the bounding surface of the solid from one side of the stratum of transition to the other. Let n be the normal to the bounding surface of the solid drawn inwards, and let α', β', γ', H' be the components and the magnitude of the magnetic force in the air. Then in the integral

$$\bar{X}=\int_{(\text{air})}^{(\text{solid})} X dn,$$

we may put

$$X=\cos(n,x)\frac{\partial}{\partial n}\left\{\left(\frac{1}{4\pi}+k+\frac{k'}{2}\right)\alpha^2-\frac{H^2}{8\pi}-\int_0^H(k-k')HdH\right\}$$
$$+\cos(n,y)\frac{\partial}{\partial n}\left\{\left(\frac{1}{4\pi}+k+\frac{k'}{2}\right)\alpha\beta\right\}$$
$$+\cos(n,z)\frac{\partial}{\partial n}\left\{\left(\frac{1}{4\pi}+k+\frac{k'}{2}\right)\gamma\alpha\right\},$$

for the space variation of a quantity in directions perpendicular to n are infinitely small compared with the space variation of the same quantity in the direction of n. Hence

$$\bar{X}=\left(\frac{1}{4\pi}+k+\frac{k''}{2}\right)\alpha\{\alpha\cos(n,x)+\beta\cos(n,y)+\gamma\cos(n,z)\}$$
$$-\frac{H^2\cos(n,x)}{8\pi}-\cos(n,x)\int_0^H(k-k')HdH$$
$$-\frac{\alpha'}{4\pi}\{\alpha'\cos(n,x)+\beta'\cos(n,y)+\gamma'\cos(n,z)\}+\frac{H'^2\cos(n,z)}{8\pi}.$$

Since the tangential components of the magnetic force and the normal components of the magnetic induction are continuous at the bounding surface, the above expressions for $\bar{X}$ can be easily put in the form,

$$\bar{X}=-2\pi k^2\{\alpha\cos(n,x)+\beta\cos(n,y)+\gamma\cos(n,z)\}^2\cos(n,x)$$

$$- \cos(n, x)\int_0^H (k - k')HdH \qquad (17)$$

$$+ \frac{k'a}{2}\{\alpha \cos(n, x) + \beta \cos(n, y) + \gamma \cos(n, z)\}.$$

Here we have, as in the expressions for the components of the bodily force reckoned per unit volume, a term

$$- \cos(n, x)\int_0^H (k - k')HdH$$

involving an integral with respect to H. When k and k' are independent of the strength of the magnetizing force, this term reduces to the form given by Kirchhoff.

We may also find stress components X_x, X_y... Y_x... in ether, and then find the expressions for the components of the surface-traction. We have, assuming the ether to be at rest,

$$\left.\begin{aligned} X &= -\frac{\partial X_x}{\partial x} - \frac{\partial X_y}{\partial y} - \frac{\partial X_z}{\partial z}. \\ &\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}\right\} \qquad (18)$$

Comparing (16) and (18) we obtain, as a particular solution of (18),

$$\left.\begin{aligned} X_x &= -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)\alpha^2 + \frac{H^2}{8\pi} + \int_0^H (k - k')HdH \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ Z_x = X_z &= -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)\gamma\alpha, \\ X_y = Y_x &= -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)\alpha\beta. \end{aligned}\right\} \qquad (19)$$

Let X_x', X_y', X_z', X_n' be the corresponding quanties in the air. We see at once

$$\begin{aligned} \bar{X} &= X_n' - X_n \\ &= X_x' \cos(n, x) + X_y' \cos(n, y) + X_z' \cos(n, z) \qquad (20) \\ &\quad - X_x \cos(n, x) - X_y \cos(n, y) - X_z \cos(n, z). \end{aligned}$$

But

$$X_x' = -\frac{\alpha'^2}{4\pi} + \frac{H'^2}{8\pi},$$

$$X_y' = -\frac{\alpha'\beta'}{4\pi},$$

$$X_z' = -\frac{\gamma'\alpha'}{4\pi},$$

as is easily seen from (19). Hence (20) may be brought to the identical form with (17).

In conclusion I desire to express my best thanks to Prof. A. Tanakadate for his useful suggestions and valuable criticisms. I also wish to thank Mr. K. Honda, who directed my attention to this problem.

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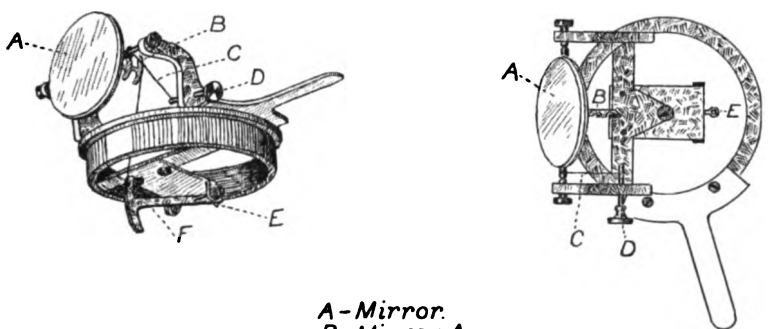
THE VOWEL A^E (AS IN HAT).

By Louis Bevier, Jr.

IN a former article (June, 1900) the acoustic character of the vowel *a* (as in *father*) was briefly discussed, as studied from the phonographic record. Since then these researches have been continued on lines similar to those there described. The apparatus employed has been somewhat improved as shown in the accompanying figure. (Fig. 1.) This exhibits the essential parts of the tracer which I have used for the past year. The vibrating mirror has been more carefully mounted than before to secure motion about one axis only. The introduction of an extra lever



A-Mirror.
B-Mirror Arm.
C-Tension Spring.
D-Tension Screw.
E-Sapphire Knob.
F-Lever.

Fig. 1.

provides, on the one hand, for a greater degree of enlargement, and transcribes the elevations and depressions of the phonograph trace without inversion, *i. e.*, right side up. In all other respects the method is the same as in that described in the article to which allusion has been made.

The curves shown in this and the succeeding articles are all made with the same degree of enlargement of ordinates. The lens used

brings the focus to a distance of about 15½ feet from the mirror, giving a beam of light to write with 186 inches long. The arm of the mirror is three-eighths of an inch long. The lever has a short arm of one-eighth of an inch, and a long arm of six-eighths of an inch. Since the long arm of the lever is attached to the arm of the mirror the system gives an enlargement of 5,952 times ($\frac{6}{8} \div \frac{1}{8} = 6$; $186 \div \frac{3}{8} = 496$; $6 \times 496 \times 2 = 5{,}952$). A much greater enlargement may be obtained if desired in two ways, first by using a lens with a longer focal distance, and secondly by reducing the arm of the mirror to two-eighths or to one-eighth of an inch. For practical reasons of convenience this further enlargement has not been resorted to as yet.

The enlargement of abscissas is a matter depending obviously on the speed with which the wax cylinder revolves during the making of a record, and its transcription, and also on the speed with which the drum bearing the photographic paper is driven. Since the present studies concern only vowels at a definite registered pitch, there has been no need to control this matter absolutely, and the speeds used have not been quite the same in all cases. In general, however, the wave as recorded in the wax has been lengthened in transcription about 35 times. In the specimen curves shown in the accompanying plate there may therefore be noted differences in the apparent length of waves of the same pitch. In experiments where the pitch of utterance is not registered it is essential to control carefully the speed of each cylinder.

The present studies have been made from the records of nine different voices, securing enough variety, it is believed, to eliminate from the results conclusions based on peculiarities of utterance that are merely individual. These voices are indicated in the table by the numbers at the extreme left of the page. Numbers 1, 2 and 6 are men's voices of baritone quality; 4 and 5 are children's voices both of soprano quality, the former that of a boy, the latter of a girl; the remaining are women's voices, 8, 9 and 10 sopranos, and 7 a rich contralto.

The waves were measured under a microscope, with appropriate attachments, and were analyzed by the ordinary method from the Fourier series. Generally 36 ordinates were taken, giving the

amplitudes of the fundamental and sixteen upper partials. A few were measured in 24 places and the partials computed as far as XI. In the case of fundamentals of a high pitch, when there was clearly no object in computing the upper partials above V. or VI., two waves were measured as one in 36 places, and the analysis gave the amplitudes of the partials from I. to VIII. These cases may be seen in the following table. It will be observed that the first column contains figures indicating the voice, as above explained. The second column gives the sum of the amplitudes of all the partials, a rough measure of the loudness of utterance. The third column, under I., shows the pitch of the fundamental by means of the vibration number, and its amplitude *percentage*. The fourth column, under II., gives pitch and amplitude percentage of the first upper partial and so on for the rest. In each record the most important partials are indicated by the heavy-faced figures used in expressing their amplitude percentages.

An examination of the above table shows that, no matter what the fundamental or chord-tone may be, the upper partial that is most important by reason of its energy is that whose rate of vibration is in the neighborhood of 1,550 to the second. If it be, as is assumed, an upper partial of the chord-tone, it must of course be a multiple of the vibration-rate of the chord-tone. Save for this relation, it is independent of the pitch of the chord-tone. With a chord-tone of 152 vibrations to the second, it will, in all probability, be the ninth upper partial (X.) which is more prominent, with a vibration rate of 1,520. On the other hand, with a chord-tone of 500 vibrations to the second, the second upper partial (III.) with a vibration rate of 1,500 will be the prominent one. The pitch of the dominant upper partial will be actually a little lower in the second case than in the first. The particular upper partial which is most strongly reinforced by the mouth-resonance will be that which lies nearest to the pitch of maximum resonance of the mouth cavity for the a^e position, *i. e.*, the nearest to 1,550 vibrations to the second. Examples may be taken from the table at will. The dominant upper partial on a fundamental of 202 will be VIII. (1,616), on 226 VII. (1,582), on 256 VI. (1,536), on 320 V. (1,600), and so on. The regularity with which this follows, no matter what the character of the voice examined, is convincing and conclusive.

ANALYSES OF THE A^{II} CURVE.

		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII
		128	256	384	512	640	768	896	1024	1152	1280	1408	1536	1664	1792	1920	2048	2176
1	118.8	24.9	6.7	2.0	2.0	6.1	**6.6**	5.2	4.2	3.7	0.8	1.9	7.7	8.8	8.1	4.9	4.4	2.0
		136	272	408	544	680	816	952	1088	1224	1360	1496	1632	1768	1904	2040	2176	2312
1	54.6	28.6	8.8	1.1	3.7	**4.0**	0.4	5.1	**10.7**	3.7	5.1	7.3	**10.7**	4.0	3.3	1.8	1.4	0.3
		144	288	432	576	720	864	1008	1152	1296	1440	1584	1728	1872	2016	2160	2204	2348
2	64.0	7.8	1.6	4.4	**41.3**	8.4	2.5	**6.6**	3.8	4.7	**9.4**	4.7	1.6	1.3	0.6	0.3	0.3	0.7
		152	304	456	612	760	912	1064	1216	1368	1520	1672						
1	98.6	27.6	2.8	7.4	5.6	**10.6**	5.8	**9.2**	7.0	5.6	**12.6**	5.8						
		160	320	480	640	800	960	1120	1280	1440	1600	1760	1920	2080	2240	2400	2560	2720
1	123.0	9.8	2.3	3.6	**29.9**	2.3	4.9	5.5	8.9	**10.7**	6.2	4.4	1.1	3.3	2.6	1.6	1.8	1.1
6	128.8	45.2	1.8	2.3	**14.7**	1.3	3.5	2.0	3.0	**18.0**	3.6	3.7	1.4	1.2	0.5	1.4	0.4	1.0
		181	362	543	724	905	1086	1267	1448	1629	1810	1991						
1	70.2	20.2	3.7	4.3	**12.5**	9.1	8.0	5.1	8.6	**15.1**	7.0	6.4						
		192	384	576	768	960	1152	1344	1536	1728	1920	2112	2304	2496	2688	2880	3072	3264
1	245.6	14.4	4.1	4.4	**17.8**	2.2	**11.8**	7.0	**11.0**	**11.7**	1.8	5.9	1.6	3.2	0.5	1.3	0.6	0.3
6	77.6	36.5	3.5	**5.8**	**4.9**	4.4	2.4	2.7	**17.8**	8.6	2.6	2.8	2.4	2.2	1.0	1.2	1.2	0.5
7	176.1	62.6	9.7	0.3	**5.0**	2.0	2.6	2.1	**8.2**	**8.4**	1.5	0.9	2.1	1.2	1.5	0.1	1.0	0.8
		202	404	606	808	1010	1212	1414	1616	1818	2020	2222						
2	117.6	6.0	6.0	3.7	**16.5**	6.3	7.1	10.4	**30.1**	1.7	5.5	6.7						
		226	452	678	904	1130	1356	1582	1808	2034	2260	2486						
1	80.8	14.4	3.2	**27.5**	9.9	**10.9**	8.4	**19.5**	3.4	1.0	0.8	1.0						
		256	512	768	1024	1280	1536	1792	2048	2304	2560	2816	3072	3328	3584	3840	4096	4352
2	63.0	14.1	3.8	8.1	**21.5**	10.2	**29.5**	3.2	0.1	2.9	4.1	2.5						
1	444.6	8.4	6.3	17.9	**21.6**	12.1	**14.9**	8.2	3.2	1.3	1.7	0.6	1.0	0.5	0.2	0.3	0.9	1.0
4	177.2	8.6	1.8	3.5	**37.1**	8.5	**10.5**	7.1	8.7	5.0	2.0	1.4	0.8	2.2	0.5	1.0	0.6	0.7

5	142.4	8.4	3.5	7.7	26.7	9.6	19.7	9.4	3.2	3.1	2.4	1.1	1.7	1.2	0.3	0.7	0.7	0.6	
6	163.0	11.0	6.1	8.1	3.0	4.2	28.3	17.1	1.9	1.8	1.5	3.3	2.1	2.1	2.4	2.6	3.3	1.2	
7	68.2	13.2	4.0	14.5	4.0	5.7	16.9	12.0	5.5	4.8	8.5	4.7	1.9	0.7	1.3	0.9	0.2	1.2	
8	122.0	3.5	5.4	18.9	17.5	6.3	25.1	15.7	4.6	1.2	0.9	0.5	1.0	1.8	0.9	0.4	0.7	0.6	
9	59.6	25.0	2.7	13.9	6.4	6.9	27.0	7.0	5.2	0.3	1.2	1.5	0.7	1.2	0.2	0.2	0.3	0.3	
		272	544	816	1088	1360	1632	1904	2176	2448	2720	2992							
1	182.8	6.0	6.0	18.6	13.6	23.7	15.0	8.7	1.0	2.8	1.9	2.7							
		320	640	960	1280	1600	1920	2240	2560	2880	3200	3520	3840	4160	4480	4800	5120	5440	
4	418.2	4.6	6.9	19.7	19.8	15.7	13.2	11.2	2.8	1.2	1.3	0.5	1.5	0.7	0.5	0.2	0.2	0.2	
5	207.0	5.5	17.9	25.2	12.5	19.5	9.5	0.6	0.6	2.7	0.4	1.9	0.1	1.8	0.6	0.7	0.2	0.5	
7	76.1	3.2	29.3	8.4	9.2	20.1	6.0	5.2	8.9	3.2	1.2	1.7	0.3	1.2	0.7	0.1	0.3	1.0	
8	83.7	6.7	21.8	26.8	11.5	20.0	9.6	0.5	0.1										
9	41.3	6.8	18.2	15.2	6.5	28.1	9.7	3.1	2.4	2.4	1.5	1.7	0.7	0.5	0.5	0.2	1.3	1.2	
10	84.3	6.5	13.1	21.7	19.6	21.5	9.4	1.4	1.8	0.6	1.7	0.5	0.5	0.1	0.6	0.2	0.1	0.7	
		384	768	1152	1536	1920	2304	2688	3072	3456	3840	4224	4608	4992	5378	5760	6144	6528	
4	250.6	2.4	7.5	51.9	10.5	9.0	9.5	1.4	0.3	1.6	1.0	1.2	0.7	0.5	0.3	0.8	0.7	0.7	
5	285.0	9.3	14.2	31.9	25.8	9.4	3.5	3.6	2.3										
7	302.7	6.6	23.4	8.3	42.8	12.8	4.3	0.7	1.1										
8	108.0	4.9	13.7	28.5	38.7	8.1	0.5	2.8	2.8										
9	128.5	7.8	13.7	17.4	42.3	13.6	1.1	0.2	0.5	0.8	0.2	0.1	0.5	0.2	0.6	0.5	0.2	0.1	
		480	960	1440	1920	2400	2880	3360	3840										
4	220.3	9.9	19.9	33.3	20.5	4.9	5.5	4.0	2.0										
		512	1024	1536	2048	2560	3072	3584	4096										
4	130.9	8.2	25.4	30.3	20.0	9.9	2.8	1.8	1.6										
5	271.0	6.4	52.0	29.4	4.8	3.6	2.6	0.6	0.6										
7	378.1	7.5	24.5	54.9	5.1	2.1	4.6	0.7	0.6										
8	270.7	6.3	52.3	26.8	6.0	4.2	2.7	1.1	0.6										
9	150.4	4.9	44.8	39.9	4.0	3.1	0.7	2.3	0.3										
		640	1280	1920	2560	3200	3840	4480	5120										
10	95.1	46.3	29.4	16.6	3.3	1.8	0.5	1.0	1.1										

It must also be noted that there are often two or even three consecutive partials that appear with considerable amplitude, *i. e.*, that are reinforced in the mouth cavity. This occurs particularly when the fundamental or chord-tone is low, and again when it is above 300. The explanation of the second case will appear later. Here let us observe the facts of the first case. When the chord-tone is of low pitch, *e. g.*, under 150 to the second, the upper partials lie of course near together. Take, for example, the fundamental 128 in the table. Here XII., XIII., and XIV. are all reinforced. In this particular utterance the pitch of maximum resonance was a little higher than an ordinary a^e, and XIII. with a vibration rate of 1,664 is most strongly reinforced, but XII. and XIV. (1,536 and 1,792 respectively) are also strongly reinforced. This is most important for the theory of the vowels, for it shows in a clear way what might well have been anticipated theoretically, viz., that the mouth cavity, with its irregular shape and walls of varying degrees of elasticity and softness, has no one particular and exact proper tone (*Eigenton*), but rather resounds to any pitch within certain not very narrow limits around a maximum. We have then a region of resonance with a not very pronounced maximum point, instead of a particular and exact pitch. For a^e this region of resonance centers at about 1,550 but often extends from 1,400 to 1,700 or even higher. Plainly then with a low fundamental we are to expect the reinforcement of adjacent upper partials.

In addition to this most characteristic resonance region, as just explained, the vowel a^e shows generally two other lower resonances. These are not so uniform and hence not so characteristic, but one or both of them *must* be present to give the a^e color. These lower regions of resonance center respectively at 1,050 and 650. For convenience I have indicated by full-faced type the important upper partials, both those due to the lower resonances, and to the upper or more characteristic one. For detailed examination take the record of voice I., on the fundamental 226. Here III. falls at 678, very near a maximum of resonance, and has in consequence a large amplitude percentage (27.5). On the other hand, neither IV. (904), nor V. (1,130) lies very near the next maximum at 1,050, and while both are reinforced, and are present with note-

worthy percentages (9.9 and 10.9 respectively), neither is relatively very important. Again VI. (1,356) is not high enough to fall within the upper region of full resonance, and is not powerfully reinforced, but VII. (1,582) coincides almost exactly with the upper maximum, and has a large amplitude percentage (19.5). Its energy percentage would of course be far greater. Many records show distinctly the three resonance regions which I have noted, as for example 1 (192) and 1 (226) in the table.

On the other hand many records, on analysis, are found to lack one or the other of these lower resonances. Both examples at 160 show IV. strong at 640, but scarcely any indication of any resonance at or near 1,050, whereas the first example at 320 shows weak resonance at 640. The example at 202 is unusual, showing a strong IV. and a weak III. and V., whereas we should normally expect a weak IV. and a strong III. and V. Such individual differences are not surprising, and our definition of any vowel must be broad enough to cover them.

We are now able to understand why it is that on high fundamentals or chord-tones we often find consecutive upper-partials strongly reinforced. It may well happen that one upper-partial will fall within one of the regions of resonance, and the next within another region of resonance, when the pitch interval between one upper-partial and the next is a wide one. At 512, for example, II. (1,024) and III. (1,536) should both of them be strongly reinforced, and at 384 it may well be that II. (768), III. (1,152) and IV. (1,536) are all strong. It must also be observed that in general the amplitude percentages are higher on a high fundamental, because there are fewer upper partials available to take the entire resonance energy. It is in accord, too, with the whole theory of resonance that the upper partials near the fundamental are favored, other things being equal. Hence if the mouth cavity resounds weakly to the vibration rate 900, on a fundamental of 150, we shall be sure to find VI. very weak, but on a fundamental of 450 II. will be fairly strong. It often happens that, where a fundamental is of high pitch, no one of the upper partials coincides well with any region of strong resonance. We have then forced resonance, the resonator adjusting itself as best it can. Thus it comes to pass that as the pitch of the

chord-tone is raised, after the tones of ordinary speech are passed, the differentiation between the vowels becomes less and less perfect, until at 1,024 (soprano high c) little differentiation is possible. This difficulty does not arise, however, at any pitch used in the ordinary speech either of men's or of women's voices.

As a result of these observations I would define our American *a*^{e} as consisting of the following acoustic elements:

(1) A chord-tone more or less strong according to the amount of reinforcement which the mouth cavity gives it, varying within great range. In general it is quite strong below 200, and weak between 250 and 600, above which last point it becomes very strong, coinciding with the lowest observed mouth-resonance of this vowel.

(2) A characteristic *a*^{e} resonance in the region of 1,550, *always* present, but not absolutely stationary, because the vowel may tend more or less toward *a* or toward *e*, with endless nuances. This characteristic resonance is not a particular point, but covers a consider able region.

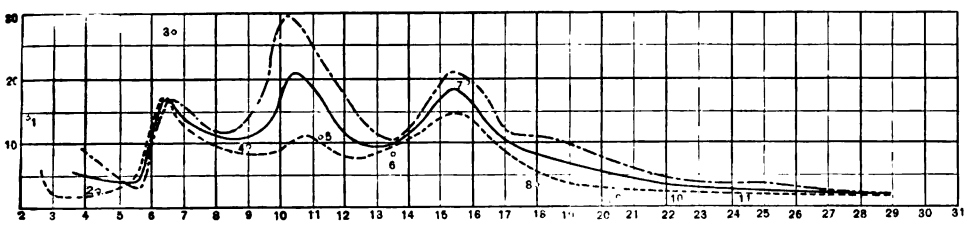
(3) A strong resonance at one or two lower regions centering in 650 and 1,050 respectively. Both *may* be present, but one at least *must* be. Great individual variation is shown in the relation of these resonances to each other and to the fundamental.

The characteristic *a*^{e} resonance at (*ca*) 1,550 largely determines the vowel character, but it must receive "body," so to speak, by one or more reinforced lower tones. This gives it to the ear a certain fullness or openness which it requires, setting it in sharp contrast to the thinness of *i* (as in *pique*), whose characteristics will be discussed later.

These facts may be exhibited diagrammatically by plotting a resonance curve for *a*^{e}. This I have done by taking all my *a*^{e} records, at least twice as many as are shown in the table of analyses, and averaging their resonance percentages. The characteristic resonance-curve of *a*^{e}, thus obtained, is shown in Fig. 2. The upper line of dots and dashes is the average of the records of women's and children's *a*^{e}'s, the lower broken line that of men. The continuous line shows the average for all taken together. This curve shows the mouth-resonance rising sharply from 550 and reaching a maximum at 650. From this point it declines, rising to a second max-

imum at 1,050. From here it once more declines and rises to its upper and most characteristic maximum at 1,550. The women's line is higher than the men's for reasons already sufficiently explained.

This resonance-curve is only an average, and as in the case of all averages is not altogether true. It represents the vowel a^e much as a composite photograph represents a human face. No individual utterance is likely to coincide closely with it in every part. To illustrate this I have introduced for comparison an individual record made on a fundamental or chord-tone of 226. The amplitude of each partial is plotted, $\odot^1$ being the fundamental, and $\odot^2$, $\odot^3$, $\odot^4$, etc., the upper partials. It will be seen that II. is weak as it should be, III. is very strong, much above the average, IV., V., VI: follow



A^e RESONANCE CURVE

the men's line closely, VII. is at a maximum, VIII. is low when it should be, and IX., X. and XI. are negligible. If the curve were plotted as an energy curve (cf. PHYS. REV., X., 4, p. 200), the dominating importance of the highest resonance would be more prominent to the eye.

It is instructive to study the actual curves obtained. The accompanying plate shows a sheet of a^e curves reduced to about $\frac{1}{3}$ actual size. The upper resonance is in general easy enough to detect by the simple process of counting the small waves embroidered upon the fundamental wave. For example at 256 in each of the specimens shown there are six crests plainly marked on each of the long or fundamental waves, and 6×256 gives the characteristic maximum of 1,536. At 512 the crests are three in number, etc. As a rough and ready means of discrimination the eye is usually quite sufficient if the pitch of the fundamental is known. For the

lower resonances the eye may still suffice sometimes after practice, *e. g.*, at 512, where it is evident at a glance (if it be a practiced glance) that II. is present in large amplitude. But for the accurate determination of relative amplitudes the mathematical analysis is indispensable. The correct analysis of a complex curve, such as either of the examples at 320, is entirely beyond the eye alone. At the lower pitches the interference of adjacent upper partials is often quite apparent to the eye but the determination of numerical relations cannot be made by the unaided eye.

It is apparent from the above why all *a*'s, whether pronounced by man, woman, or child, are heard by the ear as a unit sound. There is a large element which is substantially a constant. The variable chord-tone does not disguise the identity, for the ear has learned by long practice to disassociate this from the function of vowel-differentiation. On the other hand it is equally clear that no one *a* is ever quite like any other, and that, without violating the essential characteristics of the vowel there is room for infinite variety of individual utterance. We may learn to recognize a friend's voice in his pronunciation of a single vowel sound. Not only so, but in the whispered voice, where the chord-tone is absent, and the mouth resonances alone are present, weakly called forth by the force of expiration, and accompanied by many slight noises of rushing breath, the same essential identity is clearly possible, with the same infinite variety of individual utterance.

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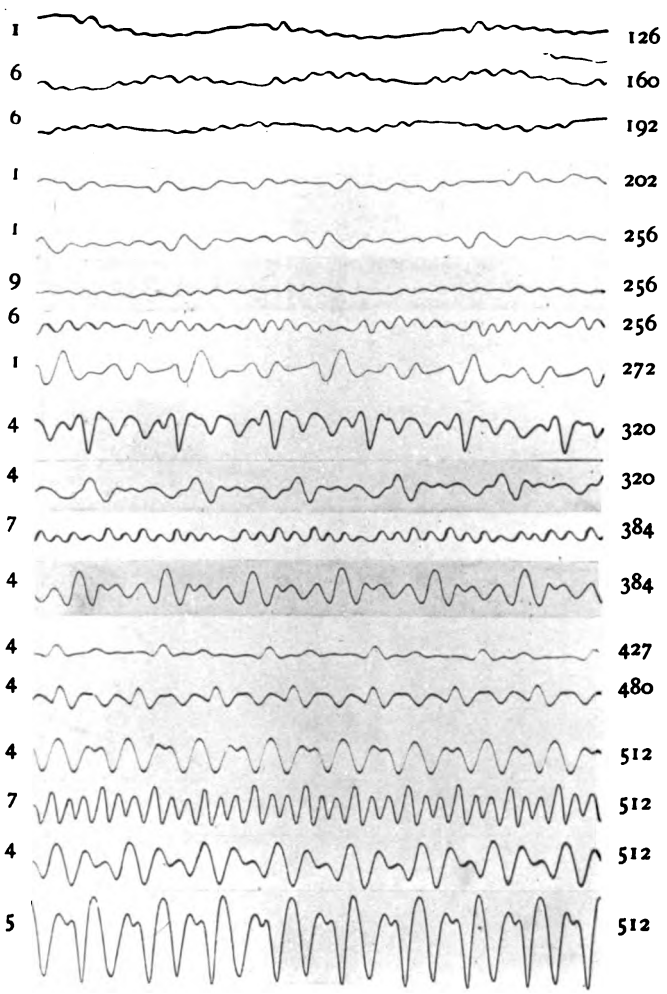
(To face p. 180.)



A°—Curves

MAGNETIZATION OF STEEL AT LIQUID AIR TEMPERATURES.

By C. C. Trowbridge.

THE present paper is an account of experiments on the magnetic moments of small bars of steel magnetized at the temperature of boiling liquid air.

The magnetic moments obtained by magnetization at approximately — 185° C. were compared with the magnetic moments of bars of the same steel obtained by magnetization at normal temperature, 20° C.

The change of magnetic strength produced by heating bar magnets to 20° C. which had been magnetized at — 185° C. was also determined, and compared with the change produced by cooling magnets of the same steel to — 185° C. which had been magnetized at 20° C. The experiments were chiefly confined to carbon magnet steel and tungsten magnet steel.

Dewar has investigated the effect of lowering the temperature of permanent magnets to — 182° C. and determining the magnetic moments at + 15° and — 182° C. through several cycles of temperature change between these limits.[1] Later, more extensive experiments of the same nature were performed by Dewar and Fleming together.[2] Among the conclusions reached by them are the following:

"1. That the sudden cooling to the temperature of liquid air usually permanently decreases the magnetic moment of short magnets made of many varieties of steel, assuming them to have been initially magnetized in a strong field.

"2. This initial decrease is found both in hardened steels having great coercive force, and also in the same steels in a soft or an-

[1] Chemical News, April 26, 1895.

[2] Proceedings Royal Society, Vol. 60, p. 57, 1896.

nealed condition and especially conspicuous in the case of the 19 per cent. nickel steel.

"3. In the case of most steels so far examined the effect of cooling magnets made of them to — 185° C. is to temporarily increase the magnetic moment after the permanent magnetic condition has been reached.

"4. The exceptions of the above rule so far noted, are the nickel steels with percentages of nickel from 19 to 29 per cent., in which case the magnetic moment is always decreased temporarily by cooling to — 185° C. after the permanent magnetic condition has been reached.

In the experiments just referred to, performed by Dewar and Fleming, the steels tested were magnetized at normal temperatures and then subjected to temperature changes through the range + 15° to — 185° C.

In the author's experiments described below, steel bars were magnetized at the temperature of boiling liquid air and their magnetic moments determined at the same temperature.

The method employed was as follows:

Short bars of steel of equal length, having been cut from the same long bar, were heated together to a red heat and then plunged into water at 20° C.; they were thus hardened to approximately the same degree.

In order to magnetize the steel bars at — 185° C., a small silvered cylindrical Dewar bulb of 100 cc. capacity, was filled with liquid air and placed within a coil having 6,500 turns of copper wire. The bar of steel to be magnetized was then put into the Dewar bulb and kept in the center of the coil by two perforated cardboard disks which fitted the interior of the bulb.

The arrangement is shown in Fig. 1, in which letters indicate the parts of the apparatus as follows: *C*, coil; *D*, Dewar bulb; *A*, liquid air; *K*, cardboard disks; *M*, magnet.

The dimensions of the coil employed were as follows:

Length, 9 cm.; inside diameter, 8 cm.; outside diameter, 14.5 cm.

The coil was made of 6,500 turns of No. 24 B. & S. copper wire, and had a resistance of 203 ohms.

The magnetizing force was very strong, .29 ampère being used; this produced 1,885 ampère turns.

The time of magnetization was made 60 seconds in each case.

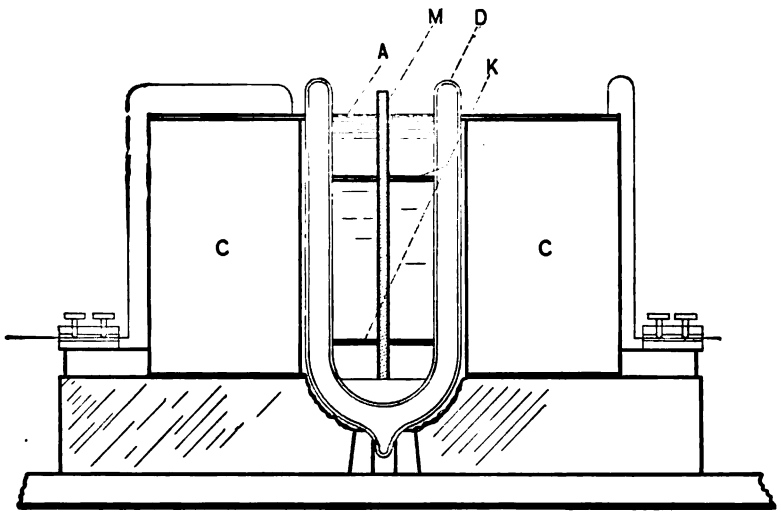


Fig. 1.

After the bars of steel were magnetized at $-185°$ C. they were kept immersed in liquid air until the magnetic moment at the low temperature was determined.

The magnetic moments of the magnets were determined with an ordinary magnetometer, the magnets resting in a brass tray, the dimensions of which were $10 \times 1.5 \times 1.5$ cm.

The method is indicated by Fig. 2, in which T is a brass tray; M, a magnet; H, a magnetometer, W; a wooden carrier for the tray; S, a meter scale.

When the magnetic moments at low temperature were determined the brass tray was filled with liquid air.

The initial change of magnetic strength, or the first gain or loss in the magnetic moment, accompanying the first change of temperature of the bars of steel after they were magnetized was determined, because it has been shown by Dewar and others that the change of magnetic strength of a permanent magnet due to a change of temperature depends partly on the previous history of the magnet.

Special care was taken not to jar the magnets throughout the experiments, and also to determine their magnetic moments immediately after magnetization; to these precautions is probably due the good agreement of the results.

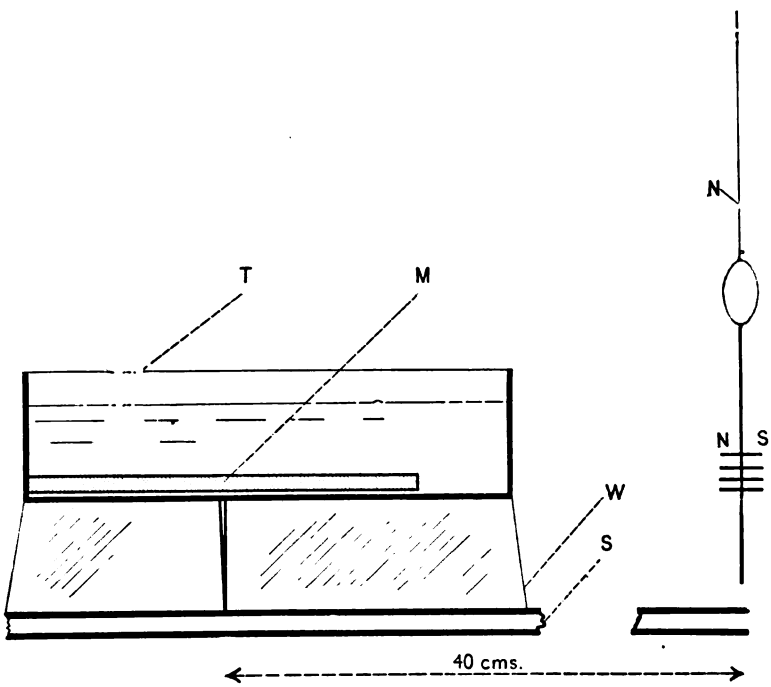


Fig. 2.

The experiments were interrupted by the failure of the company from which liquid air was produced, but the results already obtained show some interesting facts.

Two kinds of steel specially made for use as permanent magnets were tested. These steels were:

Carbon Steel, Crescent Company, Pittsburg, containing 2.40 per cent. of carbon.

Tungsten steel from Sheffield, England. (Percentage of tungsten not given.) The results of the experiments are given below in tabular form:

Carbon Steel, Crescent Company, Pittsburg — 2.4 per cent carbon.

Three bars of carbon steel, size, 83 x 10 x 3 millimeters, weighing 24.45 grams, were heated together to a red heat and then hardened in water at 20° C. They were magnetized at —185° C. and then immediately warmed to 20° C. The magnetic moments at these temperatures were as follows:

TABLE I.

Carbon Steel Magnetized at —185° *C.*

Magnet.	Magnetic Moment at —185° C.	Magnetic Moment at 20° C.	Difference (loss).	Percentage Loss.
1	211	131	80	37.9
2	202	140	62	30.6
3	211	147	64	30.3
Mean				32.9

Another bar of this steel of the same dimensions and weight as the magnets 1, 2 and 3 and hardened with them, was magnetized at 20° C. and then immediately cooled to —185° C.

The magnetic moments at 20° C. and —185° C. were as follows:

TABLE II.

Carbon Steel, Magnetized at 20° *C.*

Magnet.	Magnetic Moment at 20° C.	Magnetic Moment at —185° C.	Difference (Loss).	Percentage Loss.
4	208	189	19	9.1

It is evident that the initial loss in magnetic moment due to change of temperature when the steel is magnetized at —185° C., and then warmed to 20° C., is much greater than for the reverse operation, although in either case, a considerable loss in the magnetic moment is the result of a change in temperature of the steel of 205°.

These carbon steel magnets, 1, 2, 3 and 4, were allowed to rest undisturbed for nine days at approximately constant temperature, (room temperature in April). Their magnetic moments were then redetermined and are given in Table III, page 186.

TABLE III.

Carbon Steel; Total Change of Magnetic Moment at the end of Nine Days.

Magnet.	Original Magnetic Moment.	Magnetic Moment after 9 Days.	Difference (Loss).	Percentage Loss.
1	211	127	84	39.8
2	202	136	66	32.6
3	211	138	73	34.6
4	208	170	38	18.2

Magnets 1, 2 and 3 were magnetized at $-185°$ C. and then warmed to 20° C. The total loss of magnetic strength of these magnets since magnetization was approximately twice as much as magnet 4, which was magnetized at 20° C., cooled to $-185°$ C. (with a loss of 9.1 per cent), and then warmed to 20° C. again.

Tungsten Steel from Sheffield.—Three bars of tungsten steel, size 89 x 2 x 3 millimeters, each weighing 5.05 grams, were hardened together in water at 20° C. They were magnetized at $-185°$ C., and then warmed to 20° C.

The magnetic moments at the two temperatures were as follows:

TABLE IV.

Tungsten Steel, Magnetized at $-185°$ *C.*

Magnet.	Magnetic Moment at $-185°$ C.	Magnetic Moment at 20° C.	Difference (Loss).	Percentage Loss.
1	212	179	33	15.5
2	228	192	36	15.7
3	232	206	26	11.2
Mean				14.1

Magnet 1, nine days after showed a magnetic moment of 176.

Magnet 2 was cooled to $-185°$ C. immediately after the magnetic moment 192 was obtained and showed a moment of 196, a gain of about 2 per cent. of the value 192.

The low percentage loss (11.2), of magnet 3 may have been due to a slight mistake in manipulation that was made, which may have introduced an error of several per cent. This determination (magnet 3) is therefore questionable to that extent.

A bar of this steel hardened with magnets 1, 2, and 3 was magnetized at 20° C., and then cooled to — 185° C. The magnetic moments obtained were as follows:

TABLE V.

Tungsten Steel, Magnetized at 20° C.

Magnet.	Magnetic Moment at 20° C.	Magnetic Moment at —185° C.	Difference (Loss).	Percentage Loss.
4	213	200	13	6.1

Bundle of iron wires (not pure iron), 9 cm. long heated to a red heat and cooled in air at 20° C., magnetized at —185° C., then warmed to 20° C., showed the following loss of magnetic strength.

TABLE VI.

Iron Magnetized at —185° C.

Magnetic Moment at —185° C.	Magnetic Moment at 20° C.	Difference (Loss).	Percentage Loss.
22.3	13.5	8.8	39.4

Another fact is shown by these experiments; approximately the same magnet moment is produced, whether the steel is magnetized at 20° C., or — 185° C., other conditions being equal. This is evident from a comparison of the magnetic moments obtained after magnetizing steel bars at —185° C., with the magnetic moment obtained after magnetizing a bar of the same steel at 20° C.

There is a slight tendency towards a higher magnetic moment at low temperature than at normal temperature, but no definite statement can be made in regard to this point with the few determinations made.

TABLE VII.

Carbon Steel, Magnetized at —185° C. and at 20° C.

Magnet	Magnetized at.	Magnetic Moment.
1	—185° C.	211
2	—185° C.	202
3	—185° C.	211
4	+ 20° C.	208

TABLE VIII.

Tungsten Steel, Magnetized at −185° C. and at 20° C.

Magnet.	Magnetized at	Magnetic Moment.
1	−185° C.	212
2	−185° C.	228
3	−185° C.	232
4	+ 20° C.	213
5	−185° C. + 20 C.	232

Three bars of tungsten steel were magnetized at −185° C., but the time of magnetization was made different in each case. A bar of the same steel hardened with these magnets was magnetized at 20° C. for comparison. The magnetic moments obtained are shown in Table IX.:

TABLE IX.

Magnet.	Magnetized at	Magnetic Moment.
9	−185° C. for 1 sec.	136
10	−185° C. " 6 "	145
11	−185° C. " 60 "	145
12	20° C. " 60 "	145

These experiments were for the purpose of determining if there was any decided lag in the production of magnetic saturation at low temperature, and were mainly of a precautionary nature. It is evident that an approximate state of saturation is reached far inside the time of magnetization allowed (sixty seconds), in the tests of tungsten and carbon steels.

In the last experiments with tungsten steel the determinations were relative only, and were made with a magnetometer of special design. The instrument, a "differential astatic magnetometer," was suggested by Professor Rood and constructed by the author.

The essential part of the instrument is the suspension system, which consists of two groups of small magnets, set 23 cm. apart, rigidly connected by a very fine glass rod. The system is suspended by a single raw silk fiber 10 cm. long. By making the polarity of the two groups of magnets opposite, a system that is approximately astatic is obtained.

The object of the arrangement employed is partly to annul the effects of distant magnetic disturbing influences, such as those that arise from trolley car motors, etc., and partly to obtain a sensitive system that will act on the differential principle.

A magnet placed within a meter of the instrument and outside of the neutral plane between the two groups of magnets acts strongly on the nearest group, producing a deflection of the system.

In the foregoing experiments on the magnetization of steel at liquid air temperature, it was desirable to eliminate the magnetic effect of the liquid oxygen in the liquid air in the Dewar bulb, during the process of magnetization. It was considerd advisable to use a very strong magnetizing force. This produced approximate saturation at both low and normal temperatures, rendering the magnetic effect of the liquid oxygen on the magnetic moments obtained by magnetization at low temperature practically negligible.

The permeability of liquid oxygen is comparatively low, moreover, as can be seen from the results of an investigation of Fleming and Dewar,[1] who have studied the magnetic permeability of liquid oxygen and liquid air. They find that the magnetic permeability of liquid oxygen is 1.00287. This number is the ratio of the magnetic permeability of liquid oxygen to that of cold gaseous oxygen, and by the same method used in the given determination no difference was detected between the magnetic permeability of gaseous oxygen at — 182° C. and at normal temperatures. The value 1.0024 was obtained for air.

These values are far below that of the so-called non-magnetic iron alloys. Twelve per cent. manganese steel, usually called non-magnetic, has a magnetic permeability of 1.3 or 1.4.

As previously stated, the magnetic moments of the steel bars were determined with an ordinary magnetometer, but the magnets while being tested at low temperatures were placed in a small tray filled with liquid air. The magnetic effect of the liquid oxygen in the liquid air used in this part of the experiments was considered negligible also, for a tray containing liquid air alone was placed with several centimeters of the magnetometer and produced practically no effect on it. When the magnets were tested, they were

[1] Proc. Royal Society, 1896, Vol. 60, p. 283.

placed at 40 cms. from the magnetometer. Moreover, the liquid air in the tray being allowed to boil away, no appreciable change was noticed in the deflection of the magnetometer immediately afterwards.

It is appropriate here to call attention to the fact that in several cases mistakes have been made by writers in stating the results of investigations on magnetism at low temperature. Among these is the following statement taken from Thompson's "Elementary Lessons in Electricity and Magnetism," edition of 1901, p. 104.

"Trowbridge found severe cooling at 100° below zero to destroy the magnetism of steel magnets; but Dewar has observed that when cooled to — 200° C. in liquid oxygen, the magnetic properties of iron are nearly twice as high as at 0° C."

What Professor John Trowbridge really found was as follows:

That a bar magnet magnetized at 6° or 8° C. when cooled from 4° C. to — 25° lost less than 4 per cent. of magnetism. For a greater reduction of temperature to about — 60° C. a far greater percentage loss of magnetism was observed. In one case, a bar magnet magnetized to saturation lost 66 per cent. of its magnetism.[1]

The latter part of the statement quoted above probably refers to an investigation of Fleming and Dewar,[2] in which they found that the magnetic permeability of soft annealed Swedish iron decreased when cooled to low temperatures, but that the permeability of unannealed Swedish iron increased as the temperature diminished.

In the case of hardened iron the permeability increased several hundred per cent. when the iron was cooled to liquid air temperature.

Permanent magnets made from commercial steels containing various percentages of tungsten, etc., uncertain quantities of carbon, and having different degrees of hardness, must be regarded, in general, as individuals with special characteristics; yet the following conclusions can be drawn from the experiments described above:

1. That approximately the same magnetic moment is obtained whether a bar of tungsten steel or carbon steel is magnetized at normal or at liquid air temperature, other conditions being the same.

[1] American Journal of Science, Vol. XXI., p. 316, 1881.

[2] Proceed. Royal Soc., Vol. 60, p. 81, 1896.

2. That the initial loss in the magnetic moment, due to change of temperature, of a bar of steel magnetized at $-185°$ C. and then heated to 20° C. is much greater than when the bar is magnetized at 20° C. and then cooled to $-185°$ C., a considerable loss occurring in both cases.

3. That there is a certain amount of unstable magnetism in a newly made steel magnet, which tends to pass off at the first change of temperature in either direction from that at which magnetization takes place, much more of the unstable magnetism passing off by heating than by cooling.

It was determined that the tungsten steel magnets, while nearly the same length, but only about one-fifth of the mass of the carbon steel magnets, give a higher magnetic moment than the carbon steel. Some allowance must be made, however, for the fact that the latter, on account of their greater mass, were probably not hardened to the same degree as the tungsten magnets, the two steels being hardened in the same manner. Furthermore, the initial losses of magnetic strength due to change of temperature from $-185°$ C. to 20° C. and from 20° C. to $-185°$ C. were very much less in the case of tungsten steel magnets than for the carbon steel magnets. It is evident that there is less of the unstable magnetism, referred to above, in the case of magnets made of tungsten steel than for those made of carbon steel.

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NEW BOOKS.

Outlines of Electrochemistry. By HARRY C. JONES. New York, The Electrical Review Publishing Company, 1901. Pp. vi + 106.

In this little volume Professor Jones has brought together the series of papers recently published in the *Electrical Review* (New York) on Osmotic Pressure, the Theory of Electolytic Dissociation, Electrolysis, the Velocity of the Ions, Conductivity of Solutions, and the Electromotive Force of Voltaic Cells. This programme does not by any means cover all that is frequently included under the term Electrochemistry. The present work and the recent volume of Mr. Bertrand Blount on Practical Electrochemistry, for example, scarcely touch each other at any point. What Professor Jones gives, however, is precisely what is most valuable to the electrician. Under various heads above mentioned he brings out in clean and definite form the beautiful results obtained by the physical chemists and shows how the theory of electrolysis has been brought out of chaos. The treatment is elementary without being vague and the reader after completing the brief chapters cannot but feel impressed with the remarkable progress which has been made in this domain within the last ten years. E. L. N.

Volume XIV. *April, 1902.* *Number 4.*

THE

PHYSICAL REVIEW.

THE SPECIFIC HEAT OF SOLUTIONS. III. A FORM OF THE PFAUNDLER CALORIMETER.

By William Francis Magie.

1. In the course of my investigation of the specific heat of solutions[1] the calorimeter which was employed in the earlier work has been considerably modified and improved. As I expect to use the instrument in its present form in the future, it may not be superfluous for me to present a brief description of it, with an account of the method of using it and of the results which can be obtained with it. In the somewhat tedious course of experimentation by which this form of the instrument was worked out, I have been greatly indebted to Mr. Frederick Fisher, the mechanician of the John C. Green School of Science, who has aided me by his practical skill, and has furnished many valuable suggestions.

2. *The Calorimetric Method.*—The general plan of the instrument is that first proposed by Pfaundler.[2] The liquid to be examined and the standard liquid are contained in two similar vessels, and are heated by the same electric current in two coils of equal resistance. The specific heat is then determined from the observation of the rise of temperature in each vessel. In the study of solutions, the solvent is made the standard liquid. It is then possible to use such amounts of the solution and of the standard that the rise of temperature shall be nearly the same in each vessel. When this is the case the two most important corrections in most calorimetric work, that involving the water equivalent of the calorimeter and that

[1] This Review, Vol. IX., No. 2, Aug., 1899; Vol. XIII., No. 2, Aug., 1901.

[2] Wien. Acad. Bericht, Vol. LIX., 1869.

depending on radiation, are practically eliminated, if only the two vessels are alike in mass, in shape, and in character of surface.[1]

3. *The Calorimeter.*—The calorimeter vessel is a cylindrical cup, 6 inches high and 4 inches in diameter, made of thin sheet silver. It is furnished with a cover of fairly thick sheet brass, that fits snugly with an exterior flange over the top of the cup. Through the center of the cover passes a brass tube, projecting three fourths of an inch above the cover and 1 ½ inches below it. Within this is fitted a glass tube, the ends of which were shrunken a little in the flame of a Bunsen burner, and then ground and polished on the interior so as to furnish the bearings for the shaft of the stirrer. This shaft is a silver rod, provided above the upper bearing with a small pulley, and projecting below the cover 5 ½ inches. On the lower end is forced a cubical block of silver. Into slots cut diagonally in the surfaces of this block are forced rectangular silver plates 1 $\frac{11}{16}$ inches long and ⅝ inch wide. The paddle or stirrer thus formed is held in place by the pulley above and by a collar below its bearings. When it is turned at the usual rate it rotates about four times a second, and stirs the liquid vigorously.

In a line ½ inch from the center of the cover are placed three short brass tubes. The two end ones are 1 ½ inches apart, corresponding to the distance between the mercury cups of my Carey Foster bridge. They are fitted with rubber bushings through which pass stout copper wires, the upper ends of which carry removable binding screws. These wires within the cup are bent out and down again, so that in the cup they are 2¾ inches apart. They terminate 3 inches below the cover, where they are attached to the ends of the heating coil. This is a spiral ⅝ inch in diameter of about 40 turns of a special German Silver wire, furnished me by Mr. Edward Weston. It has a resistance of 4 ohms, and is characterized by its small or negligible temperature coefficient. This spiral is opened in the middle and held down nearly to the stirrer by a glass tube fitted in the middle one of the three brass tubes in the cover. The most convenient way to arrange this tube is to heat the lower end,

[1] These features of the instrument have been recognized by Stroud and Gee, who have described a method which is essentially the same as the one here described. El. Rev., 21, p. 262; Nature, Vol. XXXVI., p. 483. 1887.

pinch it flat, and make a small notch in the flat end. The wire is caught in this notch, and may be held there by the insulating coating.

It may be worth mentioning that it is convenient to drill the holes in the ends of the copper wires, in which the heating coil is soldered, diagonally through, so that the adjustment of the coils to equal resistance may be made without cutting the wire.

Two other tubes, 1 ¼ inches long, and of such a diameter that they will admit the thermometers used, are placed in line with the center of the cover and the tube carrying the glass rod. Their centers are ¾ inch from the edge of the cover. The thermometer is ordinarily borne in the one that is more distant from the heating coil, and the other one, which is generally stopped with a cork, is used for filling the cup, though it also receives a thermometer when the two thermometers are to be compared with each other.

The heating coils and the copper wires are coated first with asphalt varnish and then with ozokerite. Neither of these insulators could be used alone, and rubber and gutta-percha were tried also without success. The inner side of the cover and those portions of the brass tubes which project below it are coated with a thin sheathing of rubber dissolved in naphtha, to prevent possible conduction across the cover when it is covered with moisture by condensation.

Each of the cups thus constructed is set upon a network of fine cord within a cylindrical vessel of brass, 8 inches high and 6 inches in diameter, and is centered by three adjustable wooden pegs. These brass vessels or shields are placed about 4 inches from each other within a wooden box, with a cover made in two parts and cut so that the thermometers can project through it; the parts of the lid can be removed and replaced without disturbing the instrument. The conductors for the electric current enter through holes in the ends of the box. I am not sure that it would not be better to dispense with the box, when the temperature of the room can be kept constant. Thomsen[1] found it to be so; but the temperature of my work-room is so changeable that it is probably safer to protect the cups from outside changes of temperature, and to expose them only to those changes which they cause themselves. These at least are regular and similar in all observations.

[1] Pogg. Ann., Vol. 92, p. 34; Thermochem. Untersuchungen, Vol. I., p. 22.

On either side of the box rises a wooden upright sustaining a crosspiece, on which are fastened small wooden blocks, grooved to receive the upper ends of the thermometers. When the cups are in position, the thermometers are held in place in these grooves by stout brass springs. In appropriate positions are fastened to the crosspiece the magnets and circuit-breakers of two small electric bells, the hammers of which have been replaced by longer handles carrying on their ends small corks. The magnets are actuated by a pair of Edison-Lalande cell, and when in action the cork hammers keep up a lively tapping on the tops of the thermometers. The contact-maker used with this circuit is fixed in a convenient place on the side of the box.

The paddles are turned by a belt passing over both of their pulleys and over a 4-inch pulley carried on an offset in front of the box. The belt enters the box through two notches in the front, into which it falls when the front part of the cover is not in place. To admit of this the front part of the cover is made removable and is not hinged like the back part. The belt is a rubber cord about ⅛ inch thick.

4. *The Cooling Vessels.*—Before beginning an observation it is necessary to cool off the cups to some temperature below that of the room. This is done in cylindrical vessels 8½ inches in diameter and 6 inches high. Within these are placed cylinders of coarse wire netting, held in place in the vessels by three projecting arms. They are ust large enough to admit the cups, which rest on cross-pieces of wood about an inch from the bottom. The space outside these cylinders is packed with broken ice or with snow, and the vessels are filled about half full of water. When the charged cups are placed in position in these coolers, the paddles may be turned with the fingers sufficiently to bring about a regular fa ll o the thermometers, and after a little experience the cups can be removed so that the temperatures indicated after the cups have been wiped dry do not differ by more than two or three hundredths of a degree. If they differ by more than that, it is easy to equalize them by warming one of the cups with the hand.

5. *The Current.*—The current for the heating coils is furnished by a dynamo machine or a storage battery. It is led into the box

by stout copper wires, which are sufficiently flexible to admit of being pushed aside when the cups are to be removed. The connection between the cups is made by a straight piece of the same wire. The circuit contains an ammeter, a variable resistance of coarse wire, and a snap switch, placed conveniently on the front of the box. Ordinarily a current of 5 ampères is used.

6. *The Thermometers.*—The thermometers were made by Fuess. They are of Jena glass, No. 59, III., and are similar in size and shape. The graduation extends from 12° to 33° in the one and 36° in the other. A degree covers about 2 cm. and is divided directly into fiftieths. The graduation is marked in beautifully fine and even lines on a scale of opaque glass placed ust behind the thermometer tube. By the help of the reading microscopes, which are arranged to slide on the external protecting tubes, an estimate of the temperature can be made to thousandths of a degree with considerable accuracy. The bulbs are long and narrow. A short piece of rubber tubing is slipped over each thermometer and supports it at the proper height in the cup. The bottom of the bulb is brought as close as is safe to the paddle, and the upper end of it is below the surface of the liquid in the cup by 3/8 inch when 700 grams of water are used.

It is a feature of this method that the corrections of the thermometers to a standard need not be accurately known. All that is necessary is to have a comparison of one with the other. To obtain this, both thermometers were placed in the same cup, and compared at different temperatures. The curves of comparison obtained from three independent sets of observations nowhere differed from each other by more than $\frac{3}{1000}$ degree. From them a mean curve was constructed from which to correct the readings of one thermometer to the scale of the other. Since the comparison is made under the conditions of service, the corrections for stem exposure and lag are included in the results.

7. *Method of Observation.*—The course of an observation is as follows: After the cups are filled, usually with 700 grams of water and with the appropriate amount of the solution, they are placed in the coolers until the temperature falls about 5° below the room temperature (say to 15°). They are then wiped dry, the tempera-

tures being adjusted if necessary to be nearly equal, and are placed in position. At the beginning of a minute the tappers are set going, and the stirring is begun. Stirring is kept up in one sense for twenty seconds and then in the reverse sense for twenty seconds. The tappers are allowed to run five seconds longer, are then stopped, and the readings are made. This process is repeated usually twice and oftener if the differences of reading of the two thermometers show signs of a progressive change. At the beginning of a minute, after the last reading has been made, the current is thrown into the heating coils and is maintained usually for six minutes; with the current used the rise of temperature is then about 10°. During this period the stirring is continued, being changed in sense at the end of each minute. At the end of six minutes the current is cut of, the tappers set going, and readings are made as before. Nothing remains but to determine from the average temperature differences the most probable value of the rise of temperature in each cup, and to calculate the specific heat s' of the solution directly from the formula, $s' = \frac{ms\vartheta}{m'\vartheta'}$. The cups may be cooled off again and another observation made. Such an observation can be made in twenty minutes or a little over.

As an illustration we may take an observation on a $\frac{1}{200}$ milk-sugar solution. The mass of water in cup A was 700 grams, of solution in cup B was 736.57 grams. The thermometer corrections fall on the temperature in B; those due to errors in reading on that in A.

Readings.	A		Diff.	B	
		Corr. Reading.			Corr. Reading.
Before heating.	15.110		14	15.124	
	15.130	15.130	14	15.144	15.115
After heating.	25.324	25.324	−5	25.319	25.309
	25.296		−4	25.292	
	25.272		−5	25.267	
Rise of temp.		ϑ, 10.294			ϑ', 10.294

$$\frac{\vartheta}{\vartheta'} = 1.0000, \qquad s' = 0.9504$$

Heat capacity of 200 mols. water + 1 mol. milk-sugar or of 3942 grs. solution, 3746.4.
Apparent molecular heat of milk-sugar in solution, 146.4.

8. *Correction for Evaporation.*—The most important change which has been made in the construction of this instrument from that previously described[1] consists in the use of covered calorimeter cups instead of open ones. In the 25 minutes required in the earlier work for an observation, 500 mg. of water were lost from each cup by evaporation. This not only made it necessary to weigh and refill the cups after each observation, but introduced an element of uncertainty in the work that is at least partially avoided by the use of covered cups, in that any inequality in the amount of evaporation from the cups involved a considerable inequality in the amount of heat lost by evaporation. This possible difference in the amount of heat lost was included in the probable error, but it no doubt contributed to increase the magnitude of that error. Even with the covers, the evaporation is not wholly checked, as the temperature of the liquid changes, and is different from that of the cover, but it is at least rendered more uniform. In working with alcohol as a solvent or in determining the calorimeter constant with water, the first observations were always more divergent than the later ones from the mean, unless the precaution was taken to carry the temperature through its ordinary range before beginning the observations.

9. *Correction for Radiation.*—When the cups are so charged that the rise of temperature is about the same in each, the correction for radiation is negligible. This can be seen from the uniformity with which the temperatures in the cups rise or fall while the readings are being made. When the temperature changes in the cups are quite different, as in the observations for the calorimeter constant, the radiation has to be determined and allowed for. This may be done completely by determining the rate of loss of heat at different temperatures; but it is generally sufficient to judge from the differences in the temperature changes observed during the preliminary and final readings as to how much more heat one of the cups loses than the other, and to express this in terms of temperature change. This method is not suitable when the temperature changes differ greatly.

10. *Corrections for Errors in Stirring.*—In the ordinary case, the

[1] This REVIEW, Vol. IX., No. 2, August, 1899.

errors which are most difficult to control are connected with the stirring of the liquids. It is necessary to make the pulleys that turn the paddles exactly of the same diameter, and to make the paddles themselves geometrically similar; and even when they are as similar as they can be made, it is advisable to turn them in opposite senses for equal times. Even with the utmost regularity, it is not possible to be certain that the liquids are thoroughly and equally mixed, and I believe that the remaining error is more largely due to unequal mixing than to any other cause. The condition, which is important, that the paddles should extend as near the sides of the cup as possible, introduces an occasional difficulty, when the paddles, owing to imperfect fitting of the cover or to too great a strain on the belt, touch the sides of the cup. Fortunately when this happens it can usually be detected by the ear. An observation in which such a condition has existed is worthless, the heat developed by the contact being always sufficient to cause too great a rise of temperature in the cup in which it occurs.

11. *Correction for Calorimeter Constant.*—A difference in the water equivalents of the two cups may exist which must be examined and allowed for. This can scarcely be due to differences in the cups themselves, if they are carefully constructed, but may be introduced by the use of thermometers which are not sufficiently alike. The amount of this difference may be determined by a series of observations with equal quantities of water in the cups. If we represent by φ the heat capacity of the water used, by ϑ_1 the rise of temperature in cup No. 1, by C_1 the water equivalent of that cup, and use similar symbols with the subscript 2 for the quantities pertaining to cup No. 2, and remember that the same quantity of heat is contributed to each cup, we have,

$$\varphi\vartheta_1 + C_1\vartheta_1 = \varphi\vartheta_2 + C_2\vartheta_2;$$

whence

$$\varphi - \varphi\frac{\vartheta_2}{\vartheta_1} = \frac{C_2\vartheta_2}{\vartheta_1} - C_1.$$

Since ϑ_1 is nearly equal to ϑ_2, and C_1 and C_2 are small in comparison with φ, this equation gives approximately

$$\varphi\left(1 - \frac{\vartheta_2}{\vartheta_1}\right) = C_2 - C_1,$$

from which by the observations just mentioned $C_2 - C_1$ may be determined. This difference is all that is needed for correction, when the temperature change is nearly the same in each cup.

The value of the water equivalent, when it is the same for each cup, is got by using different quantities of water, so that φ_1 and φ_2 are not the same. We then have

$$C = \frac{\varphi_1\vartheta_1 - \varphi_2\vartheta_2}{\vartheta_2 - \vartheta_1}.$$

Even if the water equivalents differ, this equation will determine a value that is correct enough for use when the temperature changes are different, and the difference between the water equivalents may be got as before.

12. *Effect of Condensation.*—In the damp summer weather, the moisture which condensed on the outside of the cups, when they were cooled below the room temperature, affected the changes of temperature during the period of the initial readings, so that they were very irregular and uncertain. Successful observations could not be made at such a time.

13. *Test with Water.*—It is an advantage of this form of calorimeter that its performance can be tested by the use of equal quantities of water in the two cups. If the water equivalents and the radiation losses are the same for each, the mean ratio of the observed temperature changes should equal 1. The first set of results which follows was obtained for this case on October 10, 1901, before the laboratory was heated, and while there was a slight condensation on the cups of moisture from the atmosphere. The second set was obtained on November 9, 1901, when the laboratory was heated. The numbers given are the ratios of the temperature changes in cups A and B.

0.99987	1.00005
0.99974	1.00004
1.00004	0.99995
1.00030	1.00002
0.99963	1.00030
0.99991	
Mean, 0.99992	1.00007

The mean of both sets is 0.99999 and indicates that the correction for difference of water equivalent is negligible. In these observations the rise of temperature was over 10°. Another set of observations, in which the rise of temperature was about 3.5°, gave the following results:

	1.0003
	0.9996
	1.00025
Mean,	1.00005

Although the variations are no larger than in the other sets of measurements, and although there are good reasons for restricting the temperature change within as narrow limits as possible, yet in view of the larger effect of errors in reading the thermometers, I do not think it best to use so small a temperature change.

14. *The Calorimeter Constant.*—The water equivalent or constant of the calorimeter was determined as the mean of the following results: 700 grams of water were used in one cup, and 800 grams or 850 grams in the other.

22.6	1.0002
24.1	0.9998
24.3	1.0002
21.7	0.9997
23.4	1.0000
Mean, 23.2	

The numbers in the second column represent the specific heat of water, calculated from the observations with the mean value of the constant, and indicate the range of error when the constant is known and the difference of the temperature changes is about 1° in 10°.

15. *Examples.*—The following numbers under I. are the ratios and the molecular heats obtained from observations on a $\frac{1}{200}$ gram-molecular solution of an anhydrous form of milk-sugar. The amount of water used was 700 grams, of the milk-sugar solution 735 grams. Under II. are given the corresponding numbers for a similar solution of another form of milk-sugar. The amount of solution used in that case was 736.57 grams.

I.		II.	
0.9978	146	0.9998	145
0.9979	146	0.9999	146
0.9986	149	1.0000	146
0.9982	147	0.9997	145
0.9978	146	0.9995	144
0.9981	147	0.9999	146
0.9983	148	1.0003	147
		1.0003	147
		1.0000	146
Means, 0.9981	147	Means, 0.9999	146

The water equivalent of the calorimeter applied to such cases as those under I., where the difference between the temperature changes was about 0.02°, does not appreciably modify the results.

PHYSICAL LABORATORY, PRINCETON UNIVERSITY.

ON SOME OPTICAL PROPERTIES OF ASPHALT.

BY EDWARD L. NICHOLS.

WHEN a thin layer of asphalt varnish is spread upon glass and allowed to dry and some luminous source, such as the filament of an incandescent lamp, is observed through the film, it is found that a considerable amount of red light is transmitted, the unusual purity of which is readily ascertained by means of a spectroscope. The suddenness with which the rays beyond the red are cut off indicates the existence of a well-defined absorption band with a very steep gradient on the side toward the greater wavelengths; and one would expect to find a considerable degree of perviousness in the infra-red and anomalous dispersion in the region where the change from transparency to opacity occurs.

It is the purpose of this paper to describe some studies of this interesting substance concerning the optical properties of which nothing appears to have been recorded.

SPECTRO-PHOTOMETRIC OBSERVATIONS.

For the purpose of determining the transmitting power of asphalt in the visible spectrum, films of the requisite thickness were obtained by dipping a piece of thin plate glass into asphaltum varnish and allowing the coating to dry; after which one side of the glass was carefully cleaned. The film thus obtained covered about one half of one face of the glass plate. The instrument employed in the determination of the transmitting power of the film was a one-prism spectroscope provided with a Vierordt slit. The spectroscope was securely clamped above the table with the circle C in a vertical plane, the collimator tube horizontal, the slit horizontal and the observing telescope moving in a vertical arc. Light for the comparison spectrum was introduced through the right hand half of the slit by means of a total reflection prism P (Fig. 1). The source of light was an acetylene flame (A_1) in front of which, in order to reduce the intensity, two thicknesses of ground glass (G) were interposed.

Light from a similar acetylene flame (A_2) which was mounted in the line of the collimator, entered the left half of the slit directly. The glass plate (f), with the asphalt film, was interposed in the path of this ray near the slit. By giving the plate a lateral movement of about two centimeters, the ray could be caused to pass alternately through the glass and asphalt and through the glass alone. A comparison of the absorption spectra thus obtained with the spectrum from (A_1) furnished data for computing the transmitting power of the asphalt film.

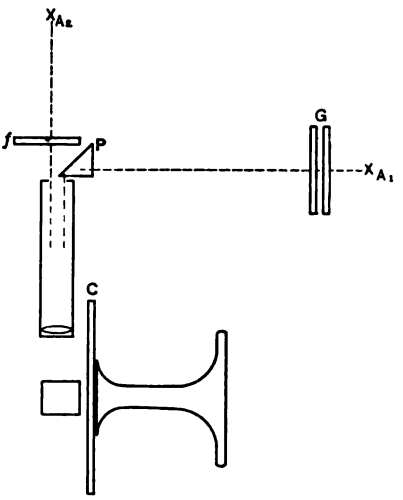


Fig. 1.

Measurements were made in five regions of the spectrum lying between .73 μ and .589 μ. The transmission of light of wave-lengths shorter than the latter was too faint to be measured. The thickness of this film, determined by measuring with a micrometer gauge the thickness of the glass and of the glass and asphalt together was found to be 0.003 cm. Table I. gives the percentage of light transmitted by the asphalt and the extinction coefficient of this substance for the regions of the spectrum in which measurements were possible.

The extinction coefficient was computed in the form used by Knut Ångström [1] with whose results with films of lampblack it was desired to compare the present measurements.

The equation has the well-known form

$$I_e = I_0 e^{-kl},$$

where I_0 is the intensity of the incident ray, I_e that of the ray transmitted by a film of thickness l and k is the extinction coefficient.

The transmission curve for the visible spectrum is shown in Fig. 2. It will be seen from these data that a film of this thickness becomes almost completely opaque in the yellow, the transmission in the

[1] Ångström, Wiedemann's Annalen, 36, p. 715 (1889).

TABLE I.

... ... visible spectrum of a layer of asphalt .003 cm. in thickness.

Wave-lengths.	$\frac{I_e}{I_o}$	Extinction Coefficients[2] (k).
...	0.0233	1256
...	0.0088	1558
...	0.0044	1813
...	0.00085	2362
...	0.00038	2631

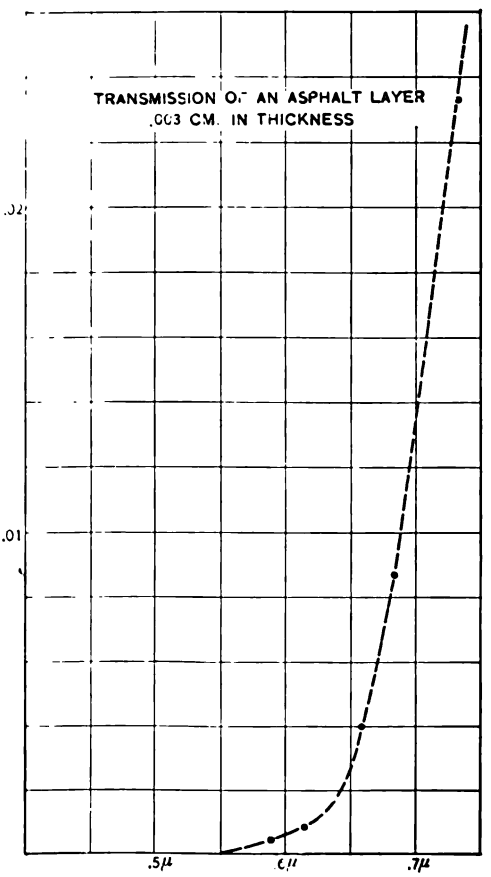


Fig. 2.

[2] To convert these coefficients into the extinction indices of Wernicke (Poggendorff's Annalen, Erg. Bd., 8, p. 67 [1877]), they must be divided by $\frac{4\pi}{\lambda}$, where λ is expressed in centimeters.

region of the D line being less than .0004 of the incident light; also that the transmission of yellow light is less than two per cent. of the transmission of the full red of wave-length .73 μ. The opacity of this film was too great to permit of the location of the position of the maximum of the absorption band and measurements were accordingly made upon much thinner layers, prepared in the following manner. A cell having the form of a hollow wedge with sides of plane glass was filled with asphalt varnish and then emptied of what could be poured out and immediately inverted and left to dry with its mouth resting upon a sheet of filter paper and the apex of the wedge uppermost.

The downward flow of the slowly drying film of asphalt gave in this case a regular gradation to the layer which at the top of the cell was so thin as to show scarcely any color, while at the bottom it was nearly opaque. The light transmitted by this cell with its double layer of asphalt at distances of 2.5 cm. and 6.5 cm. from the apex was then compared spectrophotometrically with that transmitted by an uncoated cell made of the same glass.

The color of this layer was a red, changing gradually to a pale amber, tending at the apex to a greenish yellow; indeed the variation of color with thickness might be likened to that of a dichoric liquid.

A comparison of the transmission of the two portions of these films subjected to measurement showed the substance to be optically perfect, or at least not appreciably turbid. That is to say the relation

$$\frac{n}{m}\log\frac{I_0}{I_m} = \log\frac{I_0}{I_n};$$

where n and m were the relative thicknesses in the regions measured was found to be approximately true. The ratio $\frac{n}{m}$ was determined from the distances from the apex of the wedge of the two regions, under the assumption that the films were themselves wedge-shaped, tapering uniformly towards the apex. The agreement which was at least as close as my knowledge of the relative thicknesses, was however not sufficient to establish this fact completely and measurements were accordingly made upon two layers of different thick-

nesses placed upon glass plates in a manner similar to that employed in the specimen used in obtaining the data given in Table I. The relation of transmission to thickness in this case also was found to fulfill the law for optically perfect media with a degree of approximation equal to that of my knowledge of the thickness of the films.

It was found that while each pair of films was comparable in this respect between themselves, the sets could not be compared with one another and that neither gave correct values when compared

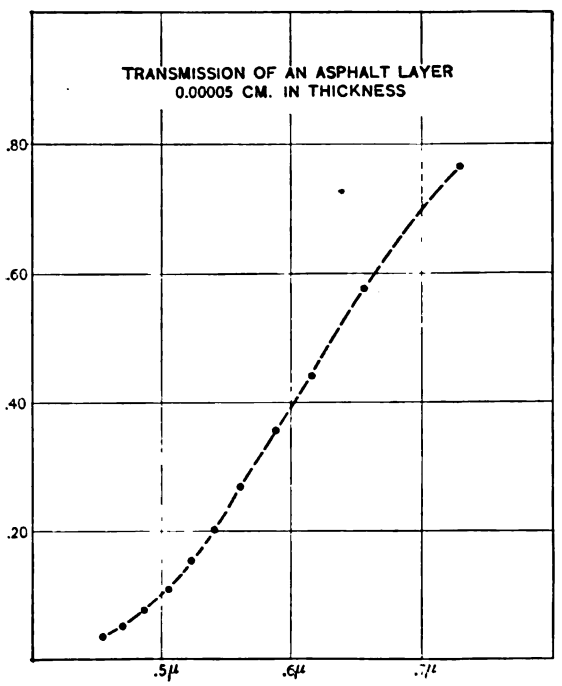


Fig. 3.

with the film of Table I. This discrepancy was due to the fact that the thickness of such films continues to diminish for a long time after they appear to be dry.

The two last mentioned films, for example, measured with the micrometer gauge about twenty-four hours after deposition, were found to be .0122 cm. and 0.0069 cm. in thickness. The next day they had fallen to 0.0026 cm. and 0.0009 cm. respectively. The

effective thickness of the former, computed from its transmitting power, in terms of that of the film of Table I., which had been measured after drying for several weeks was however only .00195 cm.

The extinction coefficients in Table I. refer to films which have been thoroughly dried and not to freshly deposited layers of asphalt varnish.

The thinner region of the double, amber-colored film near the apex of the wedge-shaped cell would have had a thickness when dry of only 0.000055 cm.

The transmission curve of this region (thickness 0.000055 cm.) is given in Fig. 3, from which it will be seen that the transparency was still diminishing at wave-length 0.45 μ. The maximum of the absorption band must therefore lie at some still shorter wave-length; perhaps in the ultra-violet.

The Anomalous Dispersion of Asphalt.

In order to determine the dispersion of asphalt for those wave-lengths for which it is transparent a prism was made by placing some fragments of solid asphaltum between two plane-parallel pieces of glass and heating carefully in a gas oven until the asphaltum began to flow, when the plates were firmly pressed together at one edge. After many trials a prism of small angle was thus obtained thin enough to transmit red, yellow and a trace of green light near its edge.

This was set up, as nearly at minimum deviation as possible in the path of the dispersed rays of a spectrometer with two prisms and the displacement of four bright lines was carefully measured.

The source of light was an electric arc plentifully supplied with lithium and sodium.

The angle of the prism was 1° 50′ 2″.

The results of these measurements are given in Table II.

Table II.

Dispersion of a prism of asphalt.

Wave-lengths.	Index of Refraction.
Li ($\lambda = 0.6708\mu$)	$n = 1.6209$
Li ($\lambda = 0.6104\mu$)	$n = 1.6282$
Na ($\lambda = 0.5896\mu$)	$n = 1.6351$
Na ($\lambda = 0.5682\mu$)	$n = 1.6339$

It will be seen that anomalous dispersion begins to manifest itself between the yellow and green. It was not possible to make measurements at shorter wave-lengths by this method on account of the opacity of the material.

MEASUREMENTS IN THE INFRA-RED.

The transmitting power of asphalt in the infra-red was studied by interposing alternately the coated portion and the uncoated portion of a strip of glass, prepared like that used in the measurements given in Table I., in the path of the rays from an acetylene flame and comparing the deflections of a Nichols radiometer exposed to the absorption spectra thus obtained with the deflections produced by the spectrum of the uninterrupted rays from the flame. The

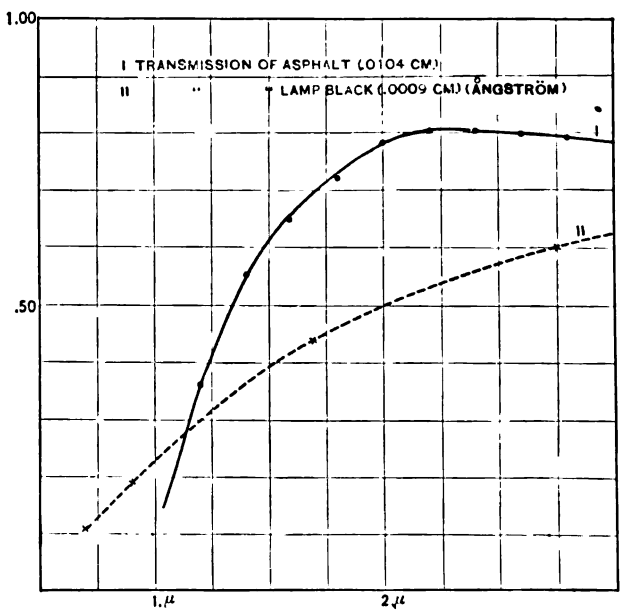


Fig. 4.

apparatus used was one employed by Mr. W. W. Coblentz in measurements of the absorption of solutions of iodine the results of which have not yet been published. It consisted of a mirror spectrometer with bisulphide prism, the radiometer being mounted in place of the eye piece so that its vanes would receive radiation of any desired

wave-length. From the observations upon the absorption in the coated and in the uncoated glass the absorption of the asphalt alone was computed. The measurements extended over the range of wave-lengths from 1.0 μ to 2.8 μ. The transmission increased rapidly from zero at 1.0 μ to about .80 at 2 μ, beyond which only slight changes in the transparency of the film were noted. There is apparently a slight maximum of transmitting power in the region 2.2 μ. Measurements could not be carried with the apparatus used beyond 2.8 μ on account of the rapidly increasing opacity of glass and carbon bisulphide, and on account of the small dispersion of the bisulphide prism for the greater wave-lengths. In order to extend the investigation to the still longer wave-lengths of the infra-red spectrum it would be necessary to repeat the measurements by means of layers of asphalt placed upon a plate of fluorite or rock salt and to make use of a prism of similar material in the spectrometer.

The results of these measurements are given in Table III. from the data contained in which the transmission curve shown in figure 4 is plotted. The thickness of the film employed was 0.010 cm. Its opacity was so complete that the disk of the sun could not be distinguished through it.

TABLE III.

Transmission of an asphalt film (0.010 *cm. in thickness*) *in the infra-red.*

Wave-length.	Transmission.	Extinction Coefficient.
1.0	.000	——
1.2	.333	104.
1.4	.550	59.8
1.6	.644	44.0
1.8	.718	33.2
2.0	.786	24.1
2.2	.809	21.2
2.4	.805	21.7
2.6	.791	23.4
2.8	.788	23.8

CONCLUSIONS.

There is much about the optical behavior of asphalt to suggest that its color may be due to the presence of carbon particles dissolved or perhaps suspended in some other medium, but the differences

between the two substances are scarcely less striking than their resemblances. We find the same transition from opacity, to transparency and this change occurs in the same region. For purposes of comparison, a portion of one of the curves given by Knut Ångström in the paper already cited has been inserted in Fig. 4. It is the transmission curve for a layer of lampblack the thickness of which was estimated at .0009 cm. in thickness. There is no significance in the comparison of a layer of asphalt with that of lampblack as regards thickness. It has been shown both by Rosicky[1] and by Stark[2] that lampblack obtained by smoking glass in the flame of a candle is of very loose structure, being equivalent to a layer of solid carbon of not more than three or four hundredths of the thickness. Ångström's values are therefore not strictly comparable with that computed for asphalt. It will be seen however that the change from opacity to transparency in the asphalt layer is much more abrupt than is the case with lampblack and that the former is more nearly transparent in those portions of the infra-red included in my measurements. A layer of lampblack .01 cm. in thickness would for example transmit only about .09 of the incident radiation of the wave-length 2 μ. Asphalt of the thickness of Ångström's layer of lampblack would on the other hand transmit .98. It has been shown by Wood[3] in a recent paper that lampblack exhibits anomalous dispersion in the visible spectrum and in this respect the two substances therefore have similar properties. The maximum of the absorption band for carbon lies in the case of some varieties of lampblack and also of layers of carbon deposited in vacuo as, for example, upon the inner surface of the bulb of incandescent lamps, in the brightest part of the visible spectrum.[4] The measurements by Rosicky and by Stark seem to indicate however that in some specimens the center of the absorption band must lie further towards the violet.

That the coloring matter in asphalt varnish consists of minute particles, possibly of carbon, which are held in suspension by the liquid may be shown by means of the following experiment:

[1] Rosicky, Wiener Berichte, 78, II., p. 407 (1878).
[2] Stark, Wiedemann's Annalen, 62, p. 351.
[3] Wood, Philosophical Magazine, 6, Vol. 1, p 405 (1901).
[4] Nichols and Blaker, PHYSICAL REVIEW, Vol. XIII., p. 378.

If a wire ring be dipped into the varnish and withdrawn it carries with it a flat film of liquid, which is very similar to a soap film in appearance. The coloring matter in this film is not uniformly distributed, as in a solution, but tends to gather into streaks and patches as if by capillary action, and it is very speedily drawn to the boundaries of the film leaving a layer which shows only the colors of interference. Upon drying it leaves behind a nearly colorless sheet of resinous material.

THE VOWEL E.

BY LOUIS BEVIER, JR.

THE vowels *a*, as in *father*, and *a*^{e}, as in *pat*, are quite uniform sounds in the mouths of most Americans. It is true that *a* is often sharped in New England, and that *a*^{e} is frequently nasalized more or less, but still it is easy, by pronouncing either in the presence of a speaker from whom one desires to get a record, to secure from him a pretty accurate reproduction.

Of the subject of the present study this is not the case. In the first place *e* itself varies considerably in different combinations of actual speech, from an open *e* (*ę*), not differing greatly from *a*^{e} to a close *e*, approaching the French *é* (*ę́*). In the second place speakers are accustomed to pronounce it short in most words, as in *net*, *pet*, etc., and it is difficult to get them to prolong it in singing at a registered pitch without a noticeable alteration of its timbre, the very matter of our study. Great care, however, has been taken in this respect, that all records analyzed should sound true, when produced, and when reproduced, and the quality is, I believe, sufficiently uniform to give trustworthy results.

The vowel here to be analyzed is the open *e* (*ę*) of *pet*. It has not seemed necessary to attempt an examination of a still more open *e*, as for example the German *ä*, because this nuance is not normal in our speech, and because the analysis of *a*^{e} and *ę* are so similar that the intermediate may be constructed theoretically with sufficient exactness. On the other hand the vowel in *pate* is not here considered because it is not a simple sound, and must be reserved until the question of diphthongs is reached.

The following table gives a sufficient number of analyses to furnish a safe basis of generalization. For full explanation of the arrangement of facts in the table the reader is referred to the discussion of the vowel *a*^{e}, printed in the March number of this journal.

An examination of the table reveals that the upper partial, which

Analyses of the *E*-Curve.

		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII
		152	304	456	608	760	912	1064	1216	1368	1520	1672	1824	1976	2128	2280	2432	2584
1	91.0	14.1	4.4	6.6	25.5	4.0	3.5	5.5	6.6	2.4	4.2	5.9	9.2	0.4	2.0	2.9	2.6	0.2
		160	320	480	640	800	960	1120	1280	1440	1600	1760	1920	2080	2240	2400	2560	2720
1	101.8	31.6	7.9	5.7	9.8	2.6	3.9	5.8	3.9	5.7	6.7	8.2	1.6	1.8	2.3	1.0	1.2	0.8
6	165.8	34.3	3.7	2.3	31.1	1.0	0.5	2.6	0.8	4.2	6.9	6.0	2.4	1.0	1.2	1.0	0.3	0.7
		181	362	543	724	905	1086	1267	1448	1629	1810	1991	2172	2353	2534	2715	2896	3177
1	82.6	25.7	3.9	11.1	1.7	3.1	3.6	5.8	6.3	8.7	14.1	4.1	3.1	3.4	2.2	1.2	1.0	1.0
		192	384	576	768	960	1152	1344	1536	1728	1920	2112	2304	2496	2688	2880	3072	3264
6	57.4	4.4	5.4	7.1	4.9	4.9	2.3	1.2	9.4	26.2	12.6	4.0	3.6	3.1	2.6	3.1	3.3	1.9
7	47.0	12.1	7.0	7.5	4.3	3.6	7.9	2.1	7.0	11.5	23.2	4.9	3.6	1.5	1.3	1.1	1.0	0.4
		202	404	606	808	1010	1212	1414	1616	1818	2020	2222	2424	2626	2828	3030	3232	3434
2	64.2	24.6	2.8	12.1	4.1	4.7	5.9	6.2	12.5	10.0	6.9	3.4	0.9	2.8	0.9	0.9	0.7	0.6
		226	452	678	904	1130	1356	1582	1808	2036	2262	2488						
1	236.0	15.7	6.9	21.9	3.5	6.3	7.6	17.3	12.5	1.5	2.6	4.2						
		256	512	768	1024	1280	1536	1792	2048	2304	2560	2816	3072	3328	3584	3840	4096	4352
1	251.6	8.8	6.6	11.4	10.6	7.8	13.6	17.6	6.2	4.5	7.3	5.6						
6	136.9	8.8	5.4	5.0	14.6	4.2	28.8	18.0	5.9	2.3	1.2	1.4	1.5	1.0	0.6	0.5	0.1	0.7
7	97.0	11.2	6.4	3.5	6.3	6.5	17.8	32.8	2.6	3.0	4.3	1.7	1.3	0.5	1.2	0.7	0.6	0.6
		272	544	816	1088	1360	1632	1904	2176	2448	2720	2992						
1	168.6	8.4	15.8	7.2	14.8	2.8	12.1	23.7	6.5	4.6	2.8	1.3						
1	94.4	12.3	10.4	14.2	10.6	7.0	28.4	10.8	3.8	1.5	1.0	0.0						

		288	576	754	1152	1440	1728	2016	2304	2592	2880	3168	3456	3744	4032	4320	4608	4896
4	153.4	12.3	9.4	25.1	7.5	8.0	10.4	13.6	5.4	1.4	1.6	0.2	1.1	0.2	0.7	1.2	0.9	1.0
		320	640	960	1280	1600	1920	2240	2560	2880	3200	3520	3840	4160	4480	4800	5120	5440
4	175.6	4.2	34.3	21.0	8.2	6.3	11.4	8.0	1.0	1.3	1.7	0.6	0.4	0.5	0.2	0.2	0.3	0.4
5	104.5	2.5	13.8	28.8	7.3	12.1	14.9	10.0	1.9	1.1	1.4	0.9	1.8	0.5	0.2	0.1	2.1	0.6
7	231.5	4.5	37.8	6.0	6.4	14.4	25.9	1.4	3.6									
8	68.1	20.0	28.2	10.6	6.8	4.1	18.2	5.4	0.6	0.9	0.7	0.9	1.3	0.9	0.2	0.4	0.3	0.5
9	39.4	15.2	30.0	9.9	3.3	6.9	19.8	2.5	3.5	1.3	2.0	1.3	0.5	0.5	0.8	0.5	0.5	1.5
		341	682	1023	1364	1705	2046	2387	2738	3079	3410	3751	4092	4433	4774	5115	5476	5817
4	194.7	7.0	15.9	28.1	7.0	7.4	12.4	4.6	5.9	3.8	2.9	2.5	1.9	2.0	1.6	0.2	0.7	1.1
		384	768	1152	1536	1920	2304	2688	3072	3456	3840	4224	4608	4992	5376	5760	6144	6528
4	79.3	9.6	25.0	11.8	12.4	16.8	7.4	7.3	1.4	0.9	1.4	0.7	1.3	0.9	1.0	0.6	0.6	0.9
5	148.8	2.2	9.6	30.8	15.9	18.2	10.5	9.1	4.2									
8	61.9	11.8	19.7	3.9	21.8	26.7	5.3	0.5	1.8	1.9	1.1	0.6	1.3	0.8	0.7	0.6	1.3	0.2
9	113.3	10.8	19.6	14.7	14.3	30.0	1.0	1.9	2.6	0.5	0.7	0.6	0.4	0.4	1.0	0.1	0.7	0.8
		480	960	1440	1920	2400	2880	3360	3840									
4	278.8	9.0	40.6	21.7	18.8	6.4	2.5	5.6	0.4									
		512	1024	1536	2048	2560	3072	3584	4096	4608	5120	5632	6144	6656	7168	7680	8192	8704
5	229.2	5.5	27.0	30.2	24.3	8.4	0.4	3.0	1.2									
7	340.0	4.5	18.9	37.5	29.0	3.8	6.2	3.3	1.8									
9	47.4	19.3	25.7	26.6	10.1	3.4	1.7	1.3	1.3	1.5	1.1	1.3	1.7	1.0	0.8	0.8	1.2	1.2
		640	1280	1920	2560	3200	3840	4180	5120									
10	107.4	68.8	8.9	21.5	2.7	1.5	0.6	0.3	0.7									

is most characteristic for *e*, and always present, lies higher in the musical scale than for a^e by an interval somewhat greater than a whole tone. The frequency of this characteristic and strongly reinforced upper partial is in the neighborhood of 1,800, whereas that of a^e lay as we saw (l. c.) at about 1,550 $\left(\frac{1,550}{1,800} < \frac{8}{9}\right)$. The actual frequency of this important upper partial varies within not very narrow limits, according to individual differences of utterance, and acccording to the pitch of the fundamental of which it must be an exact multiple. Thus in the table we find on I. 152, 1,824 (XII.) with an amplitude percentage of 9.2; on I. 160, 1,760 (XI.) with 8.2 and 6.0 respectively in the two examples; on I. 181, 1,810 (X.) with 14.1, and so on to the highest tones analyzed.

This region of resonance extends over a considerable range, so that on low fundamentals two consecutive upper partials may be strongly reinforced. For example, I. 192 of voice 6 shows both IX. and X. (1,728 and 1,920) strong, and so in many other examples. The limits may be approximately set at 1,600 and 1,950 with the maximum generally at about 1,800.

Besides this resonance there is always present at least one other, and often two others of lower pitch. These coincide very closely with those described in the study of a^e save that the lower centers at about 620 (cf. 650 for a^e). The upper has a frequency of about 1,050 as for a^e. A good example in the table which shows both of these resonances, in addition to the principal one above described, may be seen in I. 272 of voice 1. Here II. (544) is strong (15.8), IV. (1,088) is also strong (14.8), and VII. (1,904) predominatingly strong with an amplitude percentage of 23.7 The real strength of the last may be best appreciated if the same record is computed in energy percentages, which would give the following results: I. 0.2; II. 2.3; III. 1.1, IV. 8.0; V. 0.5; VI. 12.1; VII. 63.3; VIII. 6.2; IX. 3.9; X. 1.8; XI. 0.5.

More frequently the analysis shows but one of the two lower resonances at all important, either that in the neighborhood of 620 or a compromise between the two. The essential point for the character of the vowel is that there shall be, in addition to the uppermost resonance, a certain fullness of resonance at one or more

lower points. This can best be made plain by a detailed examination of several concrete cases. Take for example the two specimens at 160 by voices 1 and 6. Each shows the characteristic high resonance at XI. (1,760). This is a little below the point I have designated as the normal maximum, but is nearer to it than the next upper partial (1,920) would be. Still many examples may be found where 1,920 would be the dominant upper partial, representing presumably, other things being equal, a slightly closer *e*. Following the records down the scale toward the fundamental, X., IX. and VIII. are weak, as they should be; VII. shows in each a greater amplitude percentage, but not quite as much greater as we should usually find at that pitch (1,120) which is quite near a region of resonance. Again VI. and V. are weak, which is according to the rule, and IV. is strong. In the second case it is very strong, much more so than is usual.

Compare next the five examples by different voices on the fundamental 320. Here II., which falls near the lowest resonance maximum, is strong in all, varying from the high amplitude percentage 37.8 to the moderate 13.8. On the other hand III. falls a little too low for strong reinforcement in most normal utterances. Still the first and second examples show considerable reinforcement, particularly the second where III. is very strong, At IV. all are relatively weak, as they should be. At V. the resonance begins to be important again in the case of the second and third examples, but not in the others. VI. falls near the resonance maximum and is consequently strong in all. Even VII. in the first and second examples is quite strong, although considerably above the usual *e* resonance.

From a careful study of the table, and from numerous other analyses, the definition of *e* may be formulated as follows: The vowel *e* is a composite sound containing: (1) The chord-tone, or fundamental, varying within wide limits of loudness. In general it is strong below 200, but not so strong as that of *a*[e]. From 200 to 600 it is weak, but not so weak as that of *a*[e]. Above 600 it falls within a region of strong resonance and is very strongly reinforced.

(2) A characteristic *e* resonance in the region of 1,800. This

resonance is always present, and is most important, but its pitch varies within moderate limits, as explained.

(3) A strong resonance at one or two lower points, centering at about 620 and 1,050 respectively. Both may be present, but at least one of them must be.

As was done for *a*ᵉ, these facts may perhaps be best shown by plotting a resonance curve of amplitude percentages, obtained by averaging all the analyses made. This curve shows the women's

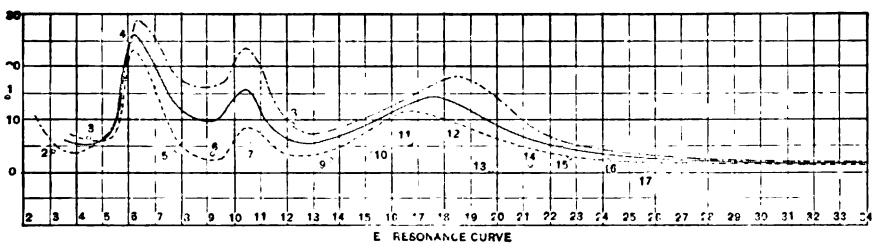


Fig. 1.

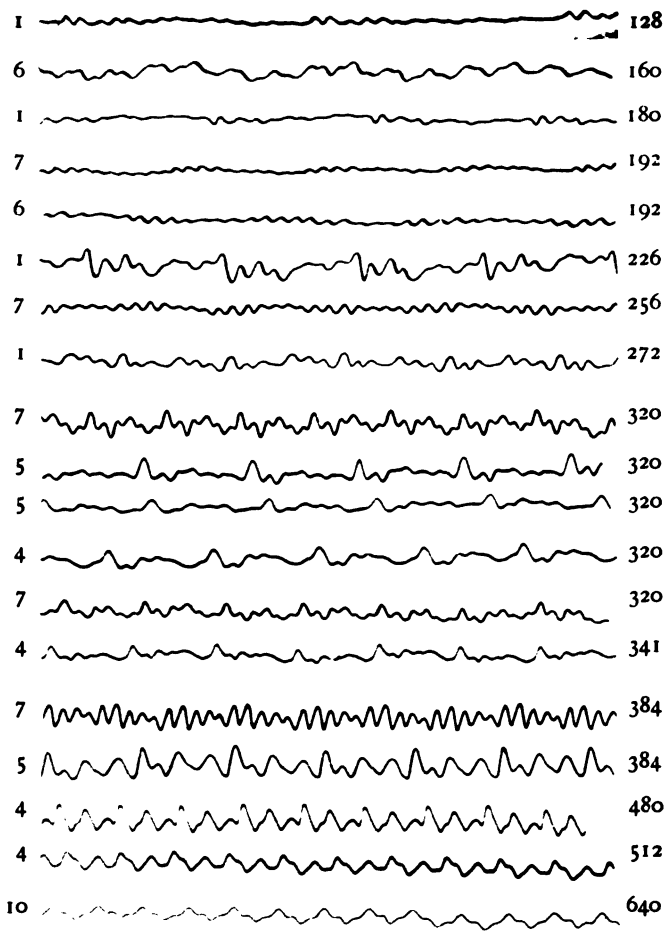
line, the men's line, and the average of both as for *a*ᵉ. We observe that the resonance begins to rise very rapidly at about 550, reaches a very pronounced maximum at about 620, descends thence and rises to its second maximum at 1,050. From this point it descends abruptly, to rise gradually once more and reach its final maximum at about 1,800. Here a noteworthy difference appears between the women's line, and that of the men's voices, the latter showing the maximum on the average at about 1,700 and the former at about 1,850. This is true as an average result, but many records of men's voices show the resonance maximum as high as 1,900, and *vice versa*, many individual examples of women's voices as low as 1,700. There does, however, seem to be a tendency on higher fundamentals, both of men's and women's voices, to pitch this resonance a little higher.

As before, (l. c.), an individual record is plotted for the sake of illustration. It follows the average very closely. The fundamental is 152. I. (152), IV. (608), VII. (1,064), VIII. (1,216), XI. (1,672), and XII. (1,824) are important, and the rest weak. The maxima are in this case at IV., VIII. and XII. respectively.

For comparison a sheet of *e*-curves is added. The most casual

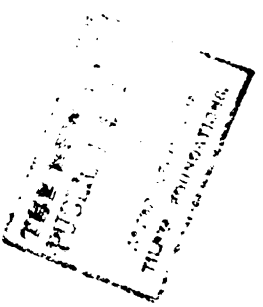
inspection will show that the characteristic resonance is of higher pitch than for a'. A simple count of the minor crests is the most obvious test. Thus at 320 these minor crests number six, giving a frequency of 1,920; at 384 five, again 1,920; at 256 seven, giving a frequency of 1,792, etc.

It may be interesting to compare two similar records lying an octave apart in fundamental pitch. The comparison will be most striking in the case of 320 by voice 7, and 640 by voice 10, the former a contralto, the latter a soprano. The record on 320 is essentially a succession of triplets, two to each fundamental wave, while that on 640 is a succession of simple triplets. The elements 640 and 1,920 are common to both, and hence their great similarity; but in the 320-record V. (1,600) is also present, and makes the one triplet different from its fellow. The fundamental at 320 is too weak to count so far as the unassisted eye can discern. These curves have for convenience of publication been reduced to one-third of their size as recorded by the transcribing instrument.



E — Curves

BEVIER.



THE VELOCITY OF IONS FROM HOT PLATINUM WIRES. PART I.

By C. D. Child.

MANY investigations have been made on the discharge from hot wires, but none on the velocity of the ions carrying this discharge. As far as the writer is aware no quantitative measurements of any kind have been taken. The fact that the discharge from a positive platinum wire is greater than from a negative one led the writer to suspect that the positive ions move more rapidly than the negative ones, and for this reason, if for no other, the subject seemed worthy of study. Since this investigation was commenced an article has been published by Rutherford[1] concerning the velocity of ions from a hot platinum foil. His article covers some of the ground which is included in this. The methods used were not, however, identical and the objects in mind were different, so that this investigation confirms his, but does not to any great extent duplicate it.

Irregularities in the Rate of Discharge.—The writer imagined before beginning the work that the discharge from a hot platinum wire through which a constant current is flowing would be constant. Examination showed that it would be much nearer the truth to say that the discharge is never constant. The investigation proved to be little more than an attempt to find the different causes for the variations.

The wire used was heated by passing through it an electric current produced by a storage battery. Resistance was placed in series with the wire and the potential was varied by varying the position of a ground connection on this resistance. A shunt was placed about the hot wire and a point midway on this shunt was assumed to have the same potential as the wire. The potential

[1] Phys. Rev., 13, 321.

difference between this point and the ground was measured by a voltmeter.

The outer cyclinder was connected to the electrometer and also to the ground. In making an observation of the discharge the connection to the ground was first broken and after a given length of time the electrometer was disconnected from the cylinder. The deflection of the electrometer would then show the amount of discharge which had passed to the cylinder.

There was one cause of difficulty which must be ascribed to lack of care in making observations. The current through the wires was produced by a storage battery. It was assumed that the electromotive force of the battery was constant. It happened, however, that it was slightly variable. It also happened that at certain temperatures very slight changes in the temperature of the wire produced large changes in the rate of discharge and the first observations were made at just these temperatures. As a result the varying changes of current through the wire made large changes in the rate of discharge. Later, of course, proper care was taken to keep the current constant.

Rate of Discharge at first Decreases.—It was found that in general the discharge from the wire was at first large, decreasing for a short time rapidly, and then increasing slowly. The following are the data taken in one case when the wire was heated for the first time. The length of the wire was 6.5 cm., the diameter was .3 mm., the diameter of the surrounding cylinder was 48 cm., its width 6 cm. The current through the wire was 3.8 ampères. This was sufficient to heat the wire to a dull red. The potential difference between the wire and the cylinder was 35 volts, the wire being charged positively. In such a case the discharge is carried by positive ions and will be called the positive discharge. This is contrary to the usage which some have adopted, but in the light of the facts to be described later it seems best to use this terminology. The discharge carried by the negative ions will be called the negative discharge. Column 1 gives the time of the observation after the current was commenced. Column 2 gives the deflection which took place in four seconds. There was a deflection of 1.9 scale divisions per volt. The capacity of the electrometer and of a small condenser placed in multiple with it was 4.43×10^{-3} micro-farads.

TABLE I.

Time.		Deflection of Electrometer.
Minutes.	Seconds.	
	10	8.3
1	30	6.2
3		5.1
5		3
6		3

It seems probable that this large discharge at the beginning was caused by impurities of some sort in the wire which were soon burnt out, or possibly by gases that had been absorbed by the platinum. The wire here used was weighed just before the observations were made and again afterwards. If there was any loss in the weight, it must have been less than .1 mg. But the fact that there was no appreciable loss of weight proves nothing, since the extra amount of discharge could be carried by an amount of matter far too small to be detected by weighing.

The decrease in the rate of discharge when the wire was first heated was very irregular, and in some cases it was scarcely noticeable. This decrease was more noticeable when the temperature was not very high.

This decrease was followed by an increase, and this increase was much more noticeable when the wire was white hot. In one case a current of 5.5 ampères was passed through a wire .3 mm. in diameter for 20 minutes and the rate of discharge increased during that time at least 50 per cent. When this increase was first observed it was thought that it might perhaps be due to an increase in the temperature of the wire occasioned by an increase in its resistance. On investigating the matter it was found that the resistance did indeed increase on account of a slight lengthening of the wire, and on account of a decrease in weight. The increase in length was, no doubt, due to a small tension that was put upon the wire to keep it straight.

However, it was afterwards found that raising the temperature did not cause an increase in the rate of discharge and other causes producing large changes in the discharge were found. The data

that were taken of the increase in length and decrease in weight have, therefore, no bearing on the matter in hand.

The real cause of the increase in the discharge can be much better considered after other experiments have been described. The discussion of this will, therefore, be postponed for the present.

This continual change in the rate of discharge made it impossible to secure accurate data in much of the work, but by allowing the current to flow through the wire for a short time before making observations it was possible to have the discharge nearly constant and to secure data approximating the truth.

Dependence of the Rate of Discharge on the Current flowing through the Wire.—As has already been stated the rate of discharge at certain temperatures of the wire was largely influenced by small changes in the current flowing through the wire. Table II. shows this dependence. Column 1 gives the current flowing through the wire. Column 2 gives the rate of discharge in ampères with positive discharge, column 3 those with the negative. The potential difference between the wire and surrounding cylinder was 40 volts. The diameter of the cylinder was 4.8 cm. and its length 6 cm.

TABLE II.

Current Through Wire.	Positive Discharge.	Current Through Wire.	Positive Discharge.	Negative Discharge.
3.5	$.13\times10^{-9}$	5.5	3.4×10^{-9}	$.13\times10^{-9}$
3.7	.25 "	5.7	3.3 "	.38 "
4.	3. "	6.	3.3 "	2.5 "
4.2	3.5 "	6.3	3.5 "	2.4 "
4.5	4. "	6.5	3.4 "	2.8 "
5.	4. "	6.8	3.3 "	2.5 "

The observations were not carried further, because with larger currents the wire melted. In no case was the negative discharge found to be as large as the positive, although these observations were taken many times.

The positive discharge passes through a maximum. Rutherford states that the discharge from a hot platinum sheet in like manner passes through a maximum. The observations regarding the rate of discharge from a wire were made before Rutherford's article was published, so that the reality of this decrease is fully established.

In explanation of the discharge the following may be said. When the temperature is barely sufficient to produce a few ions the amount of discharge is limited by the number of ions present. The number, no doubt, increases rapidly as higher temperatures are produced, but the discharge is soon checked by the effect of the ions on the field about the wire. The charges on the ions change the potential about the wire, until the potential gradient at the wire becomes very small and no more ions are then drawn out than are sufficient to keep the field in this condition. When this occurs the rate of discharge is not limited by the number of ions, but by the rapidity with which the ions move out of the field. If under these conditions the average velocity of the ions should decrease, the rate of discharge from the wire would decrease.

Rutherford explains the decrease in the rate of discharge observed at higher temperatures by supposing that at those temperatures ions begin to be sent off differing from those which had previously carried the discharge and that those ions move more slowly. Other facts bearing on this explanation will be given later.

Rutherford has investigated fully the position of the maximum rate of discharge under different conditions. The facts observed by myself were in harmony with those recorded by him, but no very extended investigation was made concerning the matter.

It is to be noticed that the negative discharge begins at nearly the same temperatures as that at which the positive decreases. This will be considered in connection with other facts.

Dependence of Discharge on Potential Difference.—The variation of the rate of discharge with the variation of the potential difference is shown in Table III. Column 1 gives the potential difference between the wire and the cylinder, column 2 the positive discharge; column 3 the negative. The current through the wire was 6.2 ampères. The other conditions were the same as those in Table II.

TABLE III.

Potential Difference.	Positive Discharge.	Negative Discharge.
46	4.5×10^{-9}	3.5×10^{-9}
30	2.1 "	1.4 "
22	1.2 "	.65 "
11	.2 "	.1 "

Velocity of Ions.—Having thus investigated some of the peculiarities of the discharge from a hot wire, the investigation of the velocity of the ions carrying this discharge was undertaken. Four different methods were employed. In general those who have studied the ionization of gases have found that the negative ions move more rapidly than the positive. The only exception to this rule which has heretofore been found was in the case of discharge produced by the electric arc. The present case, however, presents another exception. It, therefore, seemed well to have as positive a proof as was possible that this was an exception. Moreover the different methods brought out several facts of considerable interest.

These methods have all been used in the examination of ions produced by the electric arc, and were described in that work.[1]

First Method.—In the first method the only necessary measurements were those of the rate of discharge from the wire to the cylinder and of the potential gradient between these.

When discharge is carried by ions of one sign only which pass from an inner to an outer cylinder,

$$\frac{dV}{dr} = -\sqrt{\frac{4\pi I_1}{K_1} + \frac{C^2}{r^2}},$$

where V is the value of the potential at any point, k_1 is the velocity of the positive ions for unit potential gradient, r the distance from the center of the cylinder, I_1 the current through unit area at a unit's distance from the center and C a constant of integration. If there is a very large source of ions at the surface of the inner cylinder, $C = -r_0^2$ where r_0 is the radius of the inner cylinder and the equation becomes

$$\frac{dV}{dr} = -\sqrt{\frac{4\pi I_1}{K}\left(1 - \frac{r_0^2}{r^2}\right)}.$$

When a small platinum wire is used as the inner cylinder the term $\frac{r_0^2}{r^2}$ may be omitted and we have

$$\frac{dV}{dr} = -\sqrt{\frac{4\pi I_1}{K_1}},$$

and

[1] PHYS. REV., 14, 65.

$$V = -\sqrt{\frac{4\pi I_1}{K_1}}\, r,$$

if we assume zero potential at the wire, or

$$V_b = \sqrt{\frac{4\pi I_1}{K_1}}\, b,$$

where b is the radius of the outer cylinder and V_b is the potential difference between that and the wire.

If we are dealing with discharge produced by ions having different velocities we may substitute K_1 for k_1 where

$$K_1 = \frac{\Sigma\, k_1 n_1 e_1}{\Sigma\, n_1 e_1},$$

n_1 being the number of ions per unit volume which have the velocity k_1 and e_1 the charge which each one of these have, the summation being extended to all the positive ions. This K_1 we may call the average velocity of the positive ions.

Potential About the Wire.—The potential was found by a water dropper, the same that has already been described.[1] The wire was .3 mm. in diameter, the same size which has been used in the preceding experiment. A current of 6.3 ampères was passed through the wires. A cylinder 20 cm. in diameter was placed about the wire. The potential at different points is shown in Table IV. The first column gives the distance from the center of the cylinder, and the second the potential difference between the center of the cylinder and the point considered.

TABLE IV.

No. of cms. from Center.	Pd. Between Wire and Point.
1	10
2	20
3	29.5
4	41
5	51
6	59
8	78

The potential was found to fluctuate somewhat, due no doubt to the convection currents about the wire. These variations amounted

[1] PHYS. REV., 12, 68.

to about one volt on either side of the potential which was recorded. The recorded readings were, therefore, liable to an error of about one volt. This table was at first taken when positive discharge was passing from the wire, but it was practically the same when the discharge was negative. Thus it was found that if a ground on the shunt placed between the wire and the cylinder was shifted until the water dropper was at zero potential, the water dropper would remain at very nearly the same potential when the potentials of wire and cylinder were reversed. The variations caused by reversing the discharge were no greater than the continual fluctuations. A test was made in this way at each of the points given in Table VI., and the above statement was found to hold for each. These data, therefore, as truly represent the potential gradient in the case of negative discharge as in that of positive.

The potential is practically proportional to the distance from the center, showing that we have a right to say that

$$V = \frac{4\pi I_1}{K_1} r$$

and consequently

$$K_1 = \frac{4\pi I_1 b^2}{V_b}$$

In other words the current is directly proportional to the average velocity of the ions.

Ions Produced at or Near the Wire.—This shows conclusively that the ions are produced at or very near the surface of the wire. If they were produced at the surface of the cylinder surrounding the wire or between the wire and the cylinder, there would be an entirely different potential gradient from that which actually exists. That which exists could only be caused by ions starting from the wire.

Positive Ions Move the More Rapidly.—No further proof than this is needed to show that the average velocity of the positive ions is greater than that of the negative, since it has already been shown that the positive discharge at these temperatures of the wire is greater than the negative. This follows immediately from the formula

$$K_1 = \frac{4\pi I_1 b^2}{V_b^2}.$$

The potential between the wire and a cylinder 12 cm. in diameter was examined and again the potential difference between the point examined and the wire was found to be practically proportional to the distance from the wire, whether the charge was positive or negative. Other cylinders with different diameters were tried with the same result.

Rate of Discharge to Cylinders of Different Diameters.—The values of I_1 for a few different cylinders were then found. Very similar work to this has already been done by Rutherford, so that only a few observations were taken. In finding I_1 the outer cylinder was connected to an electrometer and the change of potential taking place in a given length of time was noted. The exact method of doing this has already been described.[1] From this change of potential the total current flowing to the cylinder could be found by dividing the change of potential of the electrometer by the time, here 8 seconds, and multiplying by the capacity of the electrometer. The electrometer was the same as that already used, and its capacity was 4.43×10^{-3} microfarads. From the total current I_1 is found by dividing by 2π times the width of the cylinder and from this K_1 may be computed by the use of the formula.

The results of this work are given in Table V. Column 1 gives the radius of the cylinder, column 2 the total positive current flowing to the surrounding cylinder, column 3 the total negative current, column 4 the computed velocity of the positive ions, column 5 the velocity of the negative ions. The potential difference between the wire and cylinder was 40 volts. The length of each of the cylinders was 6 cm., and the length of the hot wire was also 6 cm. The current through the wire 6.3 ampères.

TABLE V.

Radius of Cylinder.	Positive Discharge.	Negative Discharge.	K_1	K_2
2.4	3.5×10^{-9}	2.4×10^{-9}	3.8	2.6
6	.43 "	.30 "	2.4	1.7
10	.13 "	.10 "	1.95	1.5

These values for K_1 agree well with those found by Rutherford.

[1] PHYS. REV., 12, 74.

It will be noticed that the values for K_1 and K_2 decrease as the distance from the source of ionization increases. This agrees with what has already been found in the case of discharge from flames and from the electric arc and with what Rutherford has found from a hot platinum foil.

The variation of the rate of discharge with the different potential differences has already been studied in one case. According to the theory the rates of discharge should vary as the square of the potential differences. In reality it increases more rapidly than this, showing that at higher potential differences the velocity of the ions is greater than at lower. This again agrees with what has already been found in other cases. All of these facts are in agreement with the general statement that the longer the time that the ions have been in existence, the more slowly they move.

Velocity of the Ions, Second Method.—A second method used to find the velocity of the ions was a modification of one employed by Zeleny[1] to find the velocity of ions produced by X-rays.

First Modification of Zeleny's Mehod.—Practically the same modifications of Zeleny's method were necessary here as were necessary in the investigation of the electric arc.[2] In the case studied by him a lamina of gas perpendicular to the axis of the cylinders was ionized. In the case here studied a portion of the gas along the length of the inner cylinder was ionized. Consequently his method must be somewhat modified and this may be done in either of two ways. In the first let AB, Fig. 1, be the hot wire, the wires at each of its extremities having such conductivity as to remain comparatively cool. This wire is surrounded by two cylinders, CC' and DD', placed concentrically with the wire and insulated from each other at LL'. A blast of air is blown in the direction of the arrow.

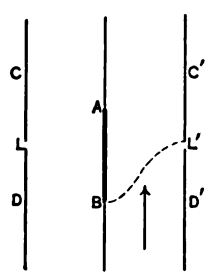


Fig. 1.

When the potential difference between the wire and the outer cylinder is small, no discharge will go to DD', but it will be blown to the upper cylinder, the ions from B moving in some such line as

[1] Phil. Trans. of Roy. Soc., 195, 193.
[2] PHYS. REV., 14, 66.

indicated by the dotted line in the diagram. As the potential difference is increased, the discharge will move more directly across to the lower cylinder. If the velocity of the ions is great per unit potential gradient, the discharge will begin to pass to the lower cylinder with smaller potential differences than it will, if the velocity of the ions is small. In this way the velocities of the positive and negative ions can be compared.

As has been already pointed out[2] this method shows the velocity of the most rapidly moving ions. There is reason to suppose that, at least in the positive discharge, there are different kinds of ions aiding in the discharge. This will be explained more in detail shortly. If this be so, this particular arrangement will give no information concerning the slower ones. They will at all times be carried to CC'.

An exact determination of the velocity is not possible by this method because the field between B and DD' cannot be mathematically discussed. Furthermore, the point B could not be determined with accuracy. B is supposed to be the point where the hot wire ends and the cold one begins, but manifestly the heat from the hot wire will be conducted off by the cold one, so that no abrupt change in temperature occurs at the point where the two unite. But though the field is thus irregular and the point B somewhat indeterminate, it is still true that the greater the velocity of the ions, the smaller the potential difference necessary to cause them to pass to DD'.

In making observations one might either keep the potential difference between the wire and the outer cylinder constant, and move AB one way or the other, until the discharge ceases to pass to DD', or one could leave AB fixed and change the potential difference. This latter procedure is the simpler in practice.

It is not possible, however, to determine absolutely the potential at which the discharge begins to pass to the lower cylinder. As may be seen in Table VI., a very small discharge passes to DD' even with small potential differences. As the potential difference is increased this discharge does not at first increase rapidly, but at length a potential is found at which it does. This may be taken as the potential at which the discharge begins. The discharge with

[2] PHYS. REV., 14, 67.

This is the same as the preceding except that the hot wire AB is placed entirely below LL'. In this case the discharge would all pass to DD', if the air were stationary and if the field were not modified by the presence of ions. When the air is blown past AB some of the discharge will be carried to CC', provided the potential difference between the wire and the cylinder is not large. If it is large, the ions should all be drawn to DD' before the air could drive any of them past DD'. The more rapidly the ions move for the same potential gradient, the smaller will be the potential difference necessary to keep the ions from passing to DD'.

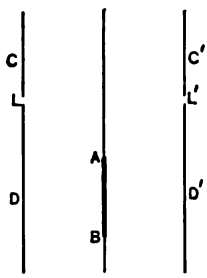


Fig. 3.

As has been previously pointed out, if there are ions present having different velocities, this method does not give us information concerning the average velocity of the ions, but concerning the velocity of the slowest moving ones.

If CC' be connected to an electrometer, and the potential difference between the wire and the cylinder be increased from zero, the electrometer would be expected to show at first a discharge passing to CC'. This should afterwards decrease and finally cease. This at least would be what we should expect if no account were taken of the modification of the field due to the ions drawn out from the wire. But as has already been shown the presence of the ions distorts the field, so that the lines of force are no longer perpendicular to the axis, but run from A up to CC'. Furthermore the field may be changed materially by the ions carried by the convection currents.

In spite of these difficulties the investigation disclosed a most interesting fact. It was found that when the wire was hot enough to produce positive discharge, but not hot enough to produce negative, the positive discharge increased at first as the potential difference was increased, and then decreased, soon ceasing altogether, but when the wire was hot enough to allow either positive or negative discharge, the rate of discharge increased continuously with increasing potential difference. That is, at lower temperatures the ions behaved very much as they did in the case studied by Zeleny, but at higher temperatures, they behaved in an entirely different manner.

In order to find the conditions under which this difference in temperature began, several sets of observations were taken with different amounts of current flowing through the wire. These are given in Table IX. The discharge here is positive in each case. The velocity of the air was 5.5 cm. per second. The distance from *A* to the plane *ee'* was 4.5 cm.

TABLE IX.

Potential Difference.	Deflection of Needle.	Potential Difference.	Deflection of Needle.	Potential Difference.	Deflection of Needle.
Current = 4.5 Ampères.		Current = 5.5 Ampères.		Current = 6.2 Ampères.	
7	1.3	7	2.9	9	3.2
10	1.8	12	3.9	14	5.9
14	1.0	20	2.6	25	8.4
17	.6	24	1.8	37	11.4
22	.2	31.5	.4	45	18.4
		34	.2		
Current = 5 Ampères.		Current = 6 Ampères.		Current = 6.5 Ampères.	
7	2.8	6	2.4	9	3.3
11.5	3	10	3.9	18.5	8.5
16	2	16	5.4	27	10.9
22	1	25	4.4	37	15.4
27	.3	37	2.4	45	22
		45	1.3		

The data of these curves are plotted in Fig. 4.

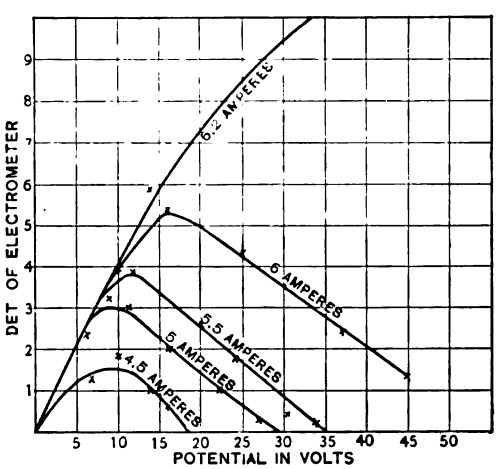


Fig. 4.

Possible Cause for Peculiarities of these Data.—There are three causes which could conceivably have produced the effect here observed. There might be more ions about the field as the wire becomes hotter and consequently greater distortion of the field; the convection currents might become stronger and more ions be carried up by them; or some of the ions might move more slowly at the higher temperatures and occasion a smaller rate of discharge.

The first explanation does not seem reasonable, for in all probability the space about the wire contains at all of these temperatures the same amount of electricity. By referring to Table II. we see that the rate of discharge from the wire ceases to increase when the amount of current passing through the wire was over 4.5 ampères. With any greater current than this the field appears to contain as many ions as was possible. But in the observations the current through the wire was at least 4.5 ampères.

Possibility of Error Due to Convection Currents.—In order to examine further the action of the convection currents, the apparatus was turned the other side up, the air from the gasometer being forced downward from above. The discharge seemed to show even less tendency to decrease with higher potential differences. This does not prove that the convection currents have no effect, since even with this condition of the apparatus the convection currents would mix the air somewhat and it might easily be that the slowly moving ions would be affected more by such currents than by the electric field. But when one considers the fact that reversing the direction of the convection currents does not change the result, that there is no reason to suppose that there is any great change in these currents at this temperature and that we have other reasons for thinking that there is a decided change in the condition of the ionization at this temperature, it seems most reasonable to explain these curves by saying that at these higher temperatures at least some ions move much more slowly than those produced at lower temperatures.

Discharge at Higher Potentials.—The distance from *A* to *ee'* was increased to 10 cm. But there was no decrease in the rate of discharge at higher potentials as compared with those of lower potentials. Still higher potentials were then used. In order to do this with the means at my command it was necessary to keep the wire

at zero potential and to change the potential of the outer cylinder by means of a water battery. This is the same method as that which has been used in studying the discharge from the electric arc.[1] Observations were taken up to 150 volts. These are given in Table X. Both the positive and negative discharges were noted. The current through the wire was 6.3 ampères. Column 1 gives the potential difference between AB and DD'. Column 2 the positive discharge and column 3 the negative.

TABLE X.

Potential Difference.	Positive Deflection.	Negative Deflection.
40	2.4	2.0
70	3.4	3.0
100	5.0	3.6
150	7.0	3.2

There is here no tendency for the rate of discharge to become smaller at the higher potentials. But the negative discharge shows more indication of decreasing than the positive. This is not in any way inconsistent with what has already been found. We have heretofore been comparing the average velocities of the positive and the negative ions or the velocities of the most rapidly moving ions. Here the velocities of those which move the least rapidly are compared, and apparently the positive discharge contains ions moving more slowly than any of the negative. It may well be that with the more rapidly moving ions of the positive discharge there are also a few very slowly moving ones. If there were but few of these they would have but little effect on the average velocities which were found by the first method and would not show at all in the second method.

Ions Produced Without a Field.—However, the fact that it was so difficult to get rid of these ions by drawing them out in a strong field suggested that we were not dealing with ions but with some sort of "emanation." The first attempt to test this was made by keeping the cylinder DD', Fig. 3, at the same potential as the wire and testing the condition of the air blown into CC'. It was thought

[1] PHYS. REV., 14, 70.

at first that any emanation would thus be blown into CC', but that no ions would leave the wire, if there was no field about it. It was found that as soon as sufficient current was sent through the wire to bring it to a temperature at which the positive ions could be produced, the air blown into the upper cylinder from about the wire allowed a discharge to pass through it. Table XI. gives the data taken in this case. Column 1 gives the current through the wire in ampères, column 2 the deflection of the electrometer produced by the positive discharge to CC', column 3 that produced by the negative. Through this series of readings the cylinder was kept at the potential of the wire. The distance from A to ee' was 4 cm. The velocity of the air was 24 cm. per second.

TABLE XI.

Current Through the Wire.	+ Discharge to CC'.	− Discharge to CC'.
5.4	.0	.0
5.8	.8	.8
6.2	2.5	2.3
6.5	3.	3.

The positive and negative discharges were practically the same.

Further thought, however, showed that this was not a proof that there was an emanation sent out from the wires, for if the air about the wire was ionized to more than molecular distances, these ions might be carried by convection currents away from the wire and, mingling with the air, be carried into cylinder CC'. This would account for the discharge which has been noted.

To test the matter further the distance from A to ee' was made 12 cm., and the size of DD' was reduced to 2 cm. in diameter. There was then no discharge to CC' that could be detected. But this on the other hand did not prove that there was no emanation, since the emanation might quickly lose its power of producing ions.

To test this still further the cylinder was divided 6 cm. above A and the upper part insulated from the lower. The lower one about the wire was kept at the same potential as before but the upper was kept at the potential of the wire, so that there was no field in this part to draw the ions. If, however, the effect was due to some kind

of emanation, this emanation would lose its power of discharging as soon as before. It was found that with the conditions as described we again had a discharge to CC'. This discharge ceased as soon as the middle cylinder was raised to the same potential as the outer cylinders. This showed conclusively that the cause of the discharge in this case was something which could be destroyed by an electric field. It does not prove that there was no emanation whatever having a very brief period of activity, but it shows that we have no evidence of such emanation. Moreover I was not able by any method with which I am acquainted to show the existence of such emanation.

The cause of the decrease in the velocity of the positive ions will be considered after the fourth method of finding the velocity of the ions has been discussed.

Fourth Method.—The last method was one which has been employed by Rutherford.[1] It depends on the distance from the wire to which ions will pass under the action of an alternating potential difference. There are not as serious difficulties to be incurred in applying this method to the hot wire as there were to the arc. There are no such irregularities in the potential about the wire as there are about the arc. Nevertheless I have not been able to find a mathematical solution determining the distance to which ions will move under the influence of a given alternating potential difference. When the number of ions between the two cylinders is so small that the potential gradient is not affected by their presence, the distance can be easily computed. But the matter is very much complicated when there are many ions present, and even if a solution could be found where only one kind of ion is present, it would help but little when both kinds of ions are present.

Ordinarily the solution for the negative current can be superimposed on that for the positive and it gives the solution for the two combined, but here when the positive ion comes in contact with a negative one, the two combine and the effect of the two combined is not the same as that of the two separate. When separate one would move a certain distance in one direction and the other a different distance in the opposite, the relative distances depending on

[1] Proc. Camb. Phil. Soc., 9, 401.

the relative velocities. When combined they would not move at all. The resultant effect of the two combined, would, therefore, be different from the resultant of the two when separate. A combination of solutions for the positive and negative ions would not give a solution for the two. These difficulties taken in connection with the fact that the velocities of the ions appear to diminish according to some law not definitely known and that we are dealing with several different kinds of ions, make a mathematical discussion of question impossible.

But while this method can not here be used to determine the absolute velocities of the ions, it may be used to compare the relative velocities. The comparison which it affords is one between the most rapidly moving of the positive and the most rapidly moving of the negative ions.

The first set of observations were made with a cylinder 9 cm. in diameter and 3 cm. wide. A wire similar to those which have been used in the previous work was placed at the center of this. The current passing through the wire was measured by an alternating current ammeter and was 6.5 ampères. Thus it is evident that both positive and negative ions were being produced. The current was given by the secondary of a transformer which was part of the village lighting system. The potential of the wire was varied by the same method that has heretofore been used. The square root of the mean square of the potential of a point at the middle of the platinum wire was found by means of an electrometer connected idiostatically. There were 130 alternations of the current per sec.

It was found that in every case the surrounding cylinder became charged positively. This of itself was sufficient to show that the more rapidly moving of the positive ions have a greater velocity than any of the negative.

To determine the potential difference necessary to cause a discharge to pass to the cylinder, the rate was observed at which the cylinder became charged when different potential differences were used. Table XII. gives the deflection of the electrometer needle in four seconds with different potential differences. Column 1 gives the potential differences and column 2 the corresponding deflections.

TABLE XII.

Potential Difference.	Deflection of Electrometer.	Potential Difference.	Deflection of Electrometer.
49	5	26	.5
43	3.5	15	.0
35	2.4		

A curve was plotted from the data here given and it was assumed that the discharge ceased at the potential difference indicated by the point at which this curve crossed the line of zero discharge. In this case the potential difference thus indicated was 23 volts.

Similar sets of observations were taken with cylinders 5 cm. and 12.5 cm. in diameter. In the former case the potential difference necessary to produce the discharge was 9 volts and the latter 38 volts.

Causes of Decrease in Velocity of Ions.—We have seen that there are reasons for believing that the average velocity of the positive ions decreases at the higher temperatures. When we attempt to suggest a cause for this decrease, there are two explanations which present themselves. One explanation would be that a new kind of ion is being produced at the higher temperatures. During most of the time that I was investigating the subject I considered this to be the correct explanation, and expressed this idea in a short article in Physikalische Zeitschrift.[1] I was led to this conclusion by the fact that at higher temperatures the gas about the wire becomes ionized. If, moreover, one were to suppose that the ions of the gas move more slowly than those coming from the metal, an explanation of the decrease in the velocity of the ions at higher temperatures is at hand.

The fact that the negative discharge began at about the same temperature as that at which the gas about the wire became ionized also seemed to show that a new kind of ionization was being produced.

I have as yet no proof that such action as this may not be taking place, but experiments to be immediately described point to a different explanation of the phenomena, namely, that the same kind of ions are being produced at higher temperatures as at lower, but that

[1] Physik. Zeitsch., 25, 3, 158.

at the higher temperatures they are loaded with particles that are sent out from the glowing platinum. It is well known that minute drops of water will diminish the velocity of the ions on which they condense. There is no reason why matter from glowing platinum should not do the same. It is also known that glowing platinum diminishes in weight[1] and that it acts as nuclei for water condensation.[2]

The experiment which points to this as being the correct explanation was one on which I stumbled while investigating the discharge in other gases than air. It was found that the discharge in CO_2 was very much smaller than I had expected it to be. In fact it was so small as to indicate a velocity of only .025 cm. per second for a potential gradient of 1 volt per cm. This was most surprising, since the velocity of ions produced by X-rays in CO_2 was approximately the same as that of ions produced in air by this means. In attempting to find some error in my measurements it was found that the discharge to a metal cylinder enclosed in a glass tube was very much smaller than it was to the same cylinder when not enclosed. The glass cylinder was carefully insulated and the insulation tested until it was certain that this decrease was not being caused by leakage to the ground.

It was then found that the rate of discharge in the enclosed cylinder was by no means constant. When the wire was first heated the discharge was large, but decreased from its first value rapidly. When the wire was heated only to red heat the rate of discharge within the tube was nearly as large as it was in the open air, and at this temperature it diminished but very little after it had heated for some time. When it was heated to a higher temperature it diminished very rapidly and the final rate of discharge was not more than 3 per cent. of what it was to the same cylinder when not enclosed in the tube.

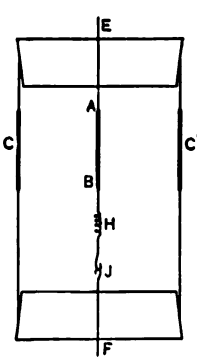


Fig. 5.

Apparatus.—These experiments were performed in a glass tube 24 cm. long, and 4.4 cm. inside diameter, closed at the ends with

[1] Wied. Ann., 31, 448.
[2] Nature, 31, 268.

rubber stoppers as shown in Fig. 5. The wire passed through the stoppers, the part between A and B being platinum. About AB was the cylinder CC'. This was 4 cm. in diameter, and 5 cm. long. H is a coil in the wire to keep AB straight. I is a hook at which the wire can be separated when it is desired to remove it. G is an opening to an air pump. The cylinder CC' was of tin.

A series of observations with different amounts of current flowing through the wire is given in Table XIII. The potential difference between the wire and the cylinder was 40 volts. Column 1 gives the time which elapsed from the starting of the current through AB to the time at which the discharge was observed, or more correctly, to the time at which this observation was commenced. Column 2 gives the positive discharge in 10^{-10} ampères when 4 ampères passed through the wire. The other columns give the rates of discharge with other amounts of current through the wire. The last two columns show the rates at which the negative discharge decreases. The rates of discharge were computed from the deflection of the electrometer needle, the length of time it was receiving the discharge and the capacity of the system.

TABLE XIII.

Time.		4 Amperes.	4.5 Amperes.	5 Amperes.	5.5 Amperes.	6 Amperes.	−6 Amperes.	−6.5 Amperes.
Minutes.	Seconds.							
	0	16	18	7.5	7.5	3.3	8.3	3.9
	20	14	10.8	4.6	4.1	2.8	2.3	1.9
	40	14.5	10.2	4.1	2.2	2.2	1.7	1.1
1	00	14	8	3.9	1.9	2.2	1.1	1.4
1	20	14	7.5	3.9	1.9	2.1	.8	1.1
1	40	14	6.6	3.9	1.4	2.2	.9	1.2

In taking the readings the air within the tube was changed after each set before the discharge with a different current was commenced. The rate at which the discharge decreased was quite different at different times. The data given in Table XIII. were taken with dry air and the wire heated for some time to remove any impurities. The smallest value given here for the positive discharge indicates a ve-

locity of only .12 cm. per second for unit potential gradient as compared with 4 cm. in the open air.

This condition of the gas within the tube existed for some time. Data bearing on this point are given in Table XIV. Six ampères were passed through the wire for some time, the circuit was broken for the length of time indicated in column 1, and then 4 ampères were sent through the wire and the rate of discharge noted. Four ampères is not sufficient of itself to produce this kind of "emanation" to an amount that could be detected by the electrometer. It could thus be used to test the condition of the gas. Column 2 gives the rate of discharge.

TABLE XIV.

Time.		Rate of Discharge.	Time.		Rate of Discharge.
Min.	Sec.		Min.	Sec.	
	0	4×10^{-10}		40	10.0×10^{-10}
	10	3.8 "	1		10.6 "
	20	7.2 "	30		12. "
	30	8.0 "			

After the tube had been opened and then closed again the rate of discharge was 14×10^{-10} ampères. The discharge with the ends of the tube open was 50×10^{-10} ampères.

There can thus be no doubt but that matter in some form is driven off from the glowing platinum and that this greatly retards the velocity of the ions when the gas is confined in the tube. This would also appear to be the cause of the smaller velocity of the ions in the open air at the higher temperatures.

The mere presence of particles driven off from a hot wire is more definitely shown by their use as nuclei for condensation to which reference has been made. A few experiments of this character were performed. The ordinary means of doing this were used. Air was drawn through a tube containing cotton wool soaked in glycerine. When the tube containing the wire was connected to an air pump from which the air had been nearly exhausted the formation of fog was plainly seen after the wire had been heated. When the air was suddenly drawn out three or four times no condensation was visible, until the wire was again heated.

It was found that two ampères sent through the wire were sufficient to produce condensation that was clearly visible. This was with a wire that required 3.5 ampères to produce any discharge that could be detected, and four ampères to become self-luminous.

With larger currents the fog became more dense but no attempt was made to take any measurements with it.

The most interesting experiments were those showing the length of time that the particles were suspended in the air. The tube could be left for several minutes after the wire had been heated and the condensation then produced appeared to be as dense as ever. When an hour passed before the fog was formed, it was found to be still plainly visible. At the end of two hours after the wire had been heated the condensation was barely visible. When the wire was heated and allowed to remain over night before the air was drawn out of the tube no condensation could be seen.

Another positive proof that matter is given off by the wire is shown by the loss of weight of the wire. With the size of wire here used no loss of weight could be detected when it was heated with five ampères for one hour. But when 6.2 ampères were passed through a piece weighing. 2888 grams for one hour, it lost .0013 gram.

There is of course nothing new in the fact that particles are driven off from a hot wire, but as far as I know no one has ever suggested the idea that these particles check the discharge from such wires. In fact the opposite has been suggested, namely that the discharge is carried by these particles. Such an idea would appear to be incorrect. The air which is enclosed and which consequently contains a greater number of the particles is the air which shows the slowest rate of discharge. Instead of aiding, the dust particles retard the motion of the ions, and may in an enclosed space almost entirely stop the discharge.

Experiments showing the behavior of the discharge in other gases than air and some bearing on the nature of the emanation will be given in the second part of the article.

It was stated at the beginning of this article that the rate of discharge increased when the wire was heated for some time. This no doubt is due to a diminution in the number of particles about the wire. It may be that the convection currents become stronger and

carry away the particles, or possibly there are fewer particles driven off after continued heating.

Summary.—The discharge from a hot platinum wire at first decreases and then slowly increases.

The positive discharge commenced a little below the temperature of red heat, increased at first rapidly as the temperature was increased, remained nearly constant through quite a range of temperature and finally decreased somewhat. The negative discharge began at nearly the same temperature as that at which the positive began to decrease. It never became as large as the positive.

Both positive and negative discharge increased rapidly as the potential difference between the wire and cylinder was increased.

Four methods were used for comparing the velocity of the positive and negative ions. One of these compares the average velocity of one with that of the other. This showed the velocity of the positive to be the greater. Two methods compare the velocities of the most rapidly moving of the positive ions with the most rapidly moving negative. Both of these show the positive ions to have the greater velocity. One compared the slowest of the positive with the slowest of the negative. This showed that at the lower temperatures the slowest of the positive ions have a much greater velocity than the slowest of the negative, but at higher temperatures a slower class of ions are produced, and the comparison of these with the slowest negative ions gave no definite results.

It was also found that at a certain temperature ionization is produced in the gas about the wire to greater than molecular distances. This is nearly the same temperature as that at which the negative discharge commences, as that at which the positive discharge materially decreases, and as that at which the very slowly moving positive ions commence to appear.

Experiments on a wire enclosed in a glass tube indicate the presence in the tube of particles which diminish very greatly the velocity of the ions. This "emanation" appears to load the ions and thus to diminish their velocity both when they are within an enclosed tube and when they are in the open space. Thus the particles driven off from a hot wire do not appear to aid in the discharge but to materially check it. This will be further discussed in the second part of this article.

A NEW FORM OF CAVENDISH BALANCE.[1]

By G. K. Burgess.

THE sensibility of Cavendish's method for the determination of the gravitation constant would be much increased if, while still preserving heavy masses hung from the balance arm, the tension which this weight exerts on the suspension wire could be suppressed. This possible, nothing would prevent realizing a measurement as sensitive as might be desired; the attraction between the two systems of spheres—the fixed and moving systems—would remain a definite quantity and allow increasing the deflection of the movable system by reducing more and more the diameter of the torsion wire. The principle that we have made use of with this object in view is the following: Compensate for the weight of the turning system by a float immersed in a mercury-bath; eliminate any capillary action that may tend to hinder the mobility of the system, and if then the zero remains constant, we have at our disposition an instrument of unlimited sensibility.

The dimensions of such an apparatus will be subordinated to considerations of precision, and ease of length measurements. If the dimensions are too large no gain in accuracy will be had; on the contrary the effects of temperature variations will be increased. On the other hand, with too small dimensions, exact measurements of length will be more difficult, and eccentricity effects increased.

We chose a length of 12 cm. for the lever-arm—a length easy to measure by the dividing-engine and not necessitating a large apparatus. From the two ends of the balance-arm are hung two lead spheres of 2 kg. each; the whole is buoyed by a cylinder immersed in mercury, whose surface is covered by dilute sulphuric acid; a rod passing through the mercury and acid connect the cylinder to the balance. The points of support for the spheres are below the center of inertia of the system, thus giving to the balance-

[1] Summary of Thesis, Recherches sur la Constants de Gravitation, Paris, 1901.

arm the form of three sides of a rectangle. Thanks to this arrangement, the torsion fiber has to support only a weight of 5 or 10 grams, instead of having to hold up a weight of 5.5 kg. and this difference allows of using a fine quartz fiber.

The attracting masses—two lead spheres of 10 kg. each—were hung from an independent support, and turned in a circle of radius of 18 cm.

The mobility of the suspended system was first carefully verified by a series of preliminary experiments. The liquid surfaces must be clean, and then as in the case of the Lippmann electrometer, the liquid surfaces were found to be mobile with respect to the glass surface of the supporting rod.

Great care had to be taken to obtain a constant zero which might be influenced by convection currents in the mercury and in the air. The *sine qua non* of this adjustment is a constant temperature throughout. We succeeded in establishing a uniform action of the mobile system only with the greatest care and only after the apparatus had been mounted several days.

Happily, as it was greater than one hour, we did not have to determine the exact period of oscillation of the system; it was sufficient to know the torsion coefficient of the fiber, established by preliminary experiments with an auxiliary cylinder, and determine the deflections of the system under definite conditions.

In our apparatus it was not necessary to place the attracted spheres at different levels to increase the sensibility. In fact, the apparatus as we constructed it, used in the most sensitive manner, gave deflections exceeding 1 meter on a scale at 2.6 meters distant.

All parts of the apparatus were symmetrically arranged about the central axis, in order to avoid corrections arising from secondary attractions. The total force of attraction was of the order of 0.01 dyne.

Besides a gravimeter, our apparatus might become accidentally both magnetometer and electrometer, thus introducing sources of error due to magnetic or electrostatic attractions. These effects were carefully tested for and eliminated.

Apparatus.

The apparatus was installed in a cellar of the Physical Research Laboratory at the Sorbonne, and mounted on a stone pillar 1 m. × 1 m. × 0.5 m. The stability conditions were sufficiently good especially at night. As to temperature variations they were a minimum: less than 0.1° C. a day, and with the change of season from winter to summer, the temperature of this cellar rose less than two degrees in three and a half months.

As to the construction of the apparatus: the four lead spheres were turned on a heavy lathe from cylinders cast with great care taken to leave them homogeneous. Their supporting wires, arsenical bronze for the mobile system and piano steel for the others, were

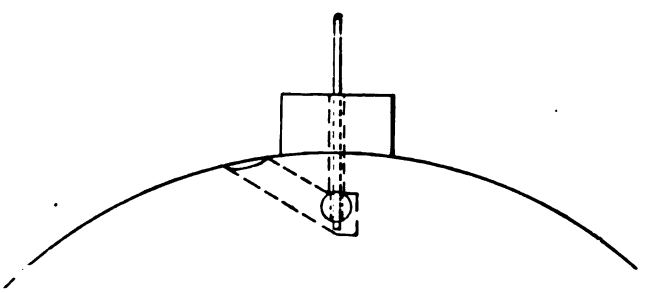


Fig. 1.

attached as shown in Fig. 1. The lead knob was equivalent in mass to the excavated portion of the sphere, and the wire was held by a small brass sphere.

The balance-arm which supported the 2 kg. masses was of aluminium bronze (7 per cent. aluminium) and of the form indicated in Fig. 2. The horizontal arm was 12 cm. long, the vertical arms 9 cm. Lead compensating weights adjustable in position were placed at the ends of the horizontal arm. Immersed in a glazed earthenware cylindrical vessel filled with mercury was a varnished hollow copper cylinder of such volume that the weights of the two spheres joined to the auxiliary weights of the balance and cylinder about equaled the weight of displaced mercury.

The cylinder was attached to the balance by a steel rod of 2 mm. diameter which was covered by a thin glass tubing from the top of the cylinder to the balance-arm, for protection against the acid layer on the mercury surface.

The earthenware vessel—12 cm. high and 9.5 cm. diameter—containing about 7 kg. of mercury, rested upon a brass plate, carried by a brass pillar movable vertically and in azimuth. This pillar was screwed into a massive circular base also adjustable in azimuth

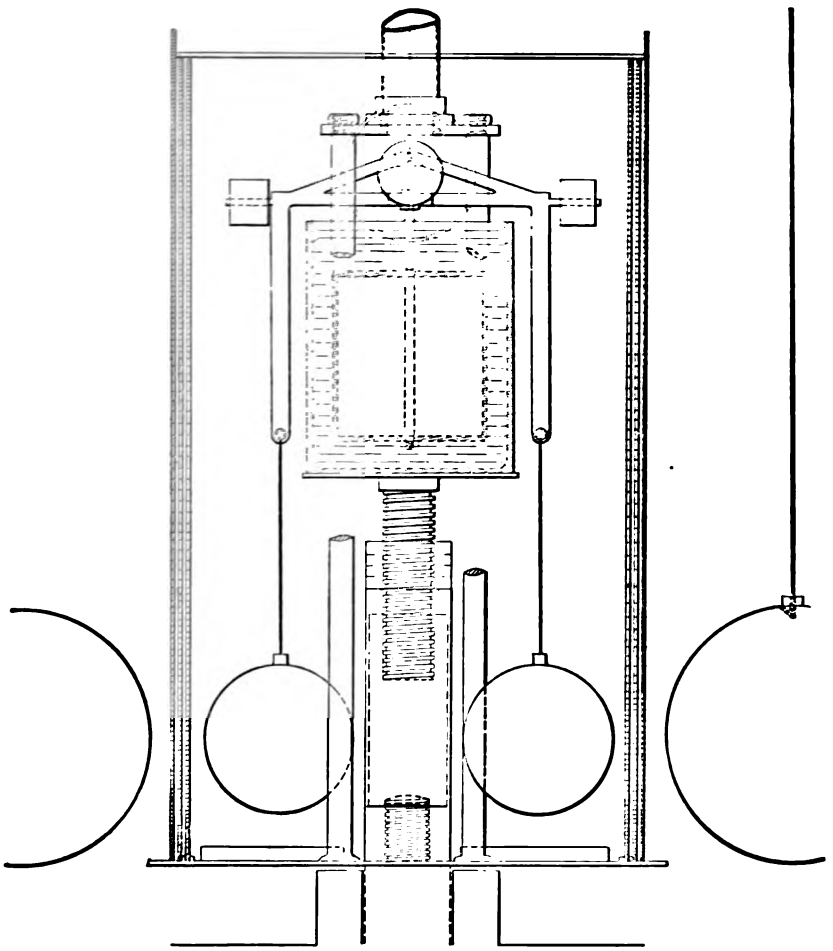


Fig. 2.

mounted on three legs with leveling screws, and carrying on the outer circumference of 24 cm. diameter a divided circle which served to measure the angles turned by the large spheres. The torsion fiber of quartz 35 cm. long and 0.03 mm. diameter passed through

a glass tube with a torsion head permitting independent vertical and azimuth movements. Two brass pillars, nearly at right angles to the plane of the balance arm, carried a brass plate into which was screwed the torsion tube. Adjustments were provided to center the fiber into the axis of symmetry of the apparatus. The mirror was plane and circular of 2.5 cm. diameter.

The support for the large outer spheres, completely independent of the inner system, consisted of a heavy oak triangular frame, carrying at its top a centrally mounted bronze arm, one end adjustable longitudinally and the other laterally, from which were hung the large spheres, adjustable vertically, so that the horizontal and vertical eccentricities of the two systems of spheres could be easily rendered a minimum.

The inner system was surrounded by a glass cylinder 20 cm. in diameter, and two of cardboard separated by a layer of sawdust. The large spheres were without this, but the whole was enveloped by a covering of thick black cloth tacked to the oak frame.

A glass window having plane faces permitted observation of the mirror with a telescope placed at 3 meters distance having a 5 cm. objective and a magnifying power sufficient to read $\frac{1}{20}$ mm. on the ground-glass scale at 2.6 m. from the apparatus. The scale was illuminated by an incandescant lamp lighted only during the instants of reading.

The dividing engine used to measure the horizontal dimensions of the apparatus by means of a small telescope mounted on the carriage was from Perreaux. Its screw was 60 cm. long, whose average pitch served as the standard of length, which compared with a Sevres meter gave 0.9998 cm. of the screw equal to 1 cm. international. Vertical distances were measured with a cathetometer reading to 0.02 mm.

An aluminium cylinder of 30.060 grm. served to determine the torsion coefficient of the quartz fiber. The ratio of its diameter to height was $1 : \sqrt{3}$ so that its ellipsoid of inertia was a sphere. This arrangement has the advantage that if the axis of suspension does not traverse the center of inertia of the cylinder, the moment of inertia rests constant.

A steel tape mounted on a board set on edge was used to measure the distance from the mirror to the scale.

THEORY.

To determine the gravitation constant, with such an apparatus, the following is the geometrical theory to a first approximation.

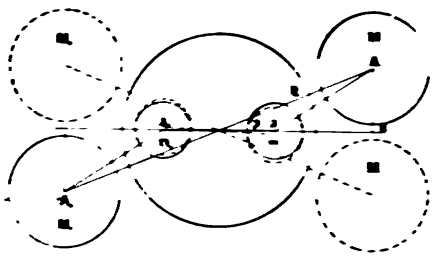


Fig. 3.

Let M and M' be the masses of the two large spheres turning about the axis of the system with radius R; m and m', the two spheres hung on the balance arm of length $2r$; $\bar{Aa}$, the distance from a large to a near small sphere; γ the torsion moment; then the attraction of M or m is:

$$MmK\ \bar{Aa}^{-2}.$$

The torsion moment:

$$\gamma = MmK \cdot \bar{Aa}r\ \bar{Aa}^{-3} = MmKRr \sin\varphi\ \bar{Aa}^{-2}.$$

But

$$\bar{Aa} = (R^2 + r^2 - 2Rr\cos\varphi)^{\frac{1}{2}}.$$

Therefore

$$\gamma = MmK \cdot Rr \sin\varphi(R^2 + r^2 - 2Rr\cos\varphi)^{-\frac{1}{2}}.$$

For the attraction of M or m', we have similarly:

$$\gamma' = Mm'K \cdot Rr \sin\varphi(R^2 + r^2 + 2Rr\cos\varphi)^{-\frac{1}{2}},$$

and for that of M' on m' and of M' on m there are analogous equations. The angle φ is composed of two parts

$$\varphi = \theta - \alpha,$$

where θ and α are the angles made by the attracting and attracted spheres with their respective equilibrium zero positions.

In an actual computation the formulæ are not quite so simple for two reasons: the eccentricity of the mounting with respect to the cen-

tral axis and the differences of level of the several spheres give rise to trigonometrical corrections easy to calculate.

The mean density of the earth Δ may be calculated from the gravitation constant K, independently of the local value of g as follows:

$$g = \tfrac{4}{3}\pi R_p \Delta K(1 + a - \tfrac{3}{2}c)\{1 + (\tfrac{5}{2}c - a)\sin^2\beta\}$$

where R_p is the polar radius, a the flattening of the earth, c the ratio of centrifugal force to that of gravity at the equator, and β the latitude.

Measurements.

Let us consider briefly the series of measurements and operations necessary to determine the gravitation constant with this apparatus.

The constant of torsion of the fiber was first determined in an auxiliary mounting by means of an aluminium cylinder. Times were taken with a Breguet (3030) chronometer by the telescope and scale method. The dimensions of the cylinder measured by the dividing engine were as follows: radius 1.31405 ± 0.00059 cm., height 2.2519 ± 0.0012 cm., mass 33.060 grm. and its moment of

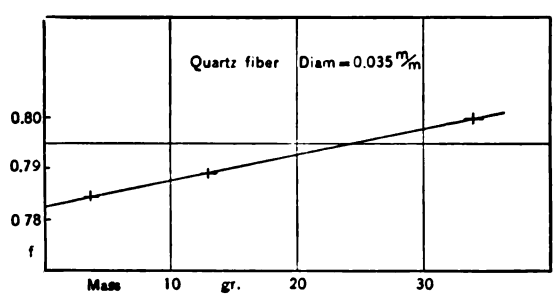


Fig. 4.

inertia 28.545 c.g.s. This latter, corrected for the knob of the cylinder, mirror and screw, was $I = 28.788$ c.g.s. units.

As the coefficient of torsion varies with the stretching force, the coefficient of the quartz fiber for any mass suspended was found by a series of auxiliary paraffine cylinders, with the results indicated in Fig. 4. Knowing the weight on the fiber in the gravitation apparatus, the proper coefficient was readily found by interpolation.

The torsion coefficient is given by $f = 4\pi^2 I / t^2$. For the quartz fiber used in three series, the preliminary determinations gave:

$$t = 51.''792 \pm 0.0005, \qquad f = 0.42367.$$

With the Aluminium Cylinder.

The gravitation apparatus was then set up with this fiber in place, and eccentricity of mounting was as nearly as possible eliminated by the adjusting screws. The zero position of the mobile system was then found and set to a convenient part of the scale. This operation took several days, for the system remained in motion so long as any disturbing cause persisted. In addition to the precautions taken to obtain constant temperature described above, numerous cardboard screens cut up into as small sections as possible the enclosed air space about the inner spheres.

When constancy of zero was obtained the outer spheres were hung in place and a series of measurements was taken, extending over several days, of the deflections of the inner system of spheres in response to the definite and subsequently measured positions of the outer attracting spheres. In general it took over two hours for the inner system to take up a very small oscillation about a new position. The period was one hour, ten minutes. The constancy of a given position is illustrated by the following, taken from the second quartz series, for three successive deflections to the right on as many days taken alternately with deflections to left:

72.205 cm.
72.318
72.135

Five minute readings were taken for four or five hours for each position. The total deflection was about 65 cm. on a scale at 2.6 m. The measurements of deflection taken, the distance from the suspension to the scale was measured, and after removing the coverings the horizontal distances between the various spheres and the torsion fiber were determined with the dividing engine placed parallel to the balance arm at 40 cm. distance. This gave the data also for horizontal eccentricity. Similarly, the vertical distances between the centers of the spheres were taken with the cathetometer, giving

vertical eccentricity corrections. The horizontal eccentricity perpendicular to the plane of the balance was also measured, and also the angles formed by the right and left and ventral positions of the outer spheres by means of the graduated circle at the base of the apparatus and the telescope and scale. During these measurements, taken in the zero position of the inner system, the smaller spheres were kept in place by stops. Phosphor-bronze was also tried as suspension but its zero is by no means constant enough for this kind of work.

For three series with the same quartz fiber, whose constants are given above, the following are the principal data for the determination of K.

	R_1	R_2	r_1	r_2	Dis Scale to Suspension.
Series I.	18.359	18.060	6.158	6.144	260.80
" II.	17.950	18.630	6.212	6.121	260.15
" III.	18.149	18.506	6.188	6.144	260.14

	Deflections = Attracted Masses		Deflections = α Attracting Masses.		Torsion Factor.
	Right.	Left	Right.	Left.	
Series I	2° 16′.09	4° 6′ .75	11°.35	28°.67	0.9812
" II.	2° 05′.11	4° 20′.62	13°.13	26°.90	1.0000
" III.	2° 05′.15	4° 48′.50	11°.47	28°.56	0.9811

There are four equations of this form to solve for each series:

$$K = f\frac{[R^2 + r^2 \pm \cos(\alpha - \varphi)]^{\frac{3}{2}}}{MmRr \cdot \sin(\alpha - \varphi)} \quad \text{where} \quad f = \frac{\gamma\pi\varphi}{180}.$$

Solving, gives $K = 6.64 \cdot 10^{-8}$ and $\Delta = 5.55$.

The measurements were so planned as to give a precision of 0.001 in the final result. The separate quantities were all determined to 0.0003, with the exception of $(\alpha - \varphi)$. This precision required a knowledge of the horizontal distances R and r (see Fig. 3), as measured on the dividing engine, to 0.002 cm. for the lesser and to 0.005 for the greater radius; the dimensions of the auxiliary cylinder to 0.002 cm., and its time of oscillation to 0.08 sec. only. This time was actually determined to 0.001 sec. and a precision

double that required in the length measurements was attained. The constancy of the deflection reading was the gravest source of error. The angles should be known to 0'.4. It had to be assumed that the elastic properties of quartz were perfect or at least that there is no change in the zero position. This assumption seems justified from the work of Threlfall and Pollock with their quartz fiber gravity balance.

The necessary tests and corrections for the use of the telescope-scale-mirror method were made as indicated by Holman.[1]

The results thus far obtained are to be considered only as tentative, and indicative of the possibilities of the method.

The apparatus may be improved by:

1. Mounting it in vacuo, only a slight advantage here as periods are not measured.
2. Making the balance-arm 10 cm. a submultiple of the meter.
3. Turn the outer spheres automatically.
4. Render the temperature of the mercury more constant by enclosing in a vacuum jacket with thick inner copper wall.
5. Avoid any but a residual deflection measurement by turning the torsion head the necessary angle, calculated beforehand, to counterbalance the gravitation moment.
6. Take readings photographically.
7. Work out in the country.

The method possesses advantages which render it perhaps the best for a determination of the gravitation constant, such as:

The possibility of increasing indefinitely the sensitiveness as has been indicated.

The non-obligation to determine the period of the system.

The possibility of using a null-method—a great advantage, obviating troublesome convection currents.

Its adaptability—due to its great sensitiveness—to the study of the effects of various interposed media and attracting masses of different kinds, or in other words for detecting, if they exist, any modifications of Newton's law.

This research was carried out under the direction of Prof. Lippmann, and to him I am greatly indebted.

[1] Holman, Telescope-Mirror-Scale Method, Tech. Quarterly, Sept., 1898.

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THE PHYSICAL REVIEW.

THE VALUE OF THE GRAVITATION CONSTANT.

By G. K. Burgess.

In 1885 Harkness[1] made a calculation by the method of least squares of the value of the gravitation constant, but the best experimental work has been done since then, and it seems desirable to-day to deduce from all the data at hand the most probable value of the gravitation constant and its analogue the mean density of the earth. Boys, in his report to the Paris Congress, 1900, has made a critical study of the various methods, but does not suggest a best value for the constants. To do this last is our object.

The gravitation constant is the quantity K in the formula which expresses the Newtonian law of attraction:

$$F = K\frac{mm'}{r^2}.$$

This constant is related to the expression for the mean density of the earth, approximately as follows:

$$\Delta = \tfrac{3}{4}\cdot\frac{g}{Kr}$$

in which g is the acceleration due to gravity of a mass at the surface of the earth whose radius is r and mean density is Δ.

We may classify the methods for measuring K and Δ in two categories: Astronomical or geodetic, as those employing the

[1] Harkness, Append. III., Washington Obs., 1885. Bibliography in Recherches sur la Constante de Gravitation, Burgess, 1901; pub. by Hermann, Paris.

plumb-line, the pendulum and any method depending upon the variation of the acceleration of gravity; and the methods employing some form of the torsion balance, the chemical balance and the metronome pendulum.

For the first, use is made of a considerable extent of ground and includes a geological survey of the rocks in the neighborhood. In the second, measurements are made in the laboratory upon known masses and distances.

The oldest measures belong, in spite of the fact that the work of Cavendish dates from the eighteenth century, to the first category. In this first group quantities are dealt with which have always been very uncertain—uncertainties of the same order as those involving the investigation of the density of the outer strata of the earth.

In the second category, all the accuracy that may be obtained is subordinate to measurements of very small length and sometimes of time, and to adjustments for symmetry.

Let us examine these various methods for the determination of our constants. We will first consider the observations based on astronomical or geodetic methods. It was Bouguer who first tried to establish such a determination by measuring the attraction of a mountain. Maskelyne and Hutton in 1775 were the first to obtain results by this method.

Two processes exist for the determination of the attraction of a mountain: In the first, the attraction of the mountain on a plumb-line is measured by joining a point on the north to a point on the south by triangulation. The latitudes of these points are determined astronomically, and their positions, compared to those furnished by the triangulation, give the value of the deviation of the plumb-line.

The second process, as does the first, requires a study of the rocks that compose the mountain and a determination of the diminution of the acceleration of gravity at the summit of the mountain as compared with measurements made at the base reduced to sea level.

This same principle of the variation of the acceleration due to gravity is applicable to measurements made in mines, as was first done by Airy.

Another method depending upon the attraction of great terrestrial masses was proposed by Thomson and Tait. This would be to

measure the variation of g at the seashore where there are very high tides as in the Bay of Fundy.

Coming to what we may call the indoor methods, Wilsing made use of a metronome pendulum of very long period, swinging at first alone and then beside a very heavy mass. This method is susceptible of more accuracy than is given by the ordinary use of a pendulum.

Such scientists as Von Jolly, Poynting, Koenig, Richarz and Menzel have employed the chemical balance for this determination. Von Jolly's apparatus was installed in a tower 21 meters high. He measured the change in weight produced in a spherical mass of mercury of 5 kg. by a lead mass of 5,775 kg. placed directly beneath, while another 5 kg. mass remained in the other balance pan at the top of the tower. This method necessitates a knowledge of the variation of the acceleration of gravity for the place of observation—a variation that is never in accord with the theoretical calculations—also changes of temperature played a most important vitiating rôle.

Poynting made two series of determinations of which the second is an improvement over the first. He had a long-arm balance of considerable sensibility which he further increased by the use of a mirror suspended by two threads: the one fixed, the other attached to the arm of the balance. The masses could be hung at two levels to eliminate the attraction of the balance-arm. The attracting mass was placed below the balance on a turn-table which a fourth smaller mass kept from tilting. The suspended masses weighed 21.5 kg. and the attracting mass 153 kg. Two independent determinations made with the same apparatus gave him 5.52 and 5.46 for Δ. The balance-beam rested continuously upon its knife-edge during a series; the temperature variations were a minimum.

Recently, Richarz and Krigar-Menzel have used the chemical balance in the following way: A rectangular mass of lead of 100,000 kg. carried a balance to which could be suspended, either above or below the lead mass, mercury spheres of 1 kg. hung from the balance-beam which was was 23 cm. long with a half-period of ¾ min. It was necessary as in Jolly's case to determine the variations of g with the height, both with and without the presence of

the lead mass. The change in weight was determined by displacing the mercury spheres alternately from the upper to the lower positions. To obtain a precision of 0.1 per cent. it would be necessary to measure a weight of 0.0001 mg.

The order of magnitude of the quantities measured was 1 mg. The greatest pains were taken to obviate suspected sources of error, but the balance was raised and lowered from its knife-edge during a series.

A balance that turns about a vertical axis may be rendered more sensible than a balance oscillating about a horizontal axis. It is for this reason that Cavendish's balance and its various modifications have been the most often used to determine the gravitation constant. This method possesses still another advantage: that of giving us the value of this constant independently of the local value of the acceleration of gravity. Besides here we do not have to guard against dust whose effects may be appreciable in Richarz's method. In the torsion method, the most serious sources of error arise from uncertainties caused by variations of temperature which give rise to convection currents, and to the imperfect elasticity of the suspension wire.

The classic set-up for the method is the following: Two small masses are hung from the extremities of a horizontal beam supported by as fine a wire as possible. Two heavy masses attract the others. The deflection produced in the zero position of the movable system by a known change in position of the heavy masses is sought. This angle, the geometrical data, the time of oscillation, the moment of inertia and the value of g being known, Δ and k may be computed.

Cavendish made his investigations in 1793 and although Baily, with a large subsidy from the British Government, took up the matter in 1841, the result of his two thousand observations is of a much less importance than that of Cavendish, who showed the important rôle played by small changes of temperature in the apparatus and its vicinity. In 1873 Cornu and Baille determined the influences of a resisting medium on a torsion balance and showed that the resistance of the air for very small velocities is rigorously proportional to the velocity. These scientists also showed the advantage in re-

ducing the dimensions of the apparatus, that by this its sensibility was in nowise reduced, rendering also its behavior more constant. They were the first to make use of a chronograph for measurements of this nature.

Boys, who commenced his measurements in 1889, made an apparatus of almost microscopic dimensions. Cavendish's balance-arm was 180 cm. long, Cornu's 50 cm., but Boys' only 2.3 cm. The masses were proportionally reduced. In the Cavendish apparatus the attracting spheres were 30 cm. in diameter, while those in Boys' apparatus were but 6 cm. in diameter. His maximum deflection was about one degree, the couple to be measured 200 times as small as that of Cornu's and 1,500 times as small as Cavendish's. It is probable that Boys went too far with such very small dimensions, for it may happen that the errors arising from the geometrical measurements and from want of symmetry play a considerable part. In discovering how to make quartz fibers and their elastic properties he acquired precious auxiliaries, the non-homogeneity of the suspension wire having always been a source of great uncertainty. In order to increase still more the sensitiveness of the system, Boys placed his two pairs of spheres at different levels, thus avoiding the attraction in opposite sense of each of the large upon the more distant small sphere. An advantage gained by the smallness of dimensions in this apparatus was to render minimum the perturbations produced by air currents due to convection or temperature changes.

Doctor Carl Braun is the first who has succeeded in operating his apparatus in vacuo. The attracting masses are of mercury of 900 grams each, placed at a distance of 40 cm. The attracted masses weigh 54 grams and are attached at either end of an aluminium arm of 24.6 cm. length. To obtain the results already published he made use of a brass wire which he is at present replacing by quartz to continue his researches, in which he operates by two methods: the classic method of deflection and the method of oscillation, which consists in noting the time of oscillation for the positions of maximum and minimum attraction of the heavy masses. Instead of using a scale and telescope placed at several meters from the apparatus, his telescope is but a few decimeters distant. The

measurements of deflection are made by means of a scale of a few centimeters length and an ocular, and the luminous beam undergoes several reflections and refractions which may introduce indeterminable errors of considerable size.

The balance of Eötvös can be turned horizontally as a whole and the measurements of the time of oscillation for the maximum and minimum positions between two quadrangular pillars which are the attracting masses are then taken. The sensibility of this apparatus is such that it is capable of detecting the variations in the horizontal component of gravity at a point distant only 50 cm. from another. A quartz fiber is not used.

SUMMARY OF EXPERIMENTAL RESULTS.

1. *Astronomical and geodetic methods.*

Observer.	Station.	Result.	Date.
Maskelyne and Hutton.	Schehallen, Scotland.	4.71 and 4.45	1775
Carlini.	Mt. Cenis, Italy.	4.77, 4.95, 4.84	1821
Airy.	Harton, England.	6.57 and 5.48	1854
James and Clarke.	Arthur's Seat, Scotland.	5.32	1855
Pechmann.	Gerold, Alps.	6.13	1865
Mendenhall.	Fuji-yama, Japan.	5.77	1880
Sterneck.	Pibram, Bohemia.	4.77	1883
	Freiberg, Saxony.	6.77	1885
Preston.	Haleakala, S. I.	5.57	1887
	Mauna Kea.	5.13	1892

Mean : 5.60 ± 0.13, $p = 0$.

(Without Sterneck's second value.)

2. *Wilsing's pendulum* 5.5579 ± 0.018, $p = 1$ (1889).

3. *Observations with chemical balance.*

Observer.	Result.	Date.
V. Jolly.	5.692 ± 0.068	1881
Poynting.	5.493[± 0.03]	1891
Richarz and Krigar-Menzel.	5.505 ± 0.009	1898

Mean : 5.507 ± 0.014, $p = 2$.

4. *Observations with tension balance.*

Observer.	Result.	Date.
Cavendish.	5.45 [± 0.05]	1798
Reich.	5.49 ± 0.023	1837
	5.583 ± 0.015	1852
Baily.	5.67 or 5.55	1843
	[5.55 ± 0.05]	
Cornu and Baille.	5.50 and 5.56	1878
	[5.54 ± 0.02]	
Boys.	5.5270 ± 0.0019	1895
	[5.5226 ± 0.0041]	
Braun.	5.5273 ± 0.0012	1896
Eötvös.	5.53 [± 0.010]	1896

Mean: 5.5243 ± 0.0009, $p = 300$.

The mean, deduced from all these series, treated by the method of least squares, is:

$$\Delta = 5.5247 \pm 0.0013.^{1}$$

The corresponding value of the gravitation constant is:

$$K = 666.07 \cdot 10^{-10} \quad \overline{\text{cm.}^3}/\text{gr.}\ \overline{\text{sec.}^2} \pm 0.16 \cdot 10^{-10}.$$

Let us consider the above series:

Series I.—There is great uncertainty in the value found for the mean density by these methods for reasons already indicated, and the mean of all these measurements has a probable error of 4 per cent. In fact, it would seem futile to attempt exact measurements by such methods, by reason of the uncertainty of quantities, such as the density of mountains and the superficial strata of the earth, which enter into the equations. Also, no great progress has been made here since the first attempts.

Series III.—There are but three determinations by this method but their respective weights increase greatly from the first to the last. It is probable that the figure found by Richarz and Krigar-Menzel is as accurate as can be hoped to be obtained by our chemical balances of to-day. As in the first series, the problem is complicated by the variation of g with elevation. Richarz found a difference of 0.5 per cent. in this variation for his two determina-

[1] The probable error is indicated.

tions at an interval of two years—introducing a like uncertainty into his results.

Series IV.—We have here a considerable number of results which are closely grouped about the mean. The quantities in brackets are the estimated values and probable errors. Baily made a constant error in the interpretation of his results; the corrected value is given. In Boys' case the value in brackets is the result deduced from all of his series and not from the four highest as he deduced it. The most recent observations by this method—those of Boys and Braun—are practically identical with each other and with the mean deduced from all observations.

If we consider the results of the last two series for the last twelve years, we see that the chemical balance gives a value 0.5 per cent. lower than that obtained by the torsion balance for the mean density. Nevertheless, taking all things into account I think it may safely be said that we know the mean density of the earth and the gravitation constant to well within 0.001 part of their respective values, as indicated above.

If the weight assigned to the torsion-balance determinations be reduced from 300 to 3, we have:

1. Wilsing's pendulum.....	$\Delta = 5.5579$	$p = 1$
2. Chemical balance.....	5.507	$p = 2$
3. Torsion balance.....	5.5243	$p = 3$
Mean.....	$\Delta = 5.5241$	

This result is identical with the previous one.

It would seem desirable to make a new determination by Wilsing's method.

UNIVERSITY OF CALIFORNIA,
February, 1902.

THE VELOCITY OF IONS FROM HOT PLATINUM WIRES. II.

By C. D. Child.

THE second part of this article deals principally with the discharge from a hot platinum wire in other gases than air and in a vacuum. The work is not at all a complete investigation, but I shall not be able to continue it further at present, and therefore publish the experiments which have already been performed.

But first I wish to describe a few experiments with the particles driven from the wire. The presence about the wire of particles which diminish the velocity of ions has already been shown. It was noticed that the copper wires to which the platinum was attached became oxidized and that there was thus a possibility that particles of copper oxide were producing the effect. The copper wire was therefore removed and only platinum wire was used in the enclosed tube. The decrease in the amount of discharge was then even more marked than before.

A second test regarding the same point was then made by heating the copper wire and then examining the air about it for the presence of nuclei in the formation of clouds. Such nuclei were indeed found when the copper was sufficiently heated, but the amount of current which had been used in former experiments was not sufficient to heat the copper which was used, so as to give off such particles. This seems to be sufficient to show that those which produce the decrease in the rate of discharge came from the platinum itself.

The effect of these particles on the ions coming from a flame was examined. Several turns of platinum wire were placed between pieces of wood about 1.5 cm. apart, and 10 cm. long. The air arising from these wires was passed between two pieces of tin of the same length and the same distance apart and 6 cm. in width. These were kept at a potential difference of a hundred volts. They

were for the purpose of taking any ions out of the air arising from the platinum. The air was then allowed to pass between a flame kept at 50 volts and a circular plate 5 cm. in diameter surrounded by a guard ring. The plate was about 2 cm. from the flame.

A charged plate was first put in the place of the flame and no discharge was noted when the wire beneath was heated. This showed that the pieces of tin were properly serving their purpose of drawing the ions out of the ascending air.

The effect of the ascending air on the discharge from the flame was then noted. When the flame was charged positively and the wire beneath was not heated, the rate of discharge was 8×10^{-10} ampère. When the wire was heated it was 4×10^{-10} ampère, showing a decrease of 50 per cent. caused by the particles arising from the platinum. When the flame was discharged negatively, the rate of discharge was 12×10^{-10} ampère, and with the wire heated 3.2×10^{-10} ampère, a decrease of 83 per cent. This experiment was modified in various ways, but the decrease in the velocity of the negative ions was always greater than that of the positive.

It seems most reasonable that this should be so. Discharge occurs from positive platinum wires at a lower temperature than from a negative one. Apparently there is greater attraction between the platinum and negative ions than between platinum and positive ions. If this be true, we must expect a greater attraction between the particles of platinum given off and the negative ions. This experiment showing the effect of the particles on the ions from a flame bears out this idea.

And it also seems reasonable that this should be the explanation for the fact that in case of discharge from hot wires the negative ions move more slowly than the positive. In most cases of discharges through gas the negative ions move more rapidly. Some explanation of the behavior here is desirable. If the particles which are driven off have a greater affinity for the negative ions, they will naturally load them to a greater extent than the positive and cause them to move more slowly. As will be shown shortly, we have reason to suppose that the particles of platinum are not driven off to so great an extent in hydrogen as in air and there we find in general that the negative ions move the more rapidly. This fact

also indicates that the presence of the particles is the cause of the slow movement of the negative ions.

The phenomena could also be explained by assuming that the ions are formed from different substances and that the positive ions are given off from some substance which produces more rapidly moving ions. Indeed for some time I supposed that some such explanation was the correct one, but in view of all the facts which have been observed it does not seem as reasonable an explanation as the view given above.

This may throw some light on the fact that the positive ions drawn from the arc move more rapidly than the negative ones, but I hope to publish soon some experiments on the discharge from carbon and the discussion of that point may properly be left until then.

Discharge in Hydrogen.—It has been found that platinum does not decrease in weight when heated in hydrogen.[1] The discharge from a wire surrounded by hydrogen was therefore examined, but it gave decided evidence of the presence of particles which loaded the ions and diminished their velocity. The rates of discharge with different currents through the wire are given in Table XV. The potential difference was 30 volts. The same cylinder and the same glass tube were used in these experiments that have been described in connection with Fig. 5, Part I.

TABLE XV.

Current through Wire.	+ Discharge.	− Discharge.
6.5	$.07 \times 10^{-8}$	
7	.35 "	
7.5	.63 "	1.2×10^{-8}
8	1.05 "	7.1 "
8.5	1.5 "	8.8 "
9	1.5 "	8 "
9.5	1.5 "	3.4 "
10	1.5 "	1.5 "

In attempting to record a higher temperature the wire was melted.

The positive discharge occurs at lower temperatures than the negative as in air. This may possibly be due to a film of oxygen

[1] Wied. Ann., 35, 107.

about the platinum which hinders the negative discharge from passing from the wire, but this does not seem a probable explanation since the wire was heated to nearly white heat in the hydrogen for thirty minutes without producing any change in this respect. However in view of the difficulty which Spiers[1] has shown to exist in getting rid of the oxygen film one can not be positive that thus heating the wire would have the desired effect.

The negative discharge is in general larger than the positive. There was, however, a very great decrease in the amount of discharge when the current through the wire was increased from nine ampères to ten. In fact the negative discharge became as small as the positive. This seemed surprising, and another wire was substituted for the one which had been melted. This second piece required a little more current to bring it to the desired temperature, but it showed exactly the same peculiarities as the first.

The wire was heated for several minutes at as high a temperature as possible, but the discharge with the smaller current continued to be as large as ever and to decrease again when the temperature was raised.

In air the rate of discharge when the wire was first heated was larger than it was after the current had been passing for some time. I endeavored to show that the same was true in hydrogen, but the needle of the galvanometer which I was using for these observations required about fifteen seconds to come to rest, and by that time the discharge had reached a constant value.

Though other experimenters have found no decrease in the weight of platinum when heated in hydrogen, it seemed well to test the matter again, using a current as large as that which produced the smaller rate of discharge. But when the wire was heated for about twenty minutes this piece was also melted. The two parts were, however, weighed and their weight was found to be the same to within .1 mg. as that of the wire before it had been heated. It did not seem profitable to repeat further the work which has already been performed on this point.

But this does not, of course, prove that no particles are driven off, since we know that they are driven off when the wire is in air at

[1] Phil. Mag. (5), 49, 84.

mperatures too low to cause any appreciable loss in weight. The ydrogen about the wire was, therefore, examined for nuclei with ater condensation. Moist hydrogen was passed into the tube and nen suddenly drawn off and the cloud formation was as noticeable s in air, even though the wire was not heated above red heat. After the condensation had been produced a few times it ceased to be visible until the wire was again heated. Evidently the formation of cloud is a more delicate test of the presence of particles than either the loss in weight or the decrease in the velocity of the ions.

This experiment seems to make it probable that the decrease in the velocity of the ions was caused by particles driven off from the platinum and loading the ions. The particles probably are not driven off to as great an extent as they are in air, since no loss of weight has been detected, and since the velocity of the ions in the hydrogen is much greater than it is in the air.

The fact that the negative discharge in the hydrogen was greatly affected while the positive was not indicates the same thing. If a new kind of ionization was being produced at the higher temperatures, we should expect some indication of its presence in the positive discharge. On the other hand we have already had evidence to show that the particles driven from the platinum have a greater attraction for the negative ions than for the positive, and this would easily explain the phenomena as they here occur.

The particles sent off in hydrogen are no doubt platinum and not an oxide. It would seem highly improbable that a platinum oxide would be formed in the presence of hydrogen. It seems, therefore, probable that the particles sent off in the air are also particles of platinum instead of any oxide of platinum.

Observations were taken with different potential differences and the rate of discharge was found to vary much as it did in air.

This fact seems to show us that the velocities may here be computed as they were in air. When this is done the highest value of K, the velocity of the positive ions, as computed from Table XVIII. was found to be 32.6 cm. per second as compared with 4.5 cm. in air, and that of the negative ions 195 cm. as compared with 3.5. These values seem to be very large, but the data taken certainly indicate such velocities.

Discharge in Oxygen, CO_2 and CO.—With other gases than hydrogen the decrease in the rate of discharge from its first value was very much the same that it was in air. The discharge in oxygen became even smaller than in air enclosed in the same tube. The velocity of the ions was apparently only .2 cm. per second, but no importance can be given to this value, since it depended to so great an extent on the size of the tube and length of time that the wire was heated.

The discharge in CO_2 behaved very much the same as it did in air. In CO the discharge did not decrease as much as in air, but still the behavior of the discharge in this gas was much more similar to its behavior in air than to that in hydrogen. No attempt was made to secure any of these gases chemically pure except hydrogen. In that case no difference was seen between the discharge in hydrogen made from commercial HCl and zinc, and that made from chemically pure HCl and zinc.

Discharge in a Vacuum.—I then proceeded to examine the rate of discharge in a vacuum. It has been shown by others that the rate of discharge from a hot platinum wire increases as the pressure of the surrounding gas is diminished. This was first verified, but in doing so it was found that the discharge at any given pressure of the gas was greatest when the wire was heated for the first time at that particular temperature. In one of the first experiments with the pressure of about 1 mm., with a current of 5 ampères passing through the wire, and a potential difference between the wire and the cylinder of 40 volts the discharge at the beginning was fully 8.4×10^{-7} ampère. This fell in a few minutes to 4.2×10^{-8} ampère, only $\frac{1}{20}$ of its first value. These measurements were taken with a galvanometer whose constant was 4.2×10^{-8} ampère per scale division.

At first sight this would seem to be nothing different from what was found at atmospheric pressure. We there found the rate of discharge to diminish as the wire was heated in a closed tube for some time. The same thing occurs here but to a more marked degree. But further examination seemed to indicate that this phenomenon was essentially different from the other. Allowing the air to enter the tube and then drawing it off did not at all bring the

wire back to its original condition. It no longer gave off as much discharge as it did in the beginning. Allowing the wire to remain over night undisturbed did not restore its power of discharge.

Effect of Hydrogen on the Wire.—Moreover heating the wire in a flame from gasolene restored it partially to its original condition and heating it to a white heat in hydrogen almost entirely restored it. But before any exact measurements were taken with this wire it was accidentally melted and another one was put in its place. The second wire behaved quite differently from the first. The rate of discharge was but little larger when it was first heated in a vacuum from what it was afterwards. Heating it in hydrogen had but little effect upon it. Both moist and dry hydrogen were tried, but neither of them affected the wire. It seemed as if some mistake must have been made in the first observations, but upon trying different pieces of platinum it was found that there was much variety in their behavior.

Platinum was accordingly secured from three different sources. It all contained more or less iridium. Two pieces which were said to contain approximately 1½ per cent. and 5 per cent. of iridium showed a decrease from about 7×10^{-7} ampère to 1.7×10^{-7} ampère when first heated, the potential difference being 50 volts and the current through the wire 5 ampères. The third, containing approximately 10 per cent. of iridium gave a discharge at first of over 8.4×10^{-6} ampère, and then decreased to about the same as the others. All three of these were brought back to nearly their original condition by being heated in hydrogen and they were not affected by being heated in air.

All of the wires were heated in the air at atmospheric pressure for a few minutes before they were heated in a vacuum, so that we would expect any dirt to have been burnt off. But even if one were to assume that the effect was due to some impurity in the wire this alone would not explain the fact that the hydrogen had so great an effect in restoring the wire to its original condition.

A piece of wire having 5 per cent. of impurity, the kind of impurity not being given, was then tried and this was found to have a greater rate of discharge at the beginning than any of the others and a greater subsequent decrease. Heating it in hydrogen produced a

large increase in the rate of discharge, although it lacked much of bringing it back to its original condition.

A piece of wire having 10 per cent. of impurity showed a smaller rate of discharge at first and a smaller change from this than the wire containing only 5 per cent. of impurity.

No attempt was made to go further with this part of the work. The difference in behavior in the different pieces of wire may have been due to a very slight impurity or possibly to gases previously absorbed by the wire. It would probably be a long and tedious piece of investigation to trace out the cause of this behavior and I am unable at present to perform it.

Experiments with Palladium.—Palladium is known to absorb greater quantities of hydrogen than platinum. A few experiments were, therefore, performed with it. But palladium melts at a lower temperature than platinum and on that account no very satisfactory results could be obtained. It melted before any negative discharge was produced and it seemed to be especially liable to melt when heated in hydrogen. As far as was found palladium behaved the same as platinum. The discharge was largest at first, decreased after a short time and was largely restored by heating in hydrogen. The final rate of discharge from it was about the same as that from platinum.

The effect produced by heating the wire in hydrogen throws some light on the mechanism of discharge, but the discussion of this will be deferred until other experiments have been described.

Observations were then made on the rate of discharge in a vacuum under varying conditions. No great exactness can be expected here since the rate of discharge depends to such a large extent upon the length of time that the wire has been heated in a vacuum, but after a few minutes the rate of discharge became reasonably constant and observations could be made of the effects produced by varying the other conditions.

The Current Varied with Wire in Vacuum.—The first quantity varied was the amount of current flowing through the wire. The potential difference was 45 volts and the pressure of the air 1 mm. The other conditions were the same as in the preceding experiments. The results of these observations are given in Table XVI. Column

1 gives the amount of current flowing through the wire, column 2 the positive discharge measured in ampères, column 3 the negative.

TABLE XVI.

Current through Wire.	+ Discharge.	− Discharge.
4.3	$.42 \times 10^{-6}$	$.21 \times 10^{-6}$
4.5	.63 "	1 "
4.8	1.6 "	3.7 "
5	3.3 "	10 "
5.2	6.6 "	22 "

With larger currents the wire was liable to melt. The same current, of course, produces a higher temperature in a vacuum than in air.

The negative current is larger under these conditions. It is also noticeable that the effect of increasing the current is not the same when the wire is in a vacuum that it is in air at ordinary pressures. In air the discharge soon reached a maximum, while here it continually increases.

Potential Varied.—A still greater difference between the phenomena in air and in a vacuum was shown when the potential difference between the wire and the cylinder was varied. Table XVII. gives the rates of discharge with different potential differences. The current was 5.2 ampères and the other conditions were the same as in the preceding experiment. Column 1 gives the potential difference between the wire and the cylinder, column 2 the rate of positive discharge measured in ampères, column 3 that of the negative.

TABLE XVII.

Potential Difference.	+ Discharge.	− Discharge.
45	4.7×10^{-6}	19×10^{-6}
36	3.6 "	15 "
26	3.6 "	10 "
18	3.1 "	8.4 "
9	2.9 "	8.4 "
5	2.5 "	8.4 "
1.5	1.9 "	8.2 "

In the last reading the ground was connected to one end of the wire running into the tube. A shunt could have been placed about

the wire and different points on this grounded, but this would have made one part of the wire positive and the rest negative. No conclusion could have been drawn from observations thus made.

The current here decreased but very little while the potential difference was changed from 45 to 1.5 volts. This was entirely different from the decrease shown when the potential difference was diminished at atmospheric pressure. The discharge at that time decreased very rapidly.

An attempt was made to determine the potential gradient between the hot wire and the surrounding cylinder by means of a second wire. The difficulty to in finding the potential by this means has already been pointed out[1] and in this case the difficulty appeared to be insuperable. All of the space within the tube becomes filled with ions. Moreover the surface of the insulator within the tube ceases to some extent to insulate when the discharge is taking place.

The observations taken seemed to indicate that the potential about the wire approaches the potential of the wire when the pressure of the gas is diminished, but after some study of the leakage to the exploring wire, I decided that it was impossible to make any definite statement concerning it.

Such a change in the potential as that which appeared to take place, could be explained by supposing that all of the space within the tube becomes partially ionized. Even at atmospheric pressures the air becomes ionized to more than molecular distances and the same thing may occur to a much greater extent in a vacuum. But it would be useless to attempt to decide this point, until we have more definite knowledge of the condition of the gas.

The Velocity of Ions in a Vacuum.—The way in which the discharge varies when the potential difference is varied is of itself sufficient to show us that the mechanism of discharge is not the same in a vacuum that it is in air at ordinary pressures. Either the velocity of the ions is so great that their presence does not greatly affect the potential about the wire, or the ionization occurs throughout the gas. In either case the task of finding the velocity of the ions in a vacuum is very difficult. The velocity can not be determined by blowing air past the wire. And until we know whether

[1] PHYS. REV., 12, 67.

the ionization all takes place at the surface of the wire or throughout the gas the employment of an alternating potential difference would give us no definite knowledge.

I had intended to use the method depending upon the determination of the rate of discharge and of the potential gradient about the wire. But evidently we have no right to assume that the potential gradient is the same here that it is in air at higher pressures. Even when the potential difference was only 1.5 volts the rate of discharge did not vary at all as it did at atmospheric pressure.

The water dropper can not be used to determine the potential gradient and the attempt to determine this by a wire has been described.

The method which has been used by J. J. Thomson[1] in finding the velocity of the ions could perhaps be used, but it would require much care. Among other measurements the kinetic energy of the ions carrying the discharge must be determined. As far as can be foreseen this would be much smaller in the case of discharge from a wire than in the case of cathode discharge. The work might possibly be performed, but I have made no attempt whatever to do so.

The Vacuum Varied.—When the attempt was made to secure a series of readings with different pressures of the gas, it was found that no two sets were ever identical. In the first place the work was complicated by the cooling effect of the convection currents about the wire. The same current through it raises it to a higher temperature when there is less gas present. In making observations the temperature should be kept constant. In order to do this a high resistance galvanometer was place in shunt with the wire, and the potential difference at the ends of the wire was measured by the galvanometer. It was assumed that the temperature would be constant, if the resistance were constant, and the resistance is proportional to the ratio of the potential difference measured by the galvanometer and the current through the wire measured by an ammeter. In making the observations this ratio was, accordingly, kept constant.

But even with this precaution I secured widely different sets of readings. At atmospheric pressure the discharge depends largely upon the number of particles about the wire, and with very low

[1] Phil. Mag. (5), 44, 302.

pressures it appears to depend on the amount of occluded hydrogen. These probably were the causes of the wide divergence in the data secured. However, the data which were taken in one case are given in Table XVIII. At each pressure of the gas the wire was heated until the discharge appeared to be constant.

It was found that with lower pressures the particles did not appear to check the motion of the ions. At least the discharge did not then become smaller when the wire was heated for some time. Thus for the particular wire used for securing the data given in Table XVIII. the discharge decreased with time until the pressure was diminished from 50 cm. to 40 cm. After that it continued to be large without regard to the length of time that the wire was heated.

Different pieces of wire behaved differently in this respect as in other respects, some of them apparently sending out the particles at lower pressures than others. In fact it became apparent that it would be useless to attempt to secure exact data on any of these phenomena without a thorough study of the impurities that may exist in the wire.

Column 1 in Table XVIII. gives the pressure of the gas in cm. of mercury. Column 2 the current through the wire, column 3 the positive discharge in ampères, column 4 the negative. The potential difference between the wire and the cylinder was 30 volts. The cylinder about the wire was the same that had been used in preceding experiments.

TABLE XVIII.

Pressure of Gas.	Current through Wire.	+ Discharge.	− Discharge.
72	6.2	$.056 \times 10^{-9}$	$.03 \times 10^{-9}$
50	6.0	.086 "	.05 "
40	5.9	.58 "	.30 "
30	5.8	1.4 "	.84 "
20	5.7	3.1 "	3.7 "
10	5.7	6.3 "	10. "
5	5.6	14. "	25. "
2	5.5	28. "	50. "
1	5.4	50. "	160. "
.5	5.3	160. "	360. "

These data agree with those of others in showing that the negative discharge is greater than the positive when the wire is in a vacuum and heated sufficiently high.

A smaller tube was hermetically sealed and the gas exhausted until cathode rays were produced. The behavior of this tube was but little different from the preceding. The first discharge was very large and continued to decrease for a long time, the final value being little larger than the value it had at 1 mm. pressure. The size of the cylinder also had little to do with the amount of discharge.

The Mechanism of Discharge.—It has been suggested that the discharge is caused by dirt on the wire. This may have some effect on the large discharge produced when the wire is first heated, but it hardly seems possible that it can be the only cause of discharge. Rubbing dirt on the wire before it was heated seemed to produce no effect on the rate of discharge. The different wires did, indeed, behave differently, and yet if the whole effect had been produced by dirt, the phenomena would not have been at all what they were.

A second explanation is one suggested by Berliner[1] that the discharge is produced by occluded hydrogen. If we modify this statement and say that the positive discharge is largely produced in this way, much may be said in favor of the suggestion. At lower temperatures the ionization appears to be produced within or very near to the surface of the metal. This part, at least, of the ionization may be produced by the occluded hydrogen.

This explains the fact that the discharge is so much larger when the wire is first heated in a vacuum. The hydrogen which is driven out from the wire would be largely ionized. Heating the wire in hydrogen would, of course, largely restore to it the power to produce a large discharge, and heating it in air would not.

We would expect the gases within the metal to be ionized at the lower temperatures. At higher temperatures the gas surrounding the wire would begin to be ionized. This would produce the ionization which we found could be blown from the wire. This ionization about the wire would allow the negative discharge to occur, since both positive and negative ions would be produced beyond the surface layer in which the fall of potential is supposed to exist.

The ions from the occluded hydrogen would undoubtedly move more rapidly than those produced in the air. In the case of dis-

[1] Wied. Ann., 33, 289.

charge produced by X-rays the positive ions of the hydogen move more rapidly than the negative of other gases. By supposing that the more rapidly moving positive ions are ions from the occluded hydrogen and that the slower ones are from the air, several other facts could be explained, as, for example, that the average velocity of the ions becomes less at higher temperatures and that the velocity of positive ions in air is greater than the negative, but the action of the particles driven off from the wire would also explain these facts, and it seems more probable that their action is the predominant one.

Possibly the particles which are driven off may ionize the air about the wire and are thus the cause of the slower ions, first, directly, by loading ions already formed, and second, indirectly, by producing a new class of ions.

The fact that a negative discharge in a vacuum is affected by the length of time that the wire is heated shows that the negative discharge is not all produced by the ions about the wire. Apparently some of the discharge is caused by something coming from within the wire.

In all probability neither the positive nor the negative discharge is produced by any one cause. It seems safe to say that the positive ions produced at the lower temperature come from within the wire, and that some at least of the negative ions are produced in the gas about the wire. It is also safe to say that some of the ions that come from within the wire are due to occluded hydrogen, and that particles driven off from the wire produce some of the phenomena. Probably the different causes enter in different amounts with different pieces of wire.

Summary.—The particles producing the decrease in the velocity of the ions come from the hot platinum and not from the copper connections.

The particles from a hot platinum wire have a greater retarding action on the negative ions drawn from a flame than on the positive.

In hydrogen the rate of discharge is much larger than in air. The positive discharge still begins at lower temperatures than the negative. The negative soon surpasses the positive, passes through a maximum and rapidly decreases, coming to have about the same value as the positive when very near the melting point. The

presence of particles about the wire after it had been heated was also shown by the cloud formation when the hydrogen was suddenly cooled, but no loss in the weight of the wire was detected. Apparently particles are driven off, but to a much smaller extent than in air. The largest value for K_1 was found to be 42.6 cm. per sec. for a potential gradient of 1 volt per cm., as compared with 4.5 cm. in air and for K_2 195 cm. as compared with 3.6 cm. As in air the rates of discharge increased very rapidly as the potential differences were increased.

The behavior of the wire in oxygen, CO_2 and CO is practically the same as in air.

The experiments seemed to indicate that the particles are driven off as platinum and not as some oxide of platinum.

When a platinum wire is first heated in a vacuum the rate of discharge is much greater than at any time afterwards. Heating the wire in hydrogen partially restored to it the power of producing a large discharge. But this did not produce any effect on the negative discharge.

The rate of discharge is much greater in a vacuum than in air at atmospheric pressures.

The positive discharge at higher temperatures is less in a vacuum than the negative. The amount of both positive and negative discharges continually increase as the temperature of the wire is increased.

The potential difference between the wire and cylinder about it has little effect on the amount of discharge in a vacuum. In this respect it behaves entirely different from the wire in air at atmospheric pressures. The size of the surrounding cylinder has little effect on the amount of discharge. No method was found by which the velocity of the ions in a vacuum could be determined.

The phenomena indicate that the ionization first occurs within the wire, and there are some reasons for thinking that this part of the ionization is produced from occluded hydrogen.

CONTRIBUTIONS TO THE STUDY OF THE INDUCTION COIL.

BY JAMES EDMUND IVES

CONTENTS.

§ 1. INTRODUCTION.

THE induction coil has been investigated in recent years by Colley, Oberbeck, Walter, Johnson and others, both theoretically and experimentally. A list of the more important recent contributions to the subject is given at the end of this paper.

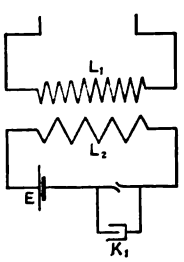

Fig. 1.

The induction coil is an electromagnetic system with two degrees of freedom. It is usually constructed by winding a *secondary* coil of a large number of turns of fine wire upon a primary of a few turns of heavy wire. (See Fig. 1.) The core of the primary may be filled with iron wires to increase its inductance. The terminals of the primary are so arranged that they can be conveniently attached to some source of electromotive force, and the primary circuit is provided with a make-and-break apparatus. A condenser of suitable capacity is put around the break, so as to diminish as far as possible the sparking at the break.

The usual method of operating the coil is as follows. A battery having been placed in the primary circuit, the circuit is opened and closed periodically. The condenser around the break is then ad-

justed so that the sparking at the break is reduced to a minimum. The secondary circuit can be left open, or can be closed through a vacuum tube, an electrolytic cell, or in any other way that is desired.

The action of the coil may be briefly stated. When the primary circuit is closed, the current begins to grow, and soon reaches what is practically its maximum value. This *make current* in the primary induces an electromotive force in the secondary, which, however is not of any importance. The primary circuit is now broken, and owing to its capacity, that is, to the capacity around the break, dies away in a series of oscillations, whose period is constant, but whose amplitude is rapidly decreasing. In a small induction coil they are so rapid that there are 1,000 to 10,000 of them in a second. These oscillations of the primary current set up an induced electromotive force in the secondary, which depends for its magnitude, as will be shown later, upon the constants of the coil, the initial strength of the primary current, the velocity of breaking the primary, and the capacity of the condenser around the break.

The usual method of using a coil is with a slow break, the primary being broken less than a hundred times a second. In this case the current in the primary rises to approximately its full value before the break occurs, and the oscillations due to the break die away before the next make. The current under these conditions would be represented by Fig. 2. This case will evidently be completely covered if a *single* break is considered.

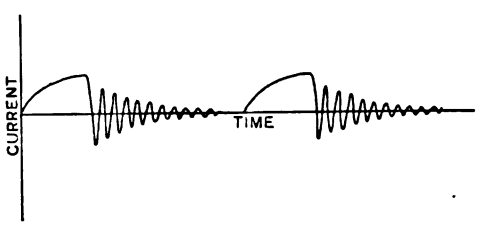

Fig. 2.

If the break is *not* slow, as in the case of the Wehnelt interruptor, the theory becomes more complicated, as the make current does not have time to rise to a steady value before the break occurs, and the break current also overlaps the next make current.

The two simplest cases will be treated in this paper, viz.: first, the case of a single break, and second, the case of a simple harmonic electromotive force in the primary.

§ 2. THEORY OF THE COIL.

In general both the primary and secondary circuits have inductance, resistance and capacity.

It will be seen from Fig. 1, that when the primary circuit is closed, its capacity may be considered infinite, but when it is broken, its capacity is that of the condenser K.

The capacity of the secondary circuit in the ordinary mode of using the induction coil is the distributed capacity due to the juxtaposition of successive turns of the coil. This, although very small, is not infinitesimal. This question of the distributed capacity of the secondary will be discussed more fully further on in this paper.

Theory of One Circuit for a Single Break.

Since the theory of a *single* circuit containing an inductance, resistance, electromotive force, and a condenser around the break is closely related to that of the primary of an induction coil, a brief discussion of the single circuit will be given before the discussion of two circuits. The same conditions will be supposed to hold as for the primary, viz: the breaks and makes will be sufficiently far apart so that the oscillations due to the break have died away before the make occurs.

We will then suppose the current to have risen to its maximum value before the break occurs, and will consider what takes place after breaking it.

The well-known equation for such a circuit is the following:

$$L\frac{dI}{dt} + RI + \frac{1}{K}\int Idt = E, \tag{1}$$

where I is the current; t, the time; L, the inductance; R, the resistance; K, the capacity around the break; and E the steady impressed electromotive force.

This equation may be written in the form

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{K} = E, \tag{2}$$

where q is the quantity of electricity which has passed any cross-section of the circuit during the time t or, what is the same thing, the quantity of electricity which has entered the condenser in the time t. If we bring E over to the left hand side of the equation, and write

$$q' = q - Q$$

where $Q = KE$ is the steady charge of the condenser under the electromotive force E, equation (2) becomes

$$L\frac{d^2q'}{dt^2} + R\frac{dq'}{dt} + \frac{q'}{K} = 0. \tag{3}$$

This is the well-known equation for a system with one degree of freedom.[1]

It is found that the charge is oscillatory or non-oscillatory according as

$$R^2 < 4\frac{L}{K} \quad \text{or} \quad R^2 > 4\frac{L}{K}.$$

In this discussion we shall consider only oscillatory discharges, since the induced electromotive force in the secondary of an induction coil due to a non-oscillatory discharge is so small that it is of no practical importance.

The solution for (3) is given by

$$q' = e^{\mu t}(A\cos \nu t + B \sin \nu t) \tag{4}$$

where

$$\mu = -\frac{R}{2L},$$

$$\nu = \sqrt{\frac{1}{KL} - \frac{R^2}{4L^2}}.$$

The arbitrary constants A and B will be determined by the initial conditions, which in the case we are considering are the following:

When $t = 0$, $q' = -Q$ and $I = I_0$.

[1] See Webster, Elect. Mag., pp. 484–488.

Whence it follows that

$$A = -Q$$

$$B = \frac{I_0 + \mu Q}{\nu}.$$

Therefore since $q' = q - Q$ we have

$$(5) \qquad q = Q - e^{\mu t}\left[Q \cos \nu t - \frac{I_0 + \mu Q}{\nu} \sin \nu t\right],$$

which may be written

$$(6) \qquad q = Q - e^{\mu t}\left[Q^2 + \left(\frac{I_0 + \mu Q}{\nu}\right)^2\right]^{\frac{1}{2}} \cos (\nu t + \alpha)$$

where

$$\alpha = \tan^{-1}\frac{I_0 + \mu Q}{\nu Q}.$$

For the current we have

$$(7) \quad I = \frac{dq}{dt} = e^{\mu t}\left[Q^2 + \left(\frac{I_0 + \mu Q}{\nu}\right)^2\right]^{\frac{1}{2}} [\nu \sin (\nu t + \alpha) - \mu \cos (\nu t + \alpha)]$$

and for the difference of potential between the two sides of the condenser

$$(8) \qquad V = \frac{q}{K} = E - \frac{e^{\mu t}}{K}\left[Q^2 + \left(\frac{I_0 + \mu Q}{\nu}\right)^2\right]^{\frac{1}{2}} \cos (\nu t + \alpha).$$

In the case which is usually considered, $E = 0$, and equations (6), (7), and (8) become

$$(9) \qquad q = \frac{I_0 e^{\mu t}}{\nu} \sin \nu t$$

$$(10) \qquad I = \frac{I_0 e^{\mu t}}{\nu} [\nu \cos \nu t + \mu \sin \nu t]$$

$$(11) \qquad V = \frac{I_0 e^{\mu t}}{K\nu} \sin \nu t.$$

Theory of one Circuit. Harmonic E.M.F.

If instead of the circuit being broken, we have impressed upon it a periodic electromotive force $E_0 \cos \omega t$ we shall have instead of equation (2)

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{K} = E_0 \cos \omega t. \tag{12}$$

The solution of which is given by

$$q = \frac{E_0}{\omega\sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}} \cos(\omega t - \alpha), \tag{13}$$

$$I = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}} \sin(\omega t - \alpha), \tag{14}$$

$$V = \frac{E_0}{\omega K\sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}} \cos(\omega t - \alpha). \tag{15}$$

V is the difference of potential between the two sides of the condenser. V_m the maximum difference of potential is given by

$$V_m = \frac{E_0}{\omega} \frac{1}{K\sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}}. \tag{16}$$

Theory of two Circuits for a Single Break.

The theory of the induction coil is the theory of two circuits. The well-known equations for two circuits, are[1]

$$\begin{aligned} L_1\frac{dI_1}{dt} + M\frac{dI_2}{dt} + R_1I_1 + \frac{1}{K_1}\int_0^t I_1dt &= E_1, \\ L_2\frac{dI_2}{dt} + M\frac{dI_1}{dt} + R_2I_2 + \frac{1}{K_2}\int_0^t I_2dt &= E_2. \end{aligned} \tag{17}$$

In the case of the induction coil, since there is no impressed electromotive force in the secondary, $E_2 = 0$. Also since the impressed electromotive force in the primary is usually small com-

[1] See Webster, Elect. Mag., pp. 491–502.

pared with the induced electromotive force in the primary, E_1 may be neglected. Then equations (17) may be written

$$(18)\qquad \begin{aligned} L_1\frac{d^2q_1}{dt^2} + M\frac{d^2q_2}{dt^2} + R_1\frac{dq_1}{dt} + \frac{q_1}{K_1} &= 0, \\ L_2\frac{d^2q_2}{dt^2} + M\frac{d^2q_1}{dt^2} + R_2\frac{dq_2}{dt} + \frac{q_2}{K_2} &= 0. \end{aligned}$$

Equations (18) are the equations for a system with two degrees of freedom, and no impressed forces.

It is well known from the theory of such equations[1] that the displacements q_1 and q_2, which are here quantities of electricity, may either die away without oscillations, or may die away with a series of oscillations. The first will happen if R_1 and R_2 are both very large. The second case is the case of the induction coil, and the one in which we are interested.

The solutions of these equations may be written down immediately, in the following form:

$$(19)\qquad \begin{aligned} q_1 &= e^{-\alpha t}(A_1\cos\beta t + B_1\sin\beta t) + e^{-\gamma t}(C_1\cos\delta t + D_1\sin\delta t), \\ q_2 &= e^{-\alpha t}(A_2\cos\beta t + B_2\sin\beta t) + e^{-\gamma t}(C_2\cos\delta t + D_2\sin\delta t). \end{aligned}$$

In which α, β, γ and δ are given by

$$(20)\qquad \begin{aligned} \alpha &= \frac{R_1L_2 + R_2L_1}{4m} + \frac{R_1[L_2n - 2K_1m] + R_2[L_1n - 2K_2m]}{4m\sqrt{n^2 + LM^2K_1K_2}}, \\ \gamma &= \frac{R_1L_2 + R_2L_1}{4m} - \frac{R_1[L_2n - 2K_1m] + R_2[L_1n - 2K_2m]}{4m\sqrt{n^2 + LM^2K_1K_2}}, \\ \alpha^2 + \beta^2 &= \frac{n + \sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}{2K_1K_2m}, \\ \gamma^2 + \delta^2 &= \frac{n - \sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}{2K_1K_2m}. \end{aligned}$$

Where

$$m = L_1L_2 - M^2,$$

$$n = L_1K_1 + L_2K_2.$$

[1] Webster, loc. cit., pp. 494–498.

In finding the values of α, β, γ, δ we have neglected the product $\alpha\gamma$ as small compared with the product $\beta\delta$. This is justified since α and γ are the reciprocals of the time constants of the two circuits and β and δ are the reciprocals of the periods multiplied by 2π, and the periods are small compared with the time constants.

Since equations (19) can be written

$$(21)\qquad \begin{aligned} q_1 &= e^{-\alpha t}A\cos(\beta t - \theta_1) + e^{-\gamma t}B\cos(\delta t - \theta_2),\\ q_2 &= e^{-\alpha t}C\cos(\beta t - \theta_3) + e^{-\gamma t}D\cos(\delta t - \theta_4), \end{aligned}$$

where

$$A = \sqrt{A_1^2 + B_1^2},\quad B = \sqrt{C_1^2 + D_1^2},\quad C = \sqrt{A_2^2 + B_2^2},\quad D = \sqrt{C_2^2 + D_2^2},$$

and

$$\theta_1 = \tan^{-1}\frac{B_1}{A_1},\quad \theta_2 = \tan^{-1}\frac{D_1}{C_1},\quad \theta_3 = \tan^{-1}\frac{B_2}{A_2},\quad \theta_4 = \tan^{-1}\frac{D_2}{C_2}.$$

we see that q_1 and q_2 and therefore the currents, are each sums of two damped harmonic oscillations. The coefficients, A, B, C, D, give the amplitudes of the component oscillations in each circuit.

The constants of equations (19) and (21) must be determined from the initial conditions, viz: when

$$t = 0,\quad q_1 = 0,\quad I_1 = \frac{dq_1}{dt} = I_0,$$

$$q_2 = 0,\quad I_2 = \frac{dq_2}{dt} = 0.$$

Boynton[1] has determined these constants for the case of the Tesla coil, and Colley, Oberbeck, Walter and Johnson have found approximate values for the case of the induction coil.

In the induction coil as ordinarily used there is no condenser in the secondary, and the only capacity it has is the distributed capacity due to the juxtaposition of the successive turns of the coil. This has been found to be very small in ordinary coils. If a coil is wound bifilarly the capacity is very much increased, and the inductance decreased.

[1] PHYS. REV., Vol. VII., pp. 35–48, 1898.

Max Wien[1] found for three coils that he examined the following results:

Coil No. 1. $R = 2921$ siemens,
$L = 3.97$ henries,
Distributed capacity = .00077 microfarads.

Coil No. 2. $R = 69$ siemens,
$L = .199$ henries,
Distributed capacity = .0019 microfarads.

Coil No. 3 (wound bifilarly).
$R = 135.1$ siemens,
$L = .767$ henries,
Distributed capacity = .0139 microfarads.

Walter[2] found for the secondary of an induction coil having an inductance of about 500 henries a capacity of only about 1.1 millionths of a microfarad; and for the secondary of a 60 cm. induction coil, having an induction of about 9,000 henries, a capacity of about 6.5 millionths of a microfarad.

Oberbeck[3] assumes that the capacity of the secondary of a small induction coil is that of a small Leyden jar, *i. e.*, about .0005 of a microfarad.

The theory of the distributed capacity of a coil has not yet been worked out, but it seems probable that the capacity is very small. The experiments which will be described later seem to indicate this.

Approximate expressions for the difference of potential in the secondary, and for the primary current have been obtained by neglecting small quantities.

Walter[4] and Oberbeck[5] have obtained the following expressions for the case of the open secondary.

[1] Wied. Ann., Bd. 44, p. 712, 1891.
[2] Wied. Ann., Bd. 66, pp. 628–630, 1898.
[3] Wied. Ann., Bd. 64, p. 204, 1898.
[4] Wied. Ann., Bd. 62, pp. 307–322; Bd. 66, pp. 623–635.
[5] Wied. Ann., loc. cit.

For the difference of potential between the terminals of the secondary they get

$$(22) \qquad V_2 = \frac{I_0 M}{L_2K_2 - L_1K_1}\left[\sqrt{L_2K_2}\, e^{-\frac{R_2}{2L_2}t} \sin \frac{t}{\sqrt{L_2K_2}} - \sqrt{L_1K_1}\, e^{-\frac{R_1}{2L_1}t} \sin \frac{t}{\sqrt{L_1K_1}}\right].$$

The periods of the component oscillations are evidently given by

$$(23) \qquad T_1 = \pi\sqrt{L_1K_1},\ T_2 = \pi\sqrt{L_2K_2}.$$

If we consider only the first few oscillations, the damping may be neglected and (22) becomes

$$(24) \quad V_2 = \frac{I_0 M}{L_2K_2 - L_1K_1}\left[\sqrt{L_2K_2} \sin \frac{t}{\sqrt{L_2K_2}} - \sqrt{L_1K_1} \sin \frac{t}{\sqrt{L_1K_1}}\right].$$

Three important cases may be distinguished according as L_1K_1 is large compared with L_2K_2; is equal to L_2K_2, or is small compared with L_2K_2.

In the first case (24) reduces to

$$(25) \qquad V_2 = -\frac{I_0 M}{\sqrt{L_1K_1}} \sin \frac{t}{\sqrt{L_1K_1}}.$$

In the second, to

$$(26) \qquad V_2 = \frac{I_0 M}{2\sqrt{L_1K_1}} \sin \frac{t}{\sqrt{L_1K_1}} = \frac{I_0 M}{2\sqrt{L_2K_2}} \sin \frac{t}{\sqrt{L_2K_2}}.$$

In the third, to

$$(27) \qquad V_2 = \frac{I_0 M}{\sqrt{L_2K_2}} \sin \frac{t}{\sqrt{L_2K_2}}.$$

Assuming that there is no magnetic leakage,

$$M = \sqrt{L_1L_2}$$

and we get for the maximum difference of potential in the three cases

$$25a \qquad V_2 \max = I_0 \sqrt{\frac{L_2}{K_1}},$$

$$26a \qquad V_2 \max = \frac{I_0}{2} \sqrt{\frac{L_2}{K_1}} = \frac{I_0}{2} \sqrt{\frac{L_1}{K_2}},$$

$$27a \qquad V_2 \max = I_0 \sqrt{\frac{L_1}{K_2}}.$$

Formula $25a$ assumes that the capacity of the secondary can be neglected. Walter[1] found by experiment that this was true for small coils. The larger the coil the more will the potential deviate from this value and approach those given by $26a$ and $27a$.

For the primary current, neglecting the reaction of the secondary, Walter[2] obtains

$$(28) \qquad I_1 = I_0 e^{-\frac{R}{2L_1}} \cos \frac{t}{\sqrt{L_1 K_1}}.$$

The period of the primary current is therefore given by

$$(29) \qquad T_1 = 2\pi \sqrt{L_1 K_1}.$$

For the maximum difference of potential between the plates of the primary condenser, during the oscillations of the primary current, Walter[3] gives the following expression:

$$(30) \qquad V_{1\,\max} = I_0 \sqrt{\frac{L_1}{K_1}}.$$

Equation (30) shows that if the capacity of the primary circuit is very small, the induced electromotive force in this circuit is very great.

[1] Loc. cit.
[2] Wied. Ann., Bd. 62, p. 310, 1897.
[3] Loc. cit., p. 322.

Johnson[1] makes a correction for the reaction of the secondary and writes (29) and (30) as follows:

$$T_1 = 2\pi \sqrt{L_1 K_1 \left(1 - \frac{M^2}{L_1 L_2}\right)} \tag{29a}$$

and

$$V_{1\,\max} = I_0 \sqrt{\frac{L_1}{K_1}\left(1 - \frac{M^2}{L_1 L_2}\right)}. \tag{30a}$$

Theory of two Circuits. Harmonic E.M.F.

If, instead of a constant current, which is suddenly broken, we have an alternating current in the primary, equations (17) become

$$\begin{aligned} L_1 \frac{d^2 q_1}{dt^2} + M \frac{d^2 q_2}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{q_1}{K_1} &= E \cos \omega t, \\ L_2 \frac{d^2 q_2}{dt^2} + M \frac{d^2 q_1}{dt^2} + R_2 \frac{dq_2}{dt} + \frac{q_2}{K_2} &= 0. \end{aligned} \tag{31}$$

We will only consider the case in which the primary circuit is a closed circuit containing no condenser, and in which therefore K_1 can be considered infinite.

We obtain for the secondary circuit

$$q_2 = \frac{M\omega^2 E}{\sqrt{D_1^2 + D_2^2}} \cos(\omega t - \alpha) \tag{32}$$

where

$$D_1 = (L_1 L_2 - M^2)\omega^4 - \left(R_1 R_2 + \frac{L_1}{K_2}\right)\omega^2,$$

$$D_2 = (R_2 L_1 + R_1 L_2)\omega^3 - \frac{R_1}{K_2}\omega$$

and

$$\alpha = \cot^{-1} \frac{D_1}{D_2}.$$

To get V_2 the difference of potential of the condenser in the secondary we write

$$V_2 = \frac{q_2}{K_2}.$$

[1] Wied. Ann., Bd. 3, pp. 450 and 458, 1900.

This gives for V_m, the amplitude of the potential oscillation,

$$V_m = \frac{M\omega^2 E}{K_2\sqrt{\left[(L_1L_2 - M^2)\omega^4 - \left(R_1R_2 + \frac{L_1}{K_2}\right)\omega^2\right]^2 + \left[(R_2L_1 + R_1L_2)\omega^3 - \frac{R_1}{K_2}\omega\right]^2}} \tag{33}$$

In the induction coil, as ordinarily used, with an open secondary, $K_2 = 0$. In this case (33) becomes

$$V_m = \frac{M\omega E}{\sqrt{L_1^2\omega^2 + R_1^2}}. \tag{34}$$

To find what capacity in the secondary will give the maximum amplitude for the potential oscillation, we must differentiate the denominator of (33) with respect to K_2 and put the result equal to zero. Calling this value of the capacity K_m, we get

$$K_m = \frac{L_1(L_1L_2 - M^2)\omega^2 + R_1^2L_2}{(L_1L_2 - M^2)\omega^4 + (R_1^2L_2^2 + 2R_1R_2M^2 + R_2^2L_1^2)\omega^2 + R_1^2R_2^2}. \tag{35}$$

Experiments verifying this formula will be described in § 6.

§ 3. INSTRUMENTS AND METHODS.

The induction coil used in the experiments described in the following pages, was made in separate pieces so that each part of it was independent of every other part, and the number of sections in the secondary could be varied at will.

The primary was made of two layers of heavy double-cotton-covered copper wire, No. 12, Brown & Sharpe gauge, and had the following dimensions:

Number of turns in outer layer,	109½
Number of turns in inner layer,	109
Total number of turns,	218½
Mean length,	25.63 cm.
Diameter of the outer coil,	4.47 cm.
Diameter of the inner coil,	4.11 cm.
Diameter of the wire,	.230 cm.

The primary was made by winding the wire upon a wooden cylinder, about which had been previously wrapped several layers

of heavy paper. To the ends of the paper tube were glued two wooden end pieces to keep the coil from spreading. When the coil had been wound, and the ends of the wire attached to one of the end pieces, the wooden cylinder was withdrawn. The inductance of the primary could be changed by putting iron wires in the hollow core. The iron wires used were of soft iron 35 cm. long and .0830 cm. in diameter. Most of the experiments were conducted without any iron in the core. The self inductance of the primary with an air core was found to be .000281 of a henry. Its resistance at 21° Centigrade, was .1666 of an ohm.

TABLE I.

Mark.	No. of Turns.	Inside Diameter in Centimeters.	Outside Diameter in Centimeters.	No. of Turns in a Layer.	No. of Layers.	Self-Inductance in Henries.	Mutual Inductanc with Primary in Henries.	Resistance at 21° C. in Ohms.
AI	5469	7.62	13.00	52–60	100	3.551		1527.0
AII	4600	7.62	13.00	50	92	2.594		1351.7
BI	5558	7.36	12.85	52–59	100	3.595	.00763	1516.0
BII	5000	7.36	12.49	50–57	95	2.913		1365.0
BIII	5000	7.37	12.82	50	100	2.951		1391.3
BIV	5000	7.34	12.25	51–60	90	2.934		1425.1
CI	1250	7.39	8.88	50	25	.1867		297.8
CII	"	7.36	8.85	"	"	.1896		298.5
CIII	"	7.37	8.87	"	"	.1880		297.5
CIV	"	7.40	8.95	"	"	.1893		295.8
DI	2500	7.38	10.28	50	50	.684		612.2

The secondary was made in eleven sections. Each section was independent of all the others and could be slipped at will on and off a hard rubber sleeve, which itself was slipped over the primary. The sections were all wound with No. 34 (Brown & Sharpe gauge) single-silk-covered copper wire. The diameter of the wire was .01601 cm. The sections were all of the same width, viz: 1.27 cm. (half an inch). Six of them contained about 5,000 turns each; one contained about 2,500 turns and the other four contained about 1,250 turns each. Their dimensions are given in Table I. Table I*a* contains the inductances, self and mutual, of the coils joined up in series.

TABLE I*a*.

Coils Used.	Self Inductance in Henries.	Mutual Inductance with Primary in Henries.
BII + BIII.	9.67	
BII + BIII + BIV.	18.92	
BI + BIV.	11.06	.0145
BI + BIV + BII.	20.84	.0213
CI + CII.	.5774	
CI + CII + CIII.	1.076	
CI + CII + CIII + CIV.	1.639	

All the sections were wound on a form in a lathe, a strip of Banker's Bond paper being placed between each two layers. AI and AII were wound on wooden forms, on which they were permanently retained. All the others were wound on a metal form. After the section had been wound the metal form was removed, and the section boiled in paraffin in a vacuum to exclude all moisture and air. Connections were made by soldering heavier wires to the ends of the sections, and inserting the ends in mercury cups.

The condensers used in these experiments had a total capacity of about thirteen microfarads. The smaller condensers were very carefully put together and were made of alternate sheets of mica and tinfoil. After the condenser had been assembled it was clamped by a metal clamp and boiled in beeswax. The metal clamp was retained permanently. The smaller condensers were divided into ten sections ranging from .006 to 0.15 of a microfarad. A set of 16 sections of a mica and tinfoil condenser, kindly loaned by Messrs. Morris E. Leeds & Co., of Philadelphia, was also used. Each section had a capacity of about .2 microfarad. These were boiled in paraffin and held together by a wooden clamp. Three large Stanley, paper and tinfoil, condensers, each having a capacity o about 2.5 microfarads, were also used.

The capacities of the condensers were determined by making an absolute determination of one section by the tuning fork charge and discharge method[1] and then comparing the other sections with this section by the bridge method.

[1] Thomson & Searle, Phil. Trans., 1890 Vol. A, pp. 583–621.

The *measurements of difference of potential* were all made with a modified Kelvin quadrant electrometer, used idiostatically. The deflections were therefore proportional to the squares of the potential.

The instrument used was the one used by Professor Webster, and described by him in his paper on electrical oscillations.[1] The quadrants were made of brass and the needle of aluminum. The needle was suspended by a quartz fiber and connected to the binding post by a platinum wire dipping into sulphuric acid. The instrument worked excellently as long as the acid was fresh. After the acid had been standing for some weeks it was found to be necessary to boil it for several hours, for if not fresh or recently boiled there was a lag when a reading was taken.

The differences of potential measured were usually small, less than 50 volts, since the primary was nearly always used with an air core. The electrometer as adjusted for these experiments was too sensitive to measure directly potentials greater than 50 volts. It could be arranged however to measure any potential by the introduction of an auxiliary air condenser of adjustable capacity in series with it in the electrometer circuit. In Fig. 3 *A* is the auxiliary air condenser, and *V* the electrometer.

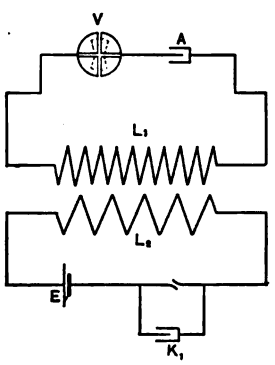

Fig. 3.

The true difference of potential can be calculated from the apparent difference given by the electrometer reading, and the known capacity of the auxiliary condenser.

In all the experiments the electrometer was used idiostatically, one side being connected to earth so as to give a constant zero.

The usual method of using the electrometer was as follows. After taking a series of observations, the instrument was calibrated by noting the deflections due to different known differences of potential. The known differences of potential used were those of a storage battery, determined by a Weston voltmeter. A calibration curve

[1] PHYS. REV., Vol. VI., p. 301, 1898.

was plotted, and the value of the potential corresponding to a given deflection read off directly from the curve.

A curious source of error observed and studied by Hallwachs[1] caused the writer some trouble in the earlier experiments. Since in the idiostatic method of using the electrometer the deflection is proportional to the square of the potential, when the sign of the potential is changed without changing its absolute value the deflection should be unchanged. It was found that this was not quite true, and investigation showed that it was due to the contact difference of potential between the brass quadrants and the aluminum needle, which increased the deflection in one case and decreased it in the other. An approximately correct value for the difference of potential is found by taking the mean between the two deflections. This was done in calibrating the instrument. Hallwachs has shown that an electrometer in which the quadrants and the needle are made of different metals can be used to find the contact difference of potential between the two metals. The difference of potential between the brass quadrants and the aluminum needle was determined in this way by the writer, and found to be equal to .42 of a volt.

In these experiments the electrometer was used idiostatically in two ways, first, with a steady deflection with an alternating current, and second, ballistically with a transient current. In both cases the deflection was proportional to the mean square of the potential. In both cases the potential performed a simple harmonic oscillation, the difference being that in the first case the oscillation was undamped, and in the second case it was strongly damped, the time constant being about a thousandth of a second.

The *measurements of inductance* were made by comparison with a standard coil, the inductance of which had been previously calculated by Maxwell's formula[2] for the self-inductance of a circular coil of rectangular cross-section.

If (see Fig. 4)

n = number of turns in coil.

a = mean radius of coil.

r = diagonal of the cross-section.

θ = angle made by diagonal with lower edge of the cross-section.

[1] Wied. Ann., Bd. XXIX., pp. 1–47, 1886.

[2] See Rayleigh, Phil. Trans., 1882, pt. II., p. 675.

We have, using the first two terms of the series

$$L = 4\pi an^2\left[\log_e \frac{8a}{r} + \tfrac{1}{12} - \tfrac{4}{3}\left(\theta - \frac{\pi}{4}\right)\cot 2\theta - \tfrac{1}{8}\pi \operatorname{cosec} 2\theta \right.$$

$$\left. - \tfrac{1}{6}\cot^2\theta \log_e \cos\theta - \tfrac{1}{6}\tan^2\theta \log_e \sin\theta\right]$$

$$+ \frac{\pi n^2 r^2}{24a}\left[\log_e \frac{8a}{r}(2\sin^2\theta + 1) + 3.45 + 27.475\cos^2\theta\right.$$

$$\left. - 3.2\left(\frac{\pi}{2} - \theta\right)\frac{\sin^3\theta}{\cos\theta} + \tfrac{1}{5}\frac{\cos^4\theta}{\sin^2\theta}\log_e \cos\theta + \tfrac{1}{8}\frac{\sin^4\theta}{\cos^2\theta}\log_e \sin\theta\right]$$

In calculating the value of L for the standard coil it was found that neglecting the second term would introduce a minus error of .4 of one per cent.

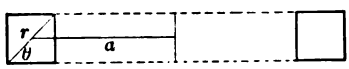

Fig. 4.

The coils used as the standard were two coils of the same dimensions, used by Taylor,[1] in his work on standard cells, as the fixed coils of an electrodynamometer. They are fully described by him in the paper referred to.

The mean value of the self inductance of each coil by calculation from the formula was found to be .001881 of a henry.

The comparison of the inductances to be measured with the standard was made by Maxwell's bridge method using the alternating current and a telephone[2] instead of a galvanometer. The comparison was usually made by comparing the unknown inductance first with the upper coil and then with the lower, and taking the mean of the two values obtained. The two values so obtained usually agreed to about a tenth of one per cent. It was found that with the alternating current used, the inductances could be measured to about a tenth of one per cent. The self inductances of the secondary segments, both separately and combined together are given in Tables I. and Ia.

The mutual inductances of the secondary segments with the primary were measured by Maxwell's[3] bridge method for comparing

[1] PHYS. REV., Vol. VII., pp. 156–159, 1898.
[2] See Max Wien, Wied. Ann., Bd. 44, pp. 681–688, 1891.
[3] See Gray, Absolute Measurements, Vol. II., pp. 465–469.

the mutual inductance of two coils with the known self inductance of one of the coils. In this case also, the alternating current and the telephone were used instead of a steady current and a galvanometer. The telephone method is much more rapid than the galvanometer method. The results are given in Table I. The inductances were all measured without any iron in the core, that is with an air core.

The *measurements of resistance* were made by the usual zero method of the Wheatstone bridge with a steady current and a galvanometer. The resistance box used was one made by Elliott Bros., of London. The resistances of the box were examined by Mr. Frank K. Bailey, of the university, and found to be accurate to a tenth of one per cent. The resistances are given in Table I. reduced to 21° centigrade.

The *measurements of currents* were made for direct currents with a Weston ammeter, and for alternating currents with a hot wire ammeter, which was calibrated by comparison with the Weston ammeter. The currents were measured to about one per cent.

To conduct the experiments described in § 6, it was necessary to obtain a harmonic current that should be free from overtones. This was done by Pupin's[1] method, which consists in inserting inductance and capacity in series in the circuit, and then adjusting the relative amounts of inductance and capacity until the circuit is in resonance for the particular frequency which is desired. When this is so, if the resistance of the circuit is relatively small, the frequency which is desired will so far predominate that the other frequencies cannot be detected. In order to use this method it is necessary to have either a large inductance or a large capacity. Since it was easier to construct a large inductance than a large capacity, a coil was built which gave the required inductance. The dimensions of a coil which would give the desired inductance were calculated from Maxwell's formula on p. 297. The cross-section of the whole coil, drawn to scale, is shown in Fig. 5.

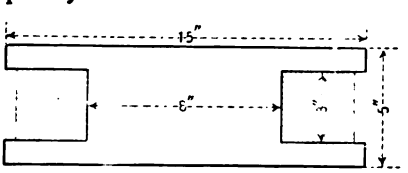

Fig. 5.

[1] Am. Jour. Sci., Vol. XLVIII., pp. 379–389, 473–485, 1894.

The portion occupied by the wire is shaded. This shaded portion was intended to be 3 inches on a side. In the coil as constructed it was 3 inches from top to bottom, and 2¾ inches from side to side. The coil was wound with about two miles of No. 18 (Brown & Sharpe gauge) double cotton insulated copper wire. Diameter, .0403 of an inch. The wire was wound in 60 layers of 60 turns each, making 3,600 turns in all. At the beginning and end of every ten layers, the wire was led off to a binding post on the top of the coil, so that the coil was divided into six independent sections.

The dimensions of the wooden form on which the coil was wound are those shown in Fig. 9, viz:

Outside diameter,	15	inches.
Inside diameter,	8	"
Height of groove,	3	"
Depth of groove,	3½	"
Height of whole coil,	5	"

The dimensions of the coil proper were

Outside diameter,	13½	inches
Inside diameter,	8	"
Height of cross-section,	3	"
Depth of cross-section,	2¾	"

The coil was designed to give the maximum inductance for the length of wire used.[1] Its inductance calculated before construction was 3.33 henries. After construction its inductance was found by comparison with the standard inductance to be 3.01 henries. This is probably partly due to the fact that the designed geometrical conditions, viz.: a rectangular cross-section was not quite fulfilled.

§ 4. The Iron Core.

To determine the effect of increasing the amount of iron in the core, the number of iron wires inserted in the core was varied, the current for any given experiment being kept constant.

An alternating current of about 60 complete periods per second was used in the primary. The difference of potential between the terminals of the secondary was measured with the electrometer, used idiostatically.

[1] See Maxwell, Elect. Mag., Vol. II., p. 316.

Tables II.–V. and Figs. 6–9 show the results obtained. In the curves the differences of potential in the secondary are plotted as ordinates, and the number of wires as abscissas.

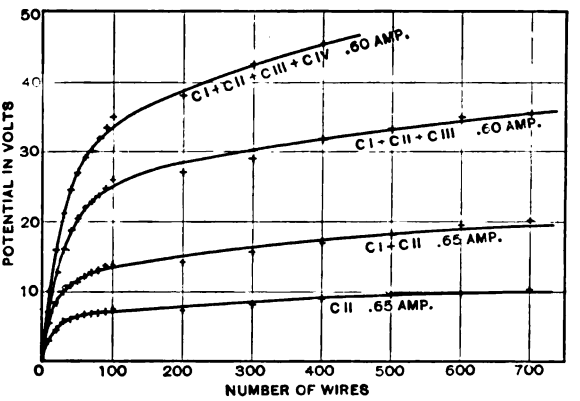

Fig. 6.

Table II. and Fig. 6 show how the form of the curve varies as the inductance in the secondary is increased by adding coils of the same number of turns and of the same diameter placed side by side.

TABLE II.

Number of Wires.	CII .65 Amp.	CI + CII .65 Amp.	CI + CII + CIII .60 Amp.	CI + CII + CIII + CIV .60 Amp.
	Volts.	Volts.	Volts.	Volts.
0	.0	.0	.0	.0
10	2.5	5.5	8.0	10.0
20	4.2	8.3	12.8	16.0
30	5.5	10.0	16.1	21.1
40	6.1	10.9	18.8	24.6
50	6.4	11.4	20.5	27.2
60	6.6	12.0	22.0	29.4
70	6.9	12.7	22.7	30.0
80	7.0	13.0	23.8	32.0
90	7.2	13.7	24.8	33.5
100	7.5	13.8	26.0	35.0
200	7.6	14.2	27.1	38.0
300	8.5	15.8	29.0	42.5
400	9.2	17.1	31.8	45.5
500	9.6	18.4	33.5	54.0
600	9.9	19.6	35.1	57.0
700	10.3	20.3	35.8	58.5

Table III. and Fig. 7 also show how the form of the curve varies as the inductance is increased, but in this case the increase

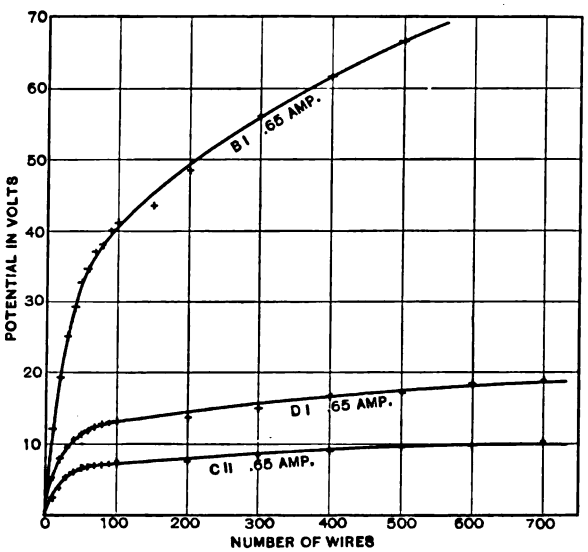

Fig. 7.

is made by replacing the original secondary by a coil of the same width, but of greater diameter and therefore of a greater number of turns.

TABLE III.

Number of Wires.	CII .65 Amp.	DI .65 Amp.	BI .65 Amp.
	Volts.	Volts.	Volts.
0	.0	.0	.0
10	2.5	5.3	12.2
20	4.2	8.0	19.2
30	5.5	9.7	25.2
40	6.1	10.5	29.3
50	6.4	11.4	32.6
60	6.6	11.8	34.5
70	6.9	12.3	37.0
80	7.0	12.7	37.8
90	7.2	12.9	40.0
100	7.5	13.2	40.7
200	7.6	13.7	48.4
300	8.5	15.1	56.0
400	9.2	16.6	62.0
500	9.6	17.3	66.8
600	9.9	18.5	68.0
700	10.3	19.0	

Table IV.

Number of Wires.	CII .65 Amp.	CII 1.05 Amp.	CII 2.3 Amp.
	Volts.	Volts.	Volts.
0	.0	.0	2.5
10	2.5	4.4	6.3
20	4.2	7.2	9.7
30	5.5	9.4	12.6
40	6.1	11.5	15.5
50	6.4	13.7	17.8
60	6.6	15.4	20.0
70	6.9	16.4	22.6
80	7.0	17.5	24.3
90	7.2	18.4	26.8
100	7.5	19.2	29.0
200	7.6	23.2	43.0
300	8.5	26.0	50.0
400	9.2	26.5	52.3
500	9.6	28.2	53.8
600	9.9	29.2	54.5
700	10.3	29.6	54.5

Tables IV., V. and Figs. 8, 9 show the change in the form of the curve for the same secondary coil as the current is increased.

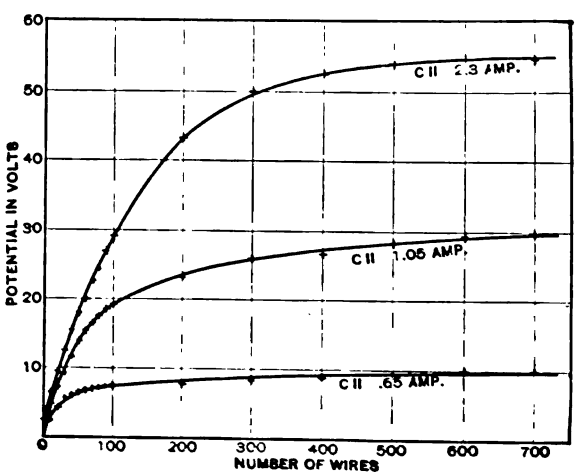

Fig. 8.

TABLE V.

Number of Wires.	DI .65 Amp.	DI 1.05 Amp.	Number of Wires.	DI .65 Amp.	DI 1.05 Amp.
	Volts.	Volts.		Volts.	Volts.
0	.0	.0	90	12.9	25.2
10	5.3	7.0	100	13.2	26.3
20	8.0	11.1	200	13.7	27.7
30	9.7	14.7	300	15.1	30.8
40	10.5	17.4	400	16.6	32.0
50	11.4	19.8	500	17.3	34.2
60	11.8	21.7	600	18.5	36.0
70	12.3	23.1	700	19.0	
80	12.7	24.3			

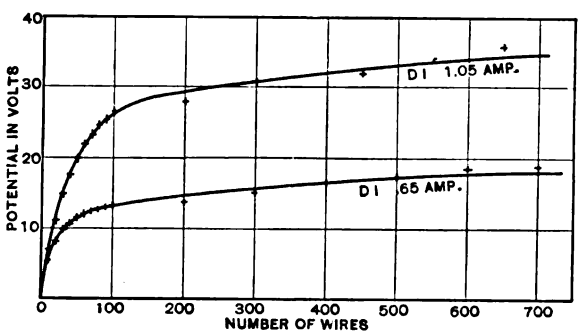

Fig. 9.

The form of these curves is given by equation (34).

Since M and L_1 are linear functions of the permeability of the core, and the permeability of air is very small compared with that of iron, we may assume that they are proportional to the number of iron wires in the core, and we may write

$$M = k_1 x,$$

$$L_1 = k_2 x,$$

where x is the number of iron wires in the core. Equation (34) then becomes

$$V_m = \frac{k_1 \omega E x}{\sqrt{k_2^2 \omega^2 x^2 + R_1^2}},$$

which is an equation of the fourth degree in V_m and x. The graph of V as a function of x gives a curve of the fourth order symmetrical with respect to both the x and y axes, and having points of inflexion at the origin. The portion of the curve plotted in the figures is for x positive and V positive. It is evident by inspection of equation (34) that when the number of wires is infinite V_m approaches a finite maximum value, $\frac{ME}{L_1}$. Also, that the potential increases very rapidly as the number of wires is increased up to a certain point after which an increase in the number of wires only makes a small difference in the potential. This is a fact of practical importance in the construction of coils.

The question arose as to whether it would make any difference whether the wires were placed together in the axis of the core, or placed loosely anywhere within the core. It was found by experiment that, within the limits of accuracy of the experiments, it made no difference whether the iron wires were placed in a bundle in the middle of the core or placed loosely anywhere within it.

§ 5. DISTRIBUTION OF THE INDUCTION.

A matter of considerable importance in the construction of coils is the knowledge of the flux of induction through the primary at different points of its length. It is important to know how the flux varies as we move along the coil from one end to the other, in order to know where to place the secondary coils so that the greatest flux shall pass through them.

To determine this, experiments were made, by placing upon the primary one of the secondary coils, and sliding it along from one end to the other. The primary was divided into ten equal parts, and the secondary was made to take eleven different positions at equal distances apart, including the ends. The terminals of the secondary coil were attached to the electrometer. The current used in the primary was the commercial alternating current, with a frequency of about 60 oscillations a second. Three different sized secondary coils were used, viz.: 5,000 turns, 2,500 turns and 1,250 turns. The coils used were BIII, DI, and CIV.

The results are given in Table VI. and Fig. 10.

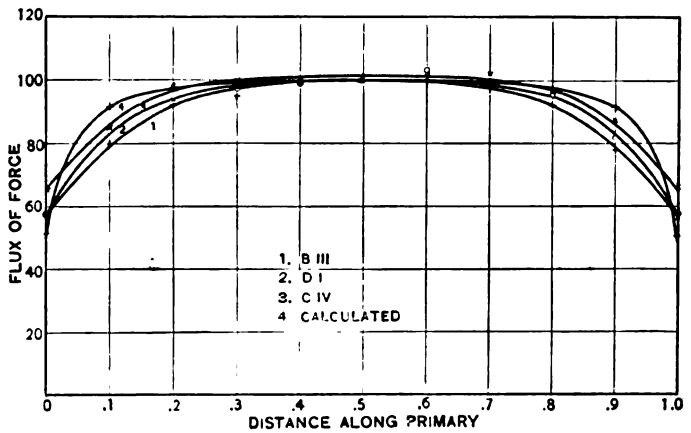

Fig. 10.

The method of performing the experiment was to move the secondary from left to right, noting the deflections for each of the eleven positions, and then move it back again from right to left noting the deflections, and finally take the mean of the two readings for the same position.

For BIII the mean primary current was 5.1 amp.; for DI, 5.6 amp.; for CIV, 8.4 amp.

TABLE VI.

Fraction of Total Length of Primary.	BIII		DI		CIV	
	Diff. of Pot. in Volts.	Reduced to 100.	Diff. of Pot. in Volts.	Reduced to 100.	Diff. of Pot. in Volts.	Reduced to 100.
.0	6.3	57	4.5	57	3.0	64
.1	8.8	80	6.6	84	4.0	85
.2	10.1	92	7.4	94	4.6	98
.3	10.5	95	7.7	97	4.7	100
.4	10.9	99	7.8	99	4.7	100
.5	11.1	101	7.9	100	4.7	100
.6	11.0	100	8.1	103	4.8	102
.7	10.8	98	7.8	99	4.8	102
.8	10.1	92	7.5	95	4.5	96
.9	8.6	78	6.5	82	4.1	87
1.0	6.2	56	4.6	58	3.1	66

The theory of this experiment is given by equation (34).

Since when we move the exploring coil along the primary everything is unchanged except M, we have V_m directly proportional to M. But M is directly proportional to the flux of induction through the solenoid at that point of its length. Therefore V_m is a measure of the flux of induction through the solenoid, and is directly proportional to it.

The flux of induction through the primary was also calculated. The variation of the field from that of an infinitely long solenoid is due to the free ends of the solenoid. The correction due to the ends was calculated from Maxwell's[1] formula for the force at any point due to a charged disk. The force was then integrated over a spherical cap, bounded by the solenoid, at different distances from the end of the solenoid. This gives the flux through the solenoid at different distances from its end. Using the first three terms of the series the following formula was obtained:

$$\text{Flux} = 4\pi^2 r^2 \left\{ \tfrac{1}{2} \frac{a^2}{r^2} \left(\frac{z}{r} - 1 \right) - \frac{3}{2.2.4} \frac{a^4}{r^4} \left[\left(\frac{z^3}{r^3} - 1 \right) - \left(\frac{z}{r} - 1 \right) \right] + \frac{3.5}{2.4.6.8} \frac{a^6}{r^6} \left[7 \left(\frac{z^5}{r^5} - 1 \right) - 10 \left(\frac{z^3}{r^3} - 1 \right) + 3 \left(\frac{z}{r} - 1 \right) \right] \right\},$$

Where a is the radius of the solenoid.

r the distance from the center of the end to the spherical cap.

z the distance from the end of that turn of the solenoid through which the cap passes.

This formula gives the flux through a turn of the solenoid at a distance z from the left hand end, due to a disc of unit density placed at the left hand end. To find the flux when unit current passes through the solenoid we must multiply by the number of turns in unit length of the coil. To get the total effect due to both ends, we must add the effects of the two ends, which are in the same direction. This gives the correction to be subtracted from the flux for an infinitely long solenoid. The solenoid used in these experiments was made of two layers, an inner and an outer layer. The flux was calculated for each separately and the total flux taken as the sum of the two. The results are given in Table VII. The results are only given for half the length of the coil, as the two

[1] Maxwell, Elect. & Mag., Vol. II., p. 284.

halves are, of course, symmetrical. The corrected flux through the solenoid is plotted as curve 4 in Fig. 10.

If the flux were calculated on the assumption that the primary was infinitely long, we should have 1,551 cm. for every point along the primary, instead of the values given in the column last but one of Table VII.

TABLE VII.

Fractional distance from left hand end of coil.	Outer Coil.				Inner Coil.				Both Coils Together.		
	Flux due to disk of unit density at left hand end.	Flux due to disk of unit density at right hand end.	Sum of both $= S_1$	$S_1 \times 4.273 = F_1$	Flux due to disk of unit density at left hand end.	Flux due to disk of unit density at right hand end.	Sum of both $= S_2$	$S_2 \times 4.253 = F_2$	Total flux for unit current due to ends $= F_1 + F_2$	Total flux through primary.	Reduced to 100.
0	98.59	.39	98.98	422.9	83.35	.26	83.61	355.7	778.6	772	50.5
.1	19.54	.47	20.01	85.5	14.97	.34	15.31	65.1	150.6	1400	91.6
.2	7.48	.57	8.05	34.4	5.47	.40	5.87	25.0	59.4	1492	97.6
.3	3.72	.76	4.48	19.1	2.62	.56	3.18	13.6	32.7	1518	99.3
.4	2.21	1.00	3.21	13.7	1.54	.75	2.29	9.8	23.5	1527	99.9
.5	1.47	1.47	2.94	12.6	1.07	1.07	2.14	9.1	21.7	1529	100.0

The flux of induction through the primary at any point of its length is of course measured by the number of lines passing through a single turn wound tightly upon it at that point. This is the quantity calculated in Table VII. Since the coils BIII, CIV and DI, used in the experiment, consist of a large number of turns, most of the turns being situated at an appreciable distance from the primary, the flux through these coils will not be directly proportional to that for a single turn. This difference in the conditions may perhaps account for the difference between the curves for the observed and calculated values in Fig 10.

§ 6. CAPACITY IN THE SECONDARY.

Experiments were made to determine the effect of inserting localized capacity in series in the secondary.

The current used in the primary was the commercial alternating current. It was analyzed by Pupin's[1] method, and was found to contain the third and fifth harmonics.

[1] See Pupin, loc. c t.

For a given frequency, ω, the amplitude of the potential oscillation in the secondary condenser, is given by equation (33). If more than one frequency is present in the current, the potential is the sum of the potentials due to the different frequencies.

Experiments were made with each of the three frequencies present in the commercial current. The frequency desired was brought out by Pupin's method which has been described in § 3. To test the accuracy of Pupin's method the large resonance coil described in § 3 was placed in series with a condenser in a circuit which had impressed upon it the electromotive force of the commercial altering current. The fundamental of the alternating current was about 60 oscillations a second. The potential of the condenser was observed as its capacity was varied in the neighborhood of that capacity which would give resonance for the fundamental. These observations are given in Table VIII. The theoretical values were then calculated from equation (16). The results are given in Table IX. The observed potentials were relative, not absolute. The results are plotted in Fig. 11. The crosses represent the calculated values, and the circles the observed values. It is hard to say why the two curves do not coincide. If we should assume that the

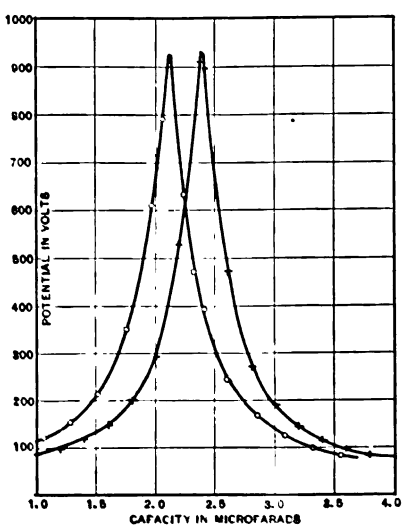

Fig. 11.

TABLE VIII.

Capacity in Microfarads.	Square Root of Deflection × 112.4.	Capacity in Microfarads.	Square Root of Deflection × 112.4.
1.06	120	2.32	470
1.26	151	2.40	393
1.51	212	2.60	248
1.76	352	2.86	167
1.97	612	3.09	126
2.06	793	3.30	101
2.23	634	3.55	82

TABLE IX.

Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.
1.00	86	2.60	473
1.20	101	2.80	271
1.40	121	3.00	191
1.60	148	3.20	143
1.80	200	3.40	117
2.00	292	3.60	97
2.20	530	3.80	84
2.37	910	4.00	74
2.40	897		

large resonance coil has a distributed capacity of about .3 microfarad, the two curves would be very nearly coincident.

TABLE X.

Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.
.000	17.1	.210	26.2	1.26	47.7
.032	17.7	.224	32.4	1.51	62.6
.038	18.0	.232	36.7	1.76	104.0
.052	18.3	.235	37.1	1.97	181.0
.059	18.4	.244	34.9	2.23	187.0
.071	19.2	.255	29.0	2.40	116.0
.080	21.9	.283	22.4	2.32	139.0
.083	24.1	.306	21.1	2.06	234.0
.090	25.0	.319	20.7	2.60	73.4
.095	22.1	.421	20.7	2.86	49.4
.107	19.5	.621	25.2	3.09	37.2
.127	19.0	.884	29.9	3.30	29.9
.155	19.4	1.060	35.5	3.55	24.3
.178	20.6				
.190	22.0				

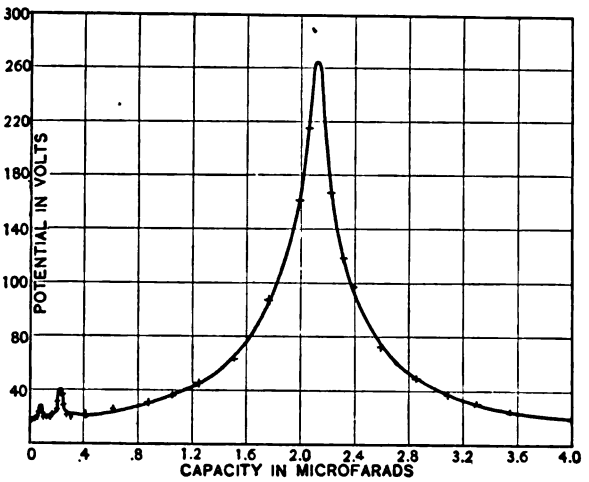

Fig. 12.

Table X. shows the results of the analysis of the commercial alternating current, made in the way described. The capacity of the circuit was gradually increased from nothing up to 3.5 microfarads. The observations are plotted in Fig. 12. The three peaks reading from right to left represent the fundamental, the third harmonic and the fifth harmonic, respectively.

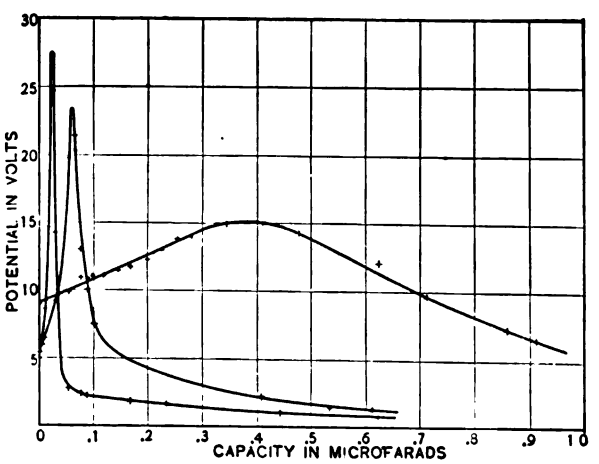

Fig. 13.

Having made these preliminary observations, the main problem was attacked, with the results given in Tables XI.–XIV., and Figs. 13–15.

TABLE XI.

BI + BIV.

Capacity in Microfarads.	Potential in Volts.		
	$n = 60$ 2.2 Amp. in Primary.	$n = 180$ .3 Amp. in Primary.	$n = 300$ .3 Amp. in Primary.
.000	9.1	5.0	4.6
.007	9.0	6.2	6.5
.008	9.2	6.0	6.7
.012	9.2	6.5	8.6
.019			14.7
.020			18.7
.027			24.7
.032	9.7	9.2	14.2
.060	9.9	19.7	2.8
.069		21.4	
.071		20.3	
.082	10.9	13.0	2.4
.095	10.8	10.0	2.2
.107	11.0	7.5	
.127	11.1		
.155	11.5		
.178	11.8		1.8
.210	12.3		
.238	13.0		
.250			1.6
.269	13.7		
.297	14.0		
.367	15.0		
.436	15.0	2.2	
.462			1.0
.506	14.2	1.7	
.659	12.0	1.2	.7
.755	9.5		
.909	7.1		
.968	6.3		

Tables XI. and XII. give the results obtained by changing the capacity of the secondary condenser, keeping the primary conditions constant. Three frequencies were used, viz.: those of the fundamental, the third overtone, and the fifth overtone. The results given in Table XI. were obtained by using BI and BIV in the

Table XII.

BI + BIV + BII.

Capacity in Microfarads.	Potential in Volts.		
	$n = 60$ 2.6 Amp. in Primary.	$n = 180$ .3 Amp. in Primary.	$n = 300$ .2 Amp. in Primary.
.000	12.0	6.8	7.0
.004		8.3	10.5
.007	12.0	9.7	15.0
.008		10.2	20.4
.012	12.4	13.0	32.5
.013			65.0
.016			27.3
.019			17.3
.020		15.4	13.0
.027		20.7	
.032	13.0	26.6	5.2
.038		28.1	
.040		26.0	
.043		22.5	
.060	14.3	9.9	1.5
.082	15.5	5.5	
.095	16.0	4.5	
.127	17.5		
.186	20.7		
.238	22.0		
.250	21.9		
.269	21.5		
.281	21.5		
.318	19.0		
.372	16.9		
.532	10.1		
.777	1.8		

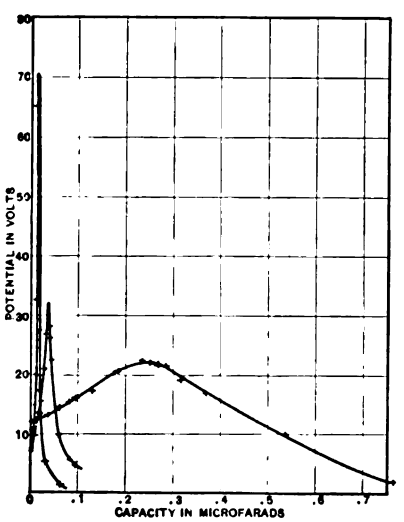

Fig. 14.

secondary, and those given in Table XII. by using BI, BIV and BII in the secondary. This gave a variation of the secondary inductance. BI + BIV and BI + BIV + BII have self inductances of 11.1 and 20.8 henries respectively. The observations are plotted in Figs. 13 and 14. The theory of these curves is given in the latter part of § 2, and the form of the resonance curves is given by equation (33).

Fig. 15 represents the "resonance curve" for the commercial current, without the weeding out of the overtones. The observations from which the curve was plotted are given in Table XIII. BI and BIV were used in the secondary, and the current in the primary was 2 amp. This curve shows, as already stated, that the commercial current contained two harmonics beside the fundamental. It leads to the same conclusion as Fig. 12, the difference being that in Fig. 12 the potential of the primary condenser is plotted, whereas in Fig. 15 the potential of the secondary condenser is plotted. Fig. 15 shows that a small induction coil and an electrometer form an excellent detector of harmonics.

TABLE XIII.

Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.
.000	16.1	.084	15.8
.026	43.8	.090	15.0
.030	30.9	.105	14.6
.031	27.8	.141	13.1
.033	25.5	.156	14.0
.038	19.7	.183	14.6
.039	18.6	.247	17.0
.044	17.4	.315	18.6
.046	17.2	.390	19.3
.050	16.3	.456	19.1
.053	18.0	.543	18.2
.057	18.1	.611	16.0
.063	18.8	.652	15.7
.068	18.8	.727	14.5
.074	18.0	.794	11.8

Figs. 13–15 show that the potential of the secondary in a small induction coil may in some cases be increased by the insertion of a condenser in series in the secondary.

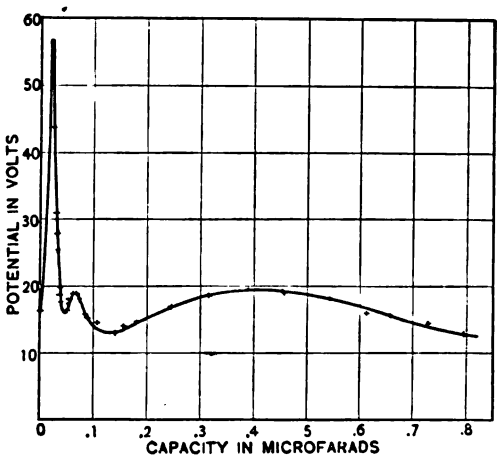

Fig. 15.

TABLE XIV.

Frequency.	Observed Value of K_m in Microfarads.	Calculated Value of K_m in Microfarads.
	$L_2 = 11.1$ henries.	
60	.403	.421
180	.066	.071
300	.024	.027
	$L_2 = 20.8$ henries.	
60	.237	.257
180	.036	.039
300	.014	.015

From these tables and curves can be found the value of the secondary capacity which will give the greatest difference of potential in the secondary for a given frequency. In Table XIV. are given the values read off from the curves, side by side with the values calculated from equation (35).

To be continued.

SURFACE COLOR.

By R. W. Wood.

So many misleading statements appear in the text-books on optics regarding the phenomenon of surface color, exhibited by certain strongly absorbing media, that it may be well to point out certain facts which are not always brought out as clearly as they should be. We frequently find the statement that these substances reflect most strongly those colors which are absent in the transmitted light, or which, as we more frequently say, are most strongly absorbed. One of the more recent of the advanced text-books refers the phenomenon to total reflection, the explanation being that for waves of certain wave-lengths the refractive index is less than one, and that therefore at a certain angle there will be total reflection for these waves, and consequent surface color. As a matter of fact the dispersion curve runs below unity only in the case of a very few substances, consequently total reflection is wholly inadequate as an explanation. Moreover the selective reflection occurs at normal incidence.

In the first place the statement that the most strongly reflected light corresponds to the light absorbed is erroneous. Take cyanine as an example. The center of the absorption band is very near the D lines, but the surface color is not yellow, but a deep plum color, not so very different in hue from that of the transmitted light. If we examine the spectrum of the reflected light we find a very dark band in the green, the center being not far from wave-length .0005. The distribution of intensity in the rest of the spectrum is not very different from what it would be in the case of reflection from glass, which shows that the peculiar color of the dye is not so much due to a very powerful reflection of certain waves, as it is to its almost complete refusal to reflect a certain region of the spectrum. If we remember that the reflecting power depends on the refractive index

as well as on the coefficient of absorption, we can easily explain this behavior.

The intensity of the light reflected at normal incidence by a transparent medium is given by the formula

$$R = \frac{(n-1)^2}{(n+1)^2}$$

As n, the refractive index, varies with the wave-length, the reflecting power will be different in different parts of the spectrum. The formula calls for a stronger reflecting power in the violet than in the red, the variation being greatest for substances with high dispersion and low mean refractive index. An experiment has recently been described by B. Walter to illustrate this type of selective reflection. Using the method of repeated reflections originated by Rubens and Nichols for the isolation of long heat waves, a remnant of blue violet light is obtained. The medium used by Walter was oil of cassia, above the surface of which a silver mirror was mounted.

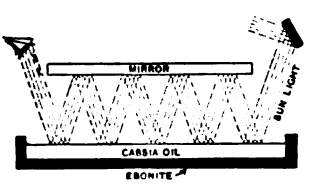

Fig. 1.

The sun's image after five reflections from the surface of the oil (the light being thrown back and forth between the surfaces as shown in Fig. 1), was found to be colored a deep blue. Though the formula calls for just such an effect, I have failed absolutely to obtain a trace of it though I employed six reflections, or one more than Walter. The image was very faint and of a neutral gray color with no suggestion of blue.

In the case of absorbing media the refractive index varies over a much wider range than it does in case of transparent substances, consequently greater variation in reflecting power with changing wave-length is to be expected. Moreover the absorption influences the reflecting power directly, the formula for media of this nature being

$$R = \frac{n^2 + n^2x^2 + 1 - 2n}{n^2 + n^2x^2 + 1 + 2n}.$$

In the case of a substance such as Hoffman's violet, the refractive index of which is unity for a certain wave-length, the amount of

light of this particular color reflected at normal incidence will be $\frac{x^2}{4+x^2}$. It is clear that this will increase as x increases, and will equal zero if x is equal to zero, as it is in the case of transparent media.

Taking the values of n and x for cyanine obtained by Pflüger in the different parts of the spectrum, we can calculate the percentages reflected in the cases of the different colors. The curve shown in

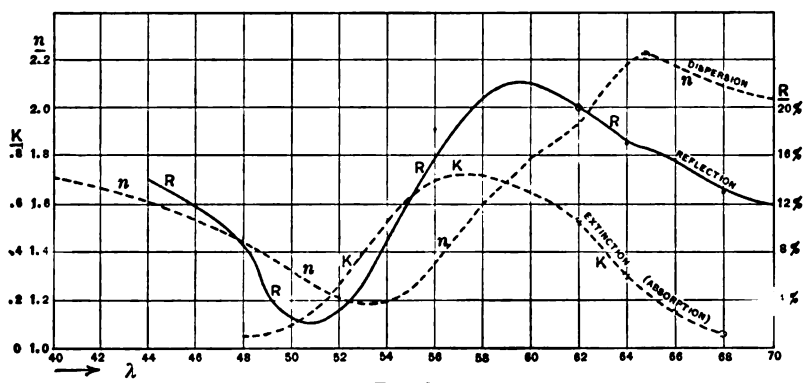

Fig. 2.

Fig. 2 was obtained in this way; it shows very clearly the cause of the peculiar surface color of the dye. Starting at the red end of the spectrum we find that the intensity of the reflected light increases with decreasing wave-length, due to the joint increase in n and x. The increase continues for some distance after n begins to decrease, owing to the continued increase of x. After reaching a maximum at $\lambda = .00059$ it turns and descends rapidly attaining its minimum at $\lambda = .0005$, where we have very small values of both n and x, and consequently very small reflecting power. Beyond this point the rise is due almost entirely to the further increase in n, since x is practically equal to zero in this portion of the spectrum.

Scarcely two per cent. of the incident light is reflected at $\lambda = .0005$ causing the dark band seen in the spectrum of the reflected light, alluded to at the beginning of the paper.

An excellent way of showing the variable reflecting power of a film of cyanine is to compare it with glass, in different parts of the

spectrum. A little of the melted dye is pressed between two plates of hot glass, which are separated when cold. A spot is selected where the film has a good optical surface, and this spot is left on the glass the rest being cleaned off. By holding the plate in the spectrum formed by a prism or grating the reflecting power of the two surfaces can be studied. In some parts of the spectrum the cyanine reflects more strongly than glass, in other regions the reverse is true, while at wave-length .0005 the cyanine refuses to reflect to such a degree that the film appears as a black spot on the blue field reflected by the glass.

It is interesting to note the difference in the surface color of the dye when the reflection takes place at the surface in contact with the glass. A very convenient way of showing the yellowish-green color in this case is to press out a film of the molten dye on one surface of a prism of 8 or 10 degrees angle.[1] In this way the light reflected from the dye can be obtained uncontaminated with the light reflected from the first glass surface. The method is analogous to that employed by Lippmann in mounting his color photographs. The calculation of the curve of reflected intensities under these conditions makes a good exercise for the student.

[1] A suitable prism can be made in half an hour by grinding down a piece of thick plate window glass. A strip of thick glass cemented along one edge will be all that is necessary to make the glass take the required form. Grind on a piece of glass with very coarse emery at first, then use finer grades, polishing with rouge at the end. Small scratches do no harm, and a high polish is not necessary.

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Several years ago we imported a number of RESISTANCE STANDARDS with REICHSANSTALT CERTIFICATES. In July, 1901, they were returned to Germany FOR RESTANDARDIZATION and were found to have changed in four years LESS THAN .02%. In April, 1902, they were RESTANDARDIZED by the U. S. NATIONAL BUREAU of STANDARDS.

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